PHY 712 Electrodynamics
9-9:50 AM MWF Olin 103

Plan for Lecture 1:

Reading: Appendix 1 and Chapters I&1

1. Course structure and expectations
2. Units – SI vs Gaussian
3. Electrostatics – Poisson equation

http://users.wfu.edu/natalie/s15phy712/

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MWF 9-9:50 AM	**OPL 103**	**http://www.wfu.edu/~natalie/s15phy712/**

Instructor: Natalie Holzwarth	**Phone:**758-5510	**Office:**300 OPL	**e-mail:**natalie@wfu.edu

- General information
- Syllabus and homework assignments
- Lecture notes
- Computer codes
- Some presentation ideas

General Information

This course is a one semester survey of Electrodyamics at the graduate level, using the textbook: **Classical Electrodyamics**, 3rd edition, by John David Jackson (John Wiley & Sons, Inc., 1999) -- "**JDJ**". link to errata The more recent textbook: **Modern Electrodynamics**, by Andrew Zangwill (Cambridge University Press, 2013) will be used as a supplement.

It is likely that your grade for the course will depend upon the following factors:

Problem sets*	45%
Presentation	10%
Exams	45%

*The schedule notes the "due" date for each assignment. Homeworks may be turned in 1 lecture past their due date without grade penalty. After that, the homework grade will be reduced by 10% for each succeeding late date. According to the honor system, all work submitted for grading purposes should represent the student's own best efforts. This means that students who work together on homework assignments should all contribute roughly equally and independently verify all derivations and results.

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Course schedule for Spring 2015

(Preliminary schedule -- subject to frequent adjustment.)

	Lecture date	JDJ Reading	Topic	Assign.	Due date
1	Mon: 01/12/2015	Chap. 1	Introduction, units and Poisson equation	#1	01/23/2015
2	Wed: 01/14/2015	Chap. 1	Electrostatic energy calculations	#2	01/23/2015
	Fri: 01/16/2015	No class	NAWH out of town		
	Mon: 01/19/2015	No class	MLK Holiday		
3	Wed: 01/21/2015	Chap. 1	Poisson equation and Green's theorm	#3	01/23/2015

Some Ideas for Computational Project

The purpose of the "Computational Project" is to provide an opportunity for you to study a topic of your choice in greater depth. The general guideline for your choice of project is that it should have something to do with electrodynamics, and there should be some degree of computation or analysis with the project. The completed project will include a short write-up and a ~20min presentation to the class. You may design your own project or use one of the following list (which will be updated throughout the term).

- Evaluate the ewald sum of various ionic crystals using Maple or a programing language. (Template available in Fortran code.)
- Work out the details of the finite difference or finite element methods.
- Work out the details of the hyperfine Hamiltonian as discussed in Chapter 5 of Jackson.
- Work out the details of Jackson problem 7.2 and related problems.
- Work out the details of reflection and refraction from birefringent materials.
- Analyze the Kramers-Kronig transform of some optical data or calculations.
- Determine the classical electrodynamics associated with an infrared or optical laser.
- Analyze the radiation intensity and spectrum from an interesting source such as an atomic or molecular transition, a free electron laser, etc.
- Work out the details of Jackson problem 14.15.

Units - SI vs Gaussian

Coulomb's Law

$$F = K_C \frac{q_1 q_2}{r_{12}^2}. \quad (1)$$

Ampere's Law

$$F = K_A \frac{i_1 i_2}{r_{12}^2}\, d\mathbf{s_1} \times d\mathbf{s_2} \times \hat{\mathbf{r}}_{\mathbf{12}}, \quad (2)$$

In the equations above, the current and charge are related by $i_1 = dq_1/dt$ for all unit systems. The two constants K_C and K_A are related so that their ratio K_C/K_A has the units of $(m/s)^2$ and it is *experimentally* known that the ratio has the value $K_C/K_A = c^2$, where c is the speed of light.

Units - SI vs Gaussian – continued

The choices for these constants in the SI and Gaussian units are given below:

	CGS (Gaussian)	SI
K_C	1	$\frac{1}{4\pi\epsilon_0}$
K_A	$\frac{1}{c^2}$	$\frac{\mu_0}{4\pi}$

Here, $\frac{\mu_0}{4\pi} \equiv 10^{-7} N/A^2$ and $\frac{1}{4\pi\epsilon_0} = c^2 \cdot 10^{-7} N/A^2 = 8.98755 \times 10^9 N \cdot m^2/C^2$.

Units - SI vs Gaussian – continued

Below is a table comparing SI and Gaussian unit systems. The fundamental units for each system are so labeled and are used to define the derived units.

Variable	SI		Gaussian		SI/Gaussian
	Unit	Relation	Unit	Relation	
length	m	fundamental	cm	fundamental	100
mass	kg	fundamental	gm	fundamental	1000
time	s	fundamental	s	fundamental	1
force	N	$kg \cdot m^2/s$	$dyne$	$gm \cdot cm^2/s$	10^5
current	A	fundamental	$statampere$	$statcoulomb/s$	$\frac{1}{10c}$
charge	C	$A \cdot s$	$statcoulomb$	$\sqrt{dyne \cdot cm^2}$	$\frac{1}{10c}$

Units - SI vs Gaussian – continued

One advantage of the Gaussian system is that the field vectors: $\mathbf{E}, \mathbf{D}, \mathbf{B}, \mathbf{H}, \mathbf{P}, \mathbf{M}$ all have the same physical dimensions., In vacuum, the following equalities hold: $\mathbf{B} = \mathbf{H}$ and $\mathbf{E} = \mathbf{D}$. Also, in the Gaussian system, the dielectric and permittivity constants ϵ and μ are dimensionless.

Basic equations of electrodynamics

CGS (Gaussian)	SI
$\nabla \cdot \mathbf{D} = 4\pi\rho$	$\nabla \cdot \mathbf{D} = \rho$
$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}$	$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$
$\mathbf{F} = q(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B})$	$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$
$u = \frac{1}{8\pi}(\mathbf{E}\cdot\mathbf{D} + \mathbf{B}\cdot\mathbf{H})$	$u = \frac{1}{2}(\mathbf{E}\cdot\mathbf{D} + \mathbf{B}\cdot\mathbf{H})$
$\mathbf{S} = \frac{c}{4\pi}(\mathbf{E}\times\mathbf{H})$	$\mathbf{S} = (\mathbf{E}\times\mathbf{H})$

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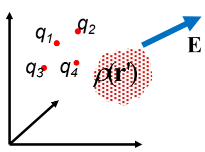
Units choice for this course:
SI units for Jackson in Chapters 1-10
Gaussian units for Jackson in Chapters 11-16

Electrostatics



$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\sum_i q_i \frac{\mathbf{r}-\mathbf{r}_i}{|\mathbf{r}-\mathbf{r}_i|^3}$$
$$= \frac{1}{4\pi\epsilon_0}\int d^3r' \rho(\mathbf{r}') \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}$$

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Electrostatics

Discrete versus continuous charge distributions

In terms of Dirac delta function:

$$\rho(\mathbf{r}) = \sum_i q_i \ \delta(\mathbf{r}-\mathbf{r}_i)$$

Digression: Note that in cartesian coordinates --

$$\delta(\mathbf{r}-\mathbf{r}_i) = \delta(x-x_i)\delta(y-y_i)\delta(z-z_i)$$

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Differential equations --

Electrostatics

$$\nabla \cdot \mathbf{E} = \rho/\epsilon_0$$

$$\nabla \times \mathbf{E} = 0$$

Electrostatic potential

$$\mathbf{E} = -\nabla\Phi(r).$$

$$\nabla^2\Phi(r) = -\rho(r)/\epsilon_0.$$

Relationship between integral and differential forms of electrostatics --

Need to show: $$\nabla^2\left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right) = -4\pi\delta^3(\mathbf{r}-\mathbf{r}').$$

Noting that

$$\int_{\substack{\text{small sphere}\\ \text{about } \mathbf{r}'}} d^3r\; \delta^3(\mathbf{r}-\mathbf{r}')f(\mathbf{r}) = f(\mathbf{r}'),$$

we see that we must show that

$$\int_{\substack{\text{small sphere}\\ \text{about } \mathbf{r}'}} d^3r\; \nabla^2\left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right) f(\mathbf{r}) = -4\pi f(\mathbf{r}').$$

We introduce a small radius a such that:

$$\frac{1}{|\mathbf{r}-\mathbf{r}'|} = \lim_{a\to 0}\frac{1}{\sqrt{|\mathbf{r}-\mathbf{r}'|^2+a^2}}.$$

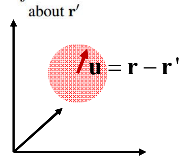
For a fixed value of a,

$$\nabla^2\frac{1}{\sqrt{|\mathbf{r}-\mathbf{r}'|^2+a^2}} = \frac{-3a^2}{(|\mathbf{r}-\mathbf{r}'|^2+a^2)^{5/2}}.$$

If the function $f(\mathbf{r})$ is continuous, we can make a Taylor expansion of it about the point $\mathbf{r} = \mathbf{r}'$, keeping only the first term. The integral over the small sphere about $\mathbf{r}'$ can be carried out analytically, by changing to a coordinate system centered at $\mathbf{r}'$;

so that

$$\int_{\substack{\text{small sphere}\\ \text{about } \mathbf{r}'}} d^3r\, \nabla^2\left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right) f(\mathbf{r}) \approx f(\mathbf{r}') \lim_{a\to 0}\int_{u<R} d^3u \frac{-3a^2}{(u^2+a^2)^{5/2}}.$$



$$\int_{u<R} d^3u \frac{-3a^2}{(u^2+a^2)^{5/2}} = 4\pi \int_0^R du\, \frac{-3a^2u^2}{(u^2+a^2)^{5/2}} = 4\pi \frac{-R^3}{(R^2+a^2)^{3/2}}.$$

$$\int_{u<R} d^3u \frac{-3a^2}{(u^2+a^2)^{5/2}} = 4\pi \int_0^R du\, \frac{-3a^2u^2}{(u^2+a^2)^{5/2}} = 4\pi \frac{-R^3}{(R^2+a^2)^{3/2}}.$$

For $a \ll R$, $4\pi \frac{-R^3}{\left(R^2+a^2\right)^{3/2}} \approx -4\pi$

→ $$\int_{\substack{\text{small sphere}\\ \text{about } \mathbf{r}'}} d^3r\, \nabla^2\left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right) f(\mathbf{r}) \approx f(\mathbf{r}')(-4\pi),$$

→ $$\nabla^2\left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right) = -4\pi\delta^3(\mathbf{r}-\mathbf{r}')$$

Example in HW1

The electrostatic potential of a neutral H atom is given by:

$$\Phi(r) = \frac{q}{4\pi\epsilon_0}\frac{e^{-\alpha r}}{r}\left(1+\frac{\alpha r}{2}\right).$$

Find the charge density (both continuous and discrete) for this potential.

Hint #1: For continuous contribution you can use the identity: $\nabla^2\Phi(r) = \frac{1}{r}\frac{\partial^2(r\Phi(r))}{\partial r^2}$

Hint #2: Don't forget to consider possible discrete contributions.