PHY 712 Electrodynamics
10-10:50 AM MWF Olin 107

Plan for Lecture 35:

Comments and problem solving advice:

- ❑ **Comment about PHY 712 final**
- ❑ **General review**

	Mon: 03/10/2014	*Spring Break*			
	Wed: 03/12/2014	*Spring Break*			
	Fri: 03/14/2014	*Spring Break*			
19	Mon: 03/17/2014	Chap. 8	Wave guides; Take-home exam due	#18	3/21/2014
20	Wed: 03/19/2014	Chap. 9	Sources of Electromagnetic Waves	#19	3/21/2014
21	Fri: 03/21/2014	Chap. 9	Sources of Electromagnetic Waves	#20	3/28/2014
22	Mon: 03/24/2014	Chap. 11	Special Theory of Relativity	#21	3/28/2014
23	Wed: 03/26/2014	Chap. 11	Special Theory of Relativity	#22	3/28/2014
24	Fri: 03/28/2014	Chap. 11	Special Theory of Relativity	#23	4/04/2014
25	Mon: 03/31/2014	Chap. 14	Radiation from moving charges	#24	4/04/2014
26	Wed: 04/02/2014	Chap. 14	Radiation from moving charges	#25	4/04/2014
27	Fri: 04/04/2014	Chap. 14	Radiation from moving charges	#26	4/11/2014
28	Mon: 04/07/2014	Chap. 14	Radiation from moving charges	#27	4/11/2014
29	Wed: 04/09/2014	Chap. 15	Radiation due to collision processes	#28	4/16/2014
30	Fri: 04/11/2014	Chap. 13	Cherenkov radiation	#29	4/16/2014
31	Mon: 04/14/2014		Special topic -- E&M of superconductivity		
32	Wed: 04/16/2014		Special topic -- E&M of superconductivity		
	Fri: 04/18/2014		Good Friday Holiday -- no class		
33	Mon: 04/21/2014		Special topics and review		
34	Wed: 04/23/2014		Special topics and review		
35	Fri: 04/25/2014		Special topics and review		
	Mon: 04/28/2014		Presentations Part I		
	Wed: 04/30/2014		Presentations Part II		

Events

Fri. Apr. 25, 2014
Jenny Dorand
WFU Ph.D. Defense
9:00 AM in Olin 107

Fri. Apr. 25, 2014
SPS Picnic
6 PM on Olin roof
(rain location: Olin 103)

Wed. Apr. 30, 2014
Honors Presentations II +
3:45 PM in Olin 101
Reception:
3:15 PM in Olin Lobby
Note early start time.

Thur. May 1, 2014
Qi Li
WFU Ph.D. Defense
1:00 PM in Olin 103

Monday 4/28/2014

Time	Presenter Name	Presenter Title
10-10:20 AM	Sam Flynn	"Group Theory and Electromagnetism"
10:25-10:40 AM	Ahmad	?????????????????????????

Wednesday 4/30/2014

Time	Presenter Name	Presenter Title
9:30-9:50 AM	Calvin Arter	"Electrodynamics and the interaction potential"
9:55-10:20 AM	Ryan Melvin	"Effects of electric fields on small strands of human RNA"
10:25-10:40 AM	Drew Onken	"The Electromagnetic Theory Behind the Free Electron Laser"

Final exam for PHY 712
- Available: Friday, May 2, 2014
- Due: Monday, May 12, 2014

Maxwell's equations

SI units; Microscopic or vacuum form ($\mathbf{P}=0$; $\mathbf{M}=0$):

Coulomb's law: $\nabla\cdot\mathbf{E}=\rho/\varepsilon_0$

Ampere-Maxwell's law: $\nabla\times\mathbf{B}-\frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t}=\mu_0\mathbf{J}$

Faraday's law: $\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0$

No magnetic monopoles: $\nabla\cdot\mathbf{B}=0$

$$\Rightarrow c^2=\frac{1}{\varepsilon_0\mu_0}$$

Maxwell's equations

SI units; Macroscopic form $(\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P} = 0;\ \mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M})$:

Coulomb's law: $\nabla \cdot \mathbf{D} = \rho_{free}$

Ampere-Maxwell's law: $\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mu_0 \mathbf{J}_{free}$

Faraday's law: $\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$

No magnetic monopoles: $\nabla \cdot \mathbf{B} = 0$

04/25/2014 PHY 712 Spring 2014 -- Lecture 35 7

Maxwell's equations

Gaussian units; Macroscopic form $(\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P} = 0;\ \mathbf{H} = \mathbf{B} - 4\pi\mathbf{M})$:

Coulomb's law: $\nabla \cdot \mathbf{D} = 4\pi\rho_{free}$

Ampere-Maxwell's law: $\nabla \times \mathbf{H} - \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} = \frac{4\pi}{c} \mathbf{J}_{free}$

Faraday's law: $\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0$

No magnetic monopoles: $\nabla \cdot \mathbf{B} = 0$

04/25/2014 PHY 712 Spring 2014 -- Lecture 35 8

Energy and power (SI units)

Electromagnetic energy density: $u \equiv \frac{1}{2}(\mathbf{E}\cdot\mathbf{D} + \mathbf{H}\cdot\mathbf{B})$

Poynting vector: $\mathbf{S} \equiv \mathbf{E} \times \mathbf{H}$

Equations for time harmonic fields:

$$\mathbf{E}(\mathbf{r},t) = \Re\left(\tilde{\mathbf{E}}(\mathbf{r},\omega)e^{-i\omega t}\right) \equiv \frac{1}{2}\left(\tilde{\mathbf{E}}(\mathbf{r},\omega)e^{-i\omega t} + \tilde{\mathbf{E}}^*(\mathbf{r},\omega)e^{i\omega t}\right)$$

$$\langle u(\mathbf{r},t)\rangle_{t\ \text{avg}} = \frac{1}{4}\Re\left(\left(\tilde{\mathbf{E}}(\mathbf{r},\omega)\cdot\tilde{\mathbf{D}}^*(\mathbf{r},\omega) + \tilde{\mathbf{B}}(\mathbf{r},\omega)\cdot\tilde{\mathbf{H}}^*(\mathbf{r},\omega)\right)\right)$$

$$\langle \mathbf{S}(\mathbf{r},t)\rangle_{t\ \text{avg}} = \frac{1}{2}\Re\left(\left(\tilde{\mathbf{E}}(\mathbf{r},\omega)\times\tilde{\mathbf{H}}^*(\mathbf{r},\omega)\right)\right)$$

04/25/2014 PHY 712 Spring 2014 -- Lecture 35 9

Solution of Maxwell's equations:

$$\nabla \cdot \mathbf{E} = \rho / \varepsilon_0 \qquad \nabla \times \mathbf{B} - \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \qquad \nabla \cdot \mathbf{B} = 0$$

Introduction of vector and scalar potentials:

$$\nabla \cdot \mathbf{B} = 0 \qquad \Rightarrow \quad \mathbf{B} = \nabla \times \mathbf{A}$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \qquad \Rightarrow \nabla \times \left(\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} \right) = 0$$

$$\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla \Phi \quad \text{or} \quad \mathbf{E} = -\nabla \Phi - \frac{\partial \mathbf{A}}{\partial t}$$

Scalar and vector potentials continued:

$\nabla \cdot \mathbf{E} = \rho / \varepsilon_0$:

$$-\nabla^2 \Phi - \frac{\partial \left(\nabla \cdot \mathbf{A} \right)}{\partial t} = \rho / \varepsilon_0$$

$$\nabla \times \mathbf{B} - \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}$$

$$\nabla \times \left(\nabla \times \mathbf{A} \right) + \frac{1}{c^2} \left(\frac{\partial \left(\nabla \Phi \right)}{\partial t} + \frac{\partial^2 \mathbf{A}}{\partial t^2} \right) = \mu_0 \mathbf{J}$$

Analysis of the scalar and vector potential equations :

$$-\nabla^2 \Phi - \frac{\partial \left(\nabla \cdot \mathbf{A} \right)}{\partial t} = \rho / \varepsilon_0$$

$$\nabla \times \left(\nabla \times \mathbf{A} \right) + \frac{1}{c^2} \left(\frac{\partial \left(\nabla \Phi \right)}{\partial t} + \frac{\partial^2 \mathbf{A}}{\partial t^2} \right) = \mu_0 \mathbf{J}$$

Lorentz gauge form -- require $\nabla \cdot \mathbf{A}_L + \frac{1}{c^2}\frac{\partial \Phi_L}{\partial t} = 0$

$$-\nabla^2 \Phi_L + \frac{1}{c^2}\frac{\partial^2 \Phi_L}{\partial t^2} = \rho / \varepsilon_0$$

$$-\nabla^2 \mathbf{A}_L + \frac{1}{c^2}\frac{\partial^2 \mathbf{A}_L}{\partial t^2} = \mu_0 \mathbf{J}$$

Solution methods for scalar and vector potentials
and their electrostatic and magnetostatic analogs:

$$-\nabla^2\Phi_L + \frac{1}{c^2}\frac{\partial^2\Phi_L}{\partial t^2} = \rho/\varepsilon_0$$

$$-\nabla^2\mathbf{A}_L + \frac{1}{c^2}\frac{\partial^2\mathbf{A}_L}{\partial t^2} = \mu_0\mathbf{J}$$

In your "bag" of tricks:

- ❑ **Direct (analytic or numerical) solution of differential equations**
- ❑ **Solution by expanding in appropriate orthogonal functions**
- ❑ **Green's function techniques**

How to choose most effective solution method --

- ❑ In general, Green's functions methods work well when source is contained in a finite region of space

Consider the electrostatic problem:

$$-\nabla^2\Phi_L = \rho/\varepsilon_0$$

Define: $\nabla'^2 G(\mathbf{r},\mathbf{r}') = -4\pi\delta^3(\mathbf{r}-\mathbf{r}')$

$$\Phi_L(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int_V d^3r'\rho(\mathbf{r}')G(\mathbf{r},\mathbf{r}') +$$

$$\frac{1}{4\pi}\int_S d^2r'\left[G(\mathbf{r},\mathbf{r}')\nabla'\Phi(\mathbf{r}') - \Phi(\mathbf{r}')\nabla' G(\mathbf{r},\mathbf{r}')\right]\cdot\hat{\mathbf{r}}'.$$

For electrostatic problems where $\rho(\mathbf{r})$ is contained in a small region of space and $S \to \infty$, $\quad G(\mathbf{r},\mathbf{r}') = \frac{1}{|\mathbf{r}-\mathbf{r}'|}$

$$\frac{1}{|\mathbf{r}-\mathbf{r}'|} = \sum_{lm}\frac{4\pi}{2l+1}\frac{r_<^l}{r_>^{l+1}}Y_{lm}(\theta,\varphi)Y_{lm}^*(\theta',\varphi')$$

Electromagnetic waves from time harmonic sources

Charge density : $\rho(\mathbf{r},t)=\Re\left(\tilde{\rho}(\mathbf{r},\omega)e^{-i\omega t}\right)$

Current density : $\mathbf{J}(\mathbf{r},t)=\Re\left(\tilde{\mathbf{J}}(\mathbf{r},\omega)e^{-i\omega t}\right)$

Note that the continuity condition :

$$\frac{\partial\rho(\mathbf{r},t)}{\partial t}+\nabla\cdot\mathbf{J}(\mathbf{r},t)=0 \quad\Rightarrow -i\omega\tilde{\rho}(\mathbf{r},\omega)+\nabla\cdot\tilde{\mathbf{J}}(\mathbf{r},\omega)=0$$

For dynamic problems where $\tilde{\rho}(\mathbf{r},\omega)$ and $\tilde{\mathbf{J}}(\mathbf{r},\omega)$ are contained in a small region of space and $S\rightarrow\infty$,

$$\tilde{G}(\mathbf{r},\mathbf{r}',\omega)=\frac{e^{i\frac{\omega}{c}|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}$$

Electromagnetic waves from time harmonic sources – continued:

For scalar potential (Lorentz gauge, $k\equiv\frac{\omega}{c}$)

$$\tilde{\Phi}(\mathbf{r},\omega)=\tilde{\Phi}_0(\mathbf{r},\omega)+\frac{1}{4\pi\varepsilon_0}\int d^3r'\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\tilde{\rho}(\mathbf{r}',\omega)$$

For vector potential (Lorentz gauge, $k\equiv\frac{\omega}{c}$)

$$\tilde{\mathbf{A}}(\mathbf{r},\omega)=\tilde{\mathbf{A}}_0(\mathbf{r},\omega)+\frac{\mu_0}{4\pi}\int d^3r'\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\tilde{\mathbf{J}}(\mathbf{r}',\omega)$$

Electromagnetic waves from time harmonic sources – continued:

Useful expansion :

$$\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}=ik\sum_{lm}j_l(kr_<)h_l(kr_>)Y_{lm}(\hat{\mathbf{r}})Y^*_{lm}(\hat{\mathbf{r}}')$$

Spherical Bessel function : $j_l(kr)$

Spherical Hankel function : $h_l(kr)=j_l(kr)+in_l(kr)$

$$\tilde{\Phi}(\mathbf{r},\omega)=\tilde{\Phi}_0(\mathbf{r},\omega)+\sum_{lm}\tilde{\phi}_{lm}(r,\omega)Y_{lm}(\hat{\mathbf{r}})$$

$$\tilde{\phi}_{lm}(r,\omega)=\frac{ik}{\varepsilon_0}\int d^3r'\tilde{\rho}(\mathbf{r}',\omega)j_l(kr_<)h_l(kr_>)Y^*_{lm}(\hat{\mathbf{r}}')$$

Model of dielectric properties of matter: Drude model

Vibrations of charged particles near equilibrium:

$$m\delta\ddot{\mathbf{r}} = q\mathbf{E}_0 e^{-i\omega t} - m\omega_0^2 \delta\mathbf{r} - m\gamma\delta\dot{\mathbf{r}}$$

For $\delta\mathbf{r} \equiv \delta\mathbf{r}_0 e^{-i\omega t}$, $\quad \delta\mathbf{r}_0 = \dfrac{q\mathbf{E}_0}{m}\dfrac{1}{\omega_0^2 - \omega^2 - i\omega\gamma}$

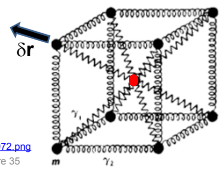
Induced dipole:

$$\mathbf{p} = q\,\delta\mathbf{r} = \frac{q^2\mathbf{E}_0}{m}\frac{1}{\omega_0^2 - \omega^2 - i\omega\gamma}e^{-i\omega t}$$

Displacement field:

$$\mathbf{D} = \varepsilon\mathbf{E} = \varepsilon_0\mathbf{E} + \mathbf{P}$$

$$\mathbf{P} = \sum_i \mathbf{p}_i \delta^3(\mathbf{r} - \mathbf{r}_i)$$



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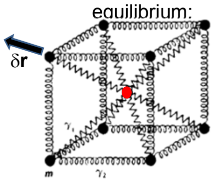
Drude model:

Vibration of particle of charge *q* and mass *m* near equilibrium:



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$$m\delta\ddot{\mathbf{r}} = q\mathbf{E}_0 e^{-i\omega t} - m\omega_0^2 \delta\mathbf{r} - m\gamma\delta\dot{\mathbf{r}}$$

Displacement field:

$$\mathbf{D} = \varepsilon\mathbf{E} = \varepsilon_0\mathbf{E} + \mathbf{P}$$

$$\mathbf{P} = \sum_i \mathbf{p}_i \delta^3(\mathbf{r} - \mathbf{r}_i) \approx N\sum_i f_i\,\mathbf{p}_i$$

$N \equiv$ number dipole/volume

$f_i \equiv$ fraction of type i dipoles

Drude model dielectric function:

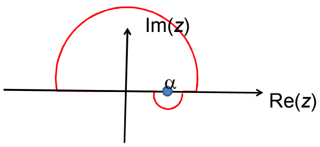
$$\frac{\varepsilon(\omega)}{\varepsilon_0} = 1 + N\sum_i f_i \frac{q_i^2}{\varepsilon_0 m_i}\frac{1}{\omega_i^2 - \omega^2 - i\omega\gamma_i}$$

$$= \frac{\varepsilon_R(\omega)}{\varepsilon_0} + i\frac{\varepsilon_I(\omega)}{\varepsilon_0}$$

$$\frac{\varepsilon_R(\omega)}{\varepsilon_0} = 1 + N\sum_i f_i \frac{q_i^2}{\varepsilon_0 m_i}\frac{\omega_i^2 - \omega^2}{(\omega_i^2 - \omega^2)^2 + \omega^2\gamma_i^2}$$

$$\frac{\varepsilon_I(\omega)}{\varepsilon_0} = N\sum_i f_i \frac{q_i^2}{\varepsilon_0 m_i}\frac{\omega\gamma_i}{(\omega_i^2 - \omega^2)^2 + \omega^2\gamma_i^2}$$

Kramers-Kronig transform – for use in dielectric analysis



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$$f(\alpha) = \frac{1}{2\pi i} \oint_{includes\ \alpha} dz \frac{f(z)}{z\text{-}\alpha} = \frac{1}{2\pi i}\left(\int_{-\infty}^{\infty} dz_R \frac{f(z_R)}{z_R\text{-}\alpha} + \int_{rest} dz \frac{f(z)}{z\text{-}\alpha} \right)$$

$$f(\alpha) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} dz_R \frac{f(z_R)}{z_R\text{-}\alpha} = \frac{1}{2\pi i} P\int_{-\infty}^{\infty} dz_R \frac{f(z_R)}{z_R\text{-}\alpha} + \frac{1}{2} f(\alpha)$$

Kramers-Kronig transform – for dielectric function:

$$\frac{\varepsilon_R(\omega)}{\varepsilon_0} - 1 = \frac{1}{\pi} P\int_{-\infty}^{\infty} d\omega' \frac{\varepsilon_I(\omega')}{\varepsilon_0} \frac{1}{\omega'\text{-}\omega}$$

$$\frac{\varepsilon_I(\omega)}{\varepsilon_0} = -\frac{1}{\pi} P\int_{-\infty}^{\infty} d\omega' \left(\frac{\varepsilon_R(\omega')}{\varepsilon_0} - 1 \right) \frac{1}{\omega'\text{-}\omega}$$

with $\varepsilon_R(-\omega) = \varepsilon_R(\omega); \quad \varepsilon_I(-\omega) = -\varepsilon_I(\omega)$

Further comments on analytic behavior of dielectric function

"Causal" relationship between **E** and **D** fields:

$$\mathbf{D}(\mathbf{r},t) = \varepsilon_0 \left\{ \mathbf{E}(\mathbf{r},t) + \int_0^{\infty} d\tau\, G(\tau) \mathbf{E}(\mathbf{r}, t-\tau) \right\}$$

$$\frac{\varepsilon(\omega)}{\varepsilon_0} - 1 = \int_0^{\infty} d\tau\, G(\tau) e^{i\omega\tau}$$