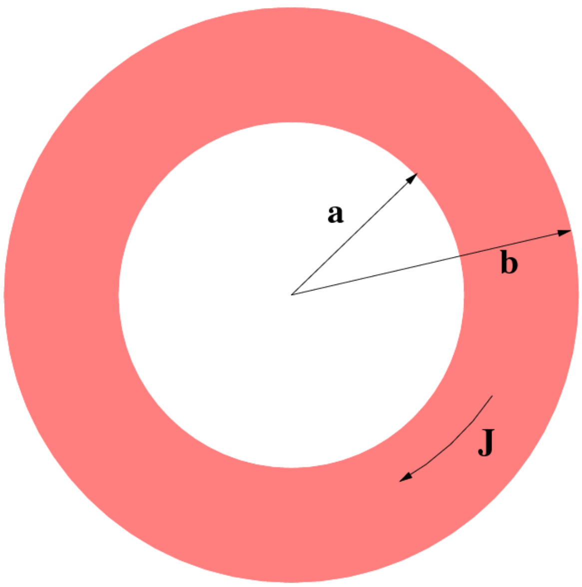
March 16, 2015

PHY 712 – Problem Set # 19

Review the mid-term exam; particularly rework problem 4 as follows.



1.

The figure above shows the cross section of a magnetostatic solenoid which is uniform in the $\hat{\mathbf{z}}$ direction (perpendicular to the page). The current flows in the azimuthal $\hat{\phi}$ direction; specifically the current density is given in cylindrical coordinates by:

$$\mathbf{J} = \begin{cases} J_0\hat{\phi} & a \leq \rho \leq b \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Here J_0 is a constant, a and b denote the inner and outer diameters of the cylinder, respectively, and $\hat{\phi} = -\sin(\phi)\hat{\mathbf{x}} + \cos(\phi)\hat{\mathbf{y}}$.

(a) Show that the vector potential $\mathbf{A}$ for this system can be written as

$$\mathbf{A} = f(\rho)\hat{\phi}, \tag{2}$$

where the scalar function $f(\rho)$ satisfies the equation

$$\left[\frac{d^2}{d\rho^2} + \frac{1}{\rho}\frac{d}{d\rho} - \frac{1}{\rho^2}\right] f(\rho) = \begin{cases} -\mu_0 J_0 & a \leq \rho \leq b \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

(b) Find the function $f(\rho)$ in the three regions: $0 \leq \rho \leq a$, $a \leq \rho \leq b$, and $\rho \geq b$.

(c) Find the $\mathbf{B}$ field in the three regions. Check to make sure that your answer is consistent with what you know about solenoids. (Hint: $\mathbf{B} \equiv \mathbf{0}$ outside the solenoid.)