PHY 712 Electrodynamics
10-10:50 AM MWF Olin 107

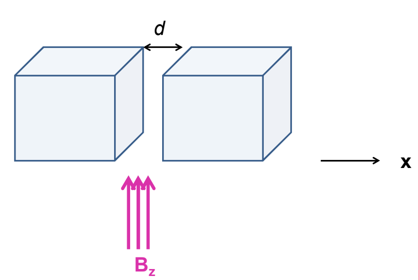
Plan for Lecture 33:

Special Topics in Electrodynamics:

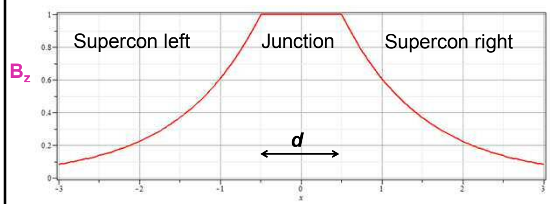
1. **Electromagnetic aspects of superconductivity – continued**
2. **Review – reflection and refraction**

	Mon: 03/10/2014	Spring Break			
	Wed: 03/12/2014	*Spring Break*			
	Fri: 03/14/2014	*Spring Break*			
19	Mon: 03/17/2014	Chap. 8	Wave guides;Take-home exam due	#18	3/21/2014
20	Wed: 03/19/2014	Chap. 9	Sources of Electromagnetic Waves	#19	3/21/2014
21	Fri: 03/21/2014	Chap. 9	Sources of Electromagnetic Waves	#20	3/28/2014
22	Mon: 03/24/2014	Chap. 11	Special Theory of Relativity	#21	3/28/2014
23	Wed: 03/26/2014	Chap. 11	Special Theory of Relativity	#22	3/28/2014
24	Fri: 03/28/2014	Chap. 11	Special Theory of Relativity	#23	4/04/2014
25	Mon: 03/31/2014	Chap. 14	Radiation from moving charges	#24	4/04/2014
26	Wed: 04/02/2014	Chap. 14	Radiation from moving charges	#25	4/04/2014
27	Fri: 04/04/2014	Chap. 14	Radiation from moving charges	#26	4/11/2014
28	Mon: 04/07/2014	Chap. 14	Radiation from moving charges	#27	4/11/2014
29	Wed: 04/09/2014	Chap. 15	Radiation due to collision processes	#28	4/16/2014
30	Fri: 04/11/2014	Chap. 13	Cherenkov radiation	#29	4/16/2014
31	Mon: 04/14/2014		Special topic -- E&M of superconductivity		
32	Wed: 04/16/2014		Special topic -- E&M of superconductivity		
	Fri: 04/18/2014		Good Friday Holiday -- no class		
3	Mon: 04/21/2014		Special topics and review		
34	Wed: 04/23/2014		Special topics and review		
35	Fri: 04/25/2014		Special topics and review		
	Mon: 04/28/2014		Presentations Part I		
	Wed: 04/30/2014		Presentations Part II		

Josephson junction -- tunneling current between two superconductors

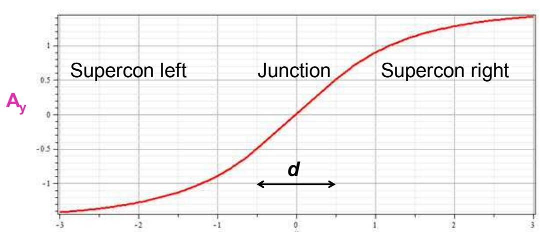


Josephson junction -- continued



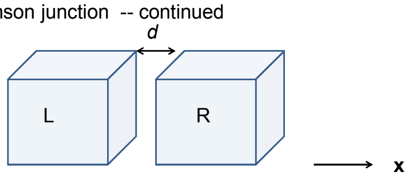
$$B_z(x) = \begin{cases} B_0 e^{(x+d/2)/\lambda_L} & x < -d/2 \\ B_0 & -d/2 < x < d/2 \\ B_0 e^{(-x+d/2)/\lambda_L} & x > d/2 \end{cases}$$

Josephson junction -- continued



$$A_y(x) = \begin{cases} B_0\left(\lambda_L e^{(x+d/2)/\lambda_L} - (\lambda_L + d/2)\right) & x < -d/2 \\ B_0 x & -d/2 < x < d/2 \\ B_0\left(-\lambda_L e^{(-x+d/2)/\lambda_L} + (\lambda_L + d/2)\right) & x > d/2 \end{cases}$$

Josephson junction -- continued



Quantum mechanical model of tunnelling current

Let $\Psi_L = \Psi_L{}^0 e^{i\phi_L}$ denote a wavefunction for a Cooper pair on left

Let $\Psi_R = \Psi_R{}^0 e^{i\phi_R}$ denote a wavefunction for a Cooper pair on right

$$-i\hbar \frac{\partial \Psi_L}{\partial t} = E_L \Psi_L + \varepsilon \Psi_R$$

$$-i\hbar \frac{\partial \Psi_R}{\partial t} = E_R \Psi_R + \varepsilon \Psi_L$$

Josephson junction -- continued

Solving for wavefunctions

$$\frac{1}{2}\frac{\partial\left|\Psi_L{}^0\right|^2}{\partial t}+i\left|\Psi_L{}^0\right|^2\frac{\partial\phi_L}{\partial t}=-\frac{i}{\hbar}\left(E_L\left|\Psi_L{}^0\right|^2+\varepsilon\Psi_L{}^0\Psi_R{}^0e^{i(\phi_R-\phi_L)}\right)$$

$$\frac{1}{2}\frac{\partial\left|\Psi_R{}^0\right|^2}{\partial t}+i\left|\Psi_R{}^0\right|^2\frac{\partial\phi_R}{\partial t}=-\frac{i}{\hbar}\left(E_R\left|\Psi_R{}^0\right|^2+\varepsilon\Psi_L{}^0\Psi_R{}^0e^{-i(\phi_R-\phi_L)}\right)$$

$$\left|\Psi_L{}^0\right|^2\equiv n_L \qquad \left|\Psi_R{}^0\right|^2\equiv n_R \qquad \phi_{LR}\equiv\phi_L-\phi_R$$

$$\frac{\partial n_L}{\partial t}=-\frac{\partial n_R}{\partial t}=-\frac{2\varepsilon}{\hbar}\sqrt{(n_Ln_R)}\sin\phi_{LR}$$

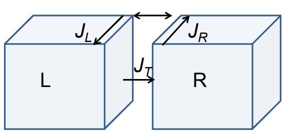
$$\frac{\partial\phi_L}{\partial t}=-\frac{E_L}{\hbar}-\varepsilon\sqrt{\frac{n_R}{n_L}}\cos\phi_{LR}$$

$$\frac{\partial\phi_R}{\partial t}=-\frac{E_R}{\hbar}-\varepsilon\sqrt{\frac{n_L}{n_R}}\cos\phi_{LR}$$

Josephson junction -- continued

Tunneling current: $J_T=2e\frac{\partial n_L}{\partial t}=-\frac{4e\varepsilon}{\hbar}\sqrt{(n_Ln_R)}\sin\phi_{LR}\equiv-J_{T0}\sin\phi_{LR}$

If $n_L = n_R$ and in absense of magnetic field, $\phi_{LR}(t)=\phi_{LR}(0)+\frac{E_R-E_L}{\hbar}t$



$$\Rightarrow J_L=\frac{2e}{2m}\left|\Psi_L{}^0\right|^2\left(\hbar\nabla\phi_L-\frac{2e}{c}\mathbf{A}\right)$$

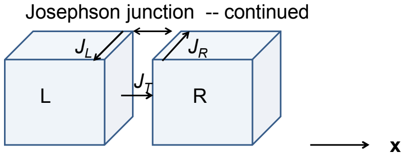
$$\Rightarrow J_R=\frac{2e}{2m}\left|\Psi_R{}^0\right|^2\left(\hbar\nabla\phi_R-\frac{2e}{c}\mathbf{A}\right)$$

$\longrightarrow$ **x**

Relationship between superconductor currents J_L and J_R and tunneling current. Within the superconductor, denote the generalize current operator acting on pair wavefunction $\Psi=\Psi^0e^{i\phi}$

$$\hat{\mathbf{v}}\equiv\frac{1}{2m}\left(-i\hbar\nabla-\frac{2e}{c}\mathbf{A}\right) \quad \text{with current } J=\frac{2e}{2}\left(\Psi^*\left(\hat{\mathbf{v}}\Psi\right)+\Psi\left(\hat{\mathbf{v}}\Psi\right)^*\right)$$

Josephson junction -- continued



$$\Rightarrow J_L=\frac{2e}{2m}\left|\Psi_L{}^0\right|^2\left(\hbar\nabla\phi_L-\frac{2e}{c}\mathbf{A}\right)\equiv 2en_L\mathbf{v}_L$$

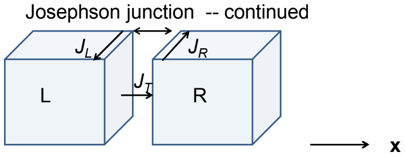
$$\Rightarrow J_R=\frac{2e}{2m}\left|\Psi_R{}^0\right|^2\left(\hbar\nabla\phi_R-\frac{2e}{c}\mathbf{A}\right)\equiv 2en_R\mathbf{v}_R$$

$$\nabla\phi_L=\frac{2m\mathbf{v}_L}{\hbar}+\frac{2e}{\hbar c}\mathbf{A} \qquad \nabla\phi_R=\frac{2m\mathbf{v}_R}{\hbar}+\frac{2e}{\hbar c}\mathbf{A}$$

Tunneling current: $J_T=2e\frac{\partial n_L}{\partial t}=-J_{T0}\sin\phi_{LR}$

Need to evaluate ϕ_{LR} in presence of magnetic field

Josephson junction -- continued



$$\nabla\phi_L = \frac{2m\mathbf{v}_L}{\hbar} + \frac{2e}{\hbar c}\mathbf{A} \qquad \nabla\phi_R = \frac{2m\mathbf{v}_R}{\hbar} + \frac{2e}{\hbar c}\mathbf{A}$$

Recall that for $x \to -\infty$ $\mathbf{v}_L \to 0$ and $\mathbf{A} \to -(\lambda_L + d/2)B_0\hat{\mathbf{y}}$

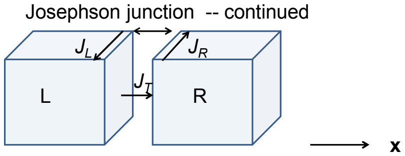
for $x \to \infty$ $\mathbf{v}_R \to 0$ and $\mathbf{A} \to (\lambda_L + d/2)B_0\hat{\mathbf{y}}$

Integrating the difference of the phase angles along y:

$$\phi_{LR} = \phi_{LR}^0 - B_0(2\lambda_L + d)y$$

Tunneling current: $J_T = -J_{T0}\sin\phi_{LR}$

Josephson junction -- continued



Integrating the difference of the phase angles along y:

$$\phi_{LR} = \phi_{LR}^0 + \frac{2e}{\hbar c}B_0(2\lambda_L + d)y$$

Tunneling current density: $J_T = -J_{T0}\sin\phi_{LR}$

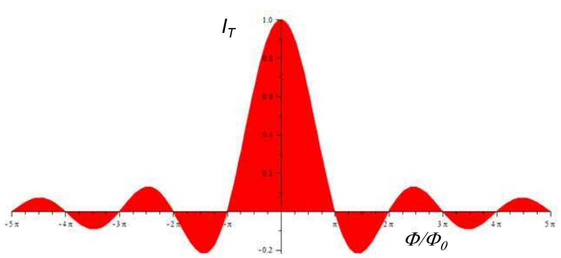
Integrating current density throughout width w and height h:

$$I_T = h\int_{-w/2}^{w/2} J_T\,dy = hwJ_{T0}\sin(\phi_{LR}^0)\frac{\sin(\pi\Phi/\Phi^0)}{\pi\Phi/\Phi^0}$$

where $\Phi = B_0 w(2\lambda_L + d)$ and $\Phi^0 = \frac{2\pi\hbar c}{2e}$

Josephson junction -- continued

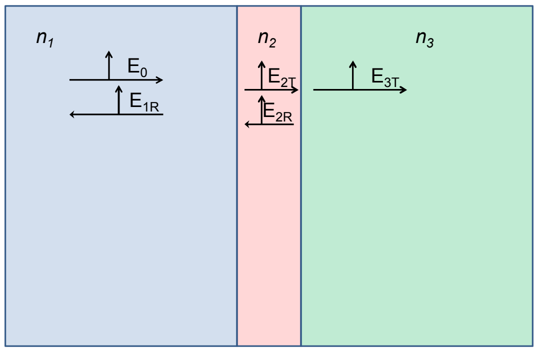
SQUID = *superconducting quantum interference device*



Note: This very sensitive "SQUID" technology has been used in scanning probe techniques. See for example, J. R. Kirtley, Rep. Prog. Physics 73, 126501 (2010).

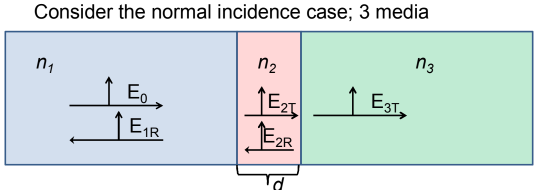
Review of reflection and refraction

Consider the normal incidence case; 3 media



Review of reflection and refraction -- continued

Consider the normal incidence case; 3 media



Note that in this steady-state formulation, we must match the tangential components of the E and H fields at each boundary

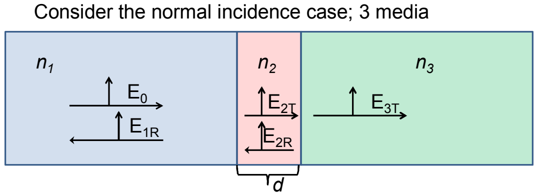
Each plane wave component has the form:

$$\mathbf{E}_j(\mathbf{r},t)=E_j\hat{\mathbf{y}}e^{i(\omega/c)(n_jx-ct)}$$

$$\mathbf{H}_j(\mathbf{r},t)=\frac{n_jE_j}{\mu_jc}\hat{\mathbf{z}}e^{i(\omega/c)(n_jx-ct)}=\frac{n_jE_j}{\mu_0c}\hat{\mathbf{z}}e^{i(\omega/c)(n_jx-ct)} \quad \text{in our case}$$

Review of reflection and refraction -- continued

Consider the normal incidence case; 3 media



Matching equations:

$$E_0+E_{1R}=E_2+E_{2R}$$

$$\frac{n_1}{n_2}\left(E_0-E_{1R}\right)=E_2-E_{2R}$$

$$E_2e^{i\theta}+E_{2R}e^{-i\theta}=E_3$$

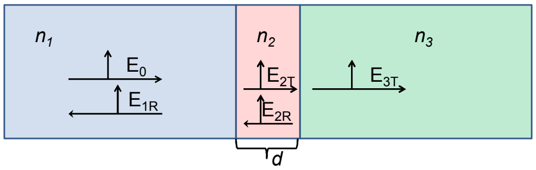
$$\frac{n_2}{n_3}\left(E_2e^{i\theta}-E_{2R}e^{-i\theta}\right)=E_3$$

Here:

$$\theta\equiv\frac{n_2\omega d}{c}$$

Review of reflection and refraction -- continued

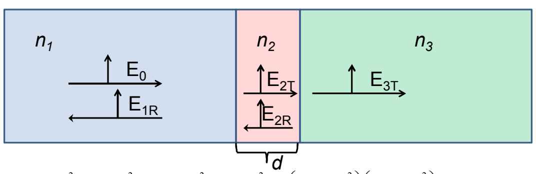
Consider the normal incidence case; 3 media



After some algebra:

$$\mathcal{R} = \frac{\left(1-\frac{n_2}{n_3}\right)^2\left(1+\frac{n_1}{n_2}\right)^2 + \left(1+\frac{n_2}{n_3}\right)^2\left(1-\frac{n_1}{n_2}\right)^2 + 2\left(1-\left(\frac{n_2}{n_3}\right)^2\right)\left(1-\left(\frac{n_1}{n_2}\right)^2\right)\cos(2\theta)}{\left(1-\frac{n_2}{n_3}\right)^2\left(1-\frac{n_1}{n_2}\right)^2 + \left(1+\frac{n_2}{n_3}\right)^2\left(1+\frac{n_1}{n_2}\right)^2 + 2\left(1-\left(\frac{n_2}{n_3}\right)^2\right)\left(1-\left(\frac{n_1}{n_2}\right)^2\right)\cos(2\theta)}$$

Review of reflection and refraction -- continued



$$\mathcal{R} = \frac{\left(1-\frac{n_2}{n_3}\right)^2\left(1+\frac{n_1}{n_2}\right)^2 + \left(1+\frac{n_2}{n_3}\right)^2\left(1-\frac{n_1}{n_2}\right)^2 + 2\left(1-\left(\frac{n_2}{n_3}\right)^2\right)\left(1-\left(\frac{n_1}{n_2}\right)^2\right)\cos(2\theta)}{\left(1-\frac{n_2}{n_3}\right)^2\left(1-\frac{n_1}{n_2}\right)^2 + \left(1+\frac{n_2}{n_3}\right)^2\left(1+\frac{n_1}{n_2}\right)^2 + 2\left(1-\left(\frac{n_2}{n_3}\right)^2\right)\left(1-\left(\frac{n_1}{n_2}\right)^2\right)\cos(2\theta)}$$

Condition for zero reflectance:

$$\left(1-\frac{n_2}{n_3}\right)^2\left(1+\frac{n_1}{n_2}\right)^2 + \left(1+\frac{n_2}{n_3}\right)^2\left(1-\frac{n_1}{n_2}\right)^2 + 2\left(1-\left(\frac{n_2}{n_3}\right)^2\right)\left(1-\left(\frac{n_1}{n_2}\right)^2\right)\cos(2\theta) = 0$$

$$\cos(2\theta) = -1 \quad \Rightarrow \frac{2n_2\omega d}{c} = \frac{4\pi n_2 d}{\lambda} = (2\nu+1)\pi \;\Rightarrow n_2 = (2\nu+1)\frac{\lambda}{4d} \text{ also } n_2 = \sqrt{n_1 n_3}$$