

PHY 712 – Problem Set # 15

Finish reading Chapter 6 and start reading Chapter 7 of **Jackson**.

1. Suppose that an electromagnetic wave of pure (real) frequency ω is traveling along the z -axis of a wave guide having a square cross section with side dimension a composed of a medium having a real permittivity constant ϵ and a real permeability constant μ . Suppose that the wave is known to have the form:

$$\mathbf{E}(\mathbf{r}, t) = \Re \left\{ H_0 e^{ikz - i\omega t} (i\mu\omega) \frac{a}{\pi} \sin\left(\frac{\pi x}{a}\right) \hat{\mathbf{y}} \right\}$$

$$\mathbf{H}(\mathbf{r}, t) = \Re \left\{ H_0 e^{ikz - i\omega t} \left[-ik \frac{a}{\pi} \sin\left(\frac{\pi x}{a}\right) \hat{\mathbf{x}} + \cos\left(\frac{\pi x}{a}\right) \hat{\mathbf{z}} \right] \right\}.$$

Here H_0 denotes a real amplitude, and the parameter k is assumed to be real and equal to

$$k \equiv \sqrt{\mu\epsilon\omega^2 - \left(\frac{\pi}{a}\right)^2},$$

for $\mu\epsilon\omega^2 > \left(\frac{\pi}{a}\right)^2$.

- (a) Show that this wave satisfies the sourceless Maxwell's equations.
- (b) Find the form of the time-averaged Poynting vector

$$\langle \mathbf{S} \rangle_{avg} \equiv \frac{1}{2} \Re \{ \mathbf{E}(\mathbf{r}, t) \times \mathbf{H}^*(\mathbf{r}, t) \}$$

for this electromagnetic wave.