PHY 712 Electrodynamics
10-10:50 AM MWF Olin 107

Plan for Lecture 31:

Special Topics in Electrodynamics:

Electromagnetic aspects of superconductivity

19	Mon: 03/17/2014	Chap. 8	Wave guides;Take-home exam due	#18	3/21/2014
20	Wed: 03/19/2014	Chap. 9	Sources of Electromagnetic Waves	#19	3/21/2014
21	Fri: 03/21/2014	Chap. 9	Sources of Electromagnetic Waves	#20	3/28/2014
22	Mon: 03/24/2014	Chap. 11	Special Theory of Relativity	#21	3/28/2014
23	Wed: 03/26/2014	Chap. 11	Special Theory of Relativity	#22	3/28/2014
24	Fri: 03/28/2014	Chap. 11	Special Theory of Relativity	#23	4/04/2014
25	Mon: 03/31/2014	Chap. 14	Radiation from moving charges	#24	4/04/2014
26	Wed: 04/02/2014	Chap. 14	Radiation from moving charges	#25	4/04/2014
27	Fri: 04/04/2014	Chap. 14	Radiation from moving charges	#26	4/11/2014
28	Mon: 04/07/2014	Chap. 14	Radiation from moving charges	#27	4/11/2014
29	Wed: 04/09/2014	Chap. 15	Radiation due to collision processes	#28	4/16/2014
30	Fri: 04/11/2014	Chap. 13	Cherenkov radiation	#29	4/16/2014
31	Mon: 04/14/2014		Special topic -- E&M of superconductivity		
32	Wed: 04/16/2014		Special topic -- E&M of superconductivity		
	Fri: 04/18/2014		Good Friday Holiday -- no class		
33	Mon: 04/21/2014				
34	Wed: 04/23/2014				
35	Fri: 04/25/2014				
	Mon: 04/28/2014		Presentations Part I		
	Wed: 04/30/2014		Presentations Part II		

Signup for PHY 712 presentations

Please note that in order to sign up, you will need to have a title. Please choose one of the empty time slots and record your title and your name.

Presentations on Monday 4/28/2014 in Olin 107

Time	Presenter Name	Presenter Title
10-10:20 AM		
10:25-10:40 AM		

Presentations on Wednesday 4/30/2014 in Olin 102 (note early start time)

Time	Presenter Name	Presenter Title
9:30-9:50 AM		
9:55-10:20 AM		
10:25-10:40 AM		

Comment on HW

PHY 712 -- Assignment #27

April 7, 2014

Continue reading Chap. 14 in **Jackson** .

1. Consider an electron moving at constant velocity βc ≈ c in a circular trajectory of radius ρ. Its total energy is E= γ m c^2. Determine the ratio of the energy lost during one full cycle to the total energy. Evaluate the expression for an electron with total energy 200 GeV in a synchroton of radius ρ=10^3 m.

The estimate of the energy loss per cycle is actually discussed in the beginning of Chap. 14. Dividing that result by the total energy of the particle $E=\gamma mc^2$:

$$\frac{\delta E}{E}=\frac{4\pi}{3}\frac{q^2}{\rho mc^2}\beta^3\gamma^3$$

Note that $\frac{q^2}{\rho}$ has the units of ergs with q measured in statcoulombs and ρ measured in cm.

$$\frac{\delta E}{E}=\frac{4\pi}{3}\frac{q^2}{\rho mc^2}\beta^3\gamma^3\approx 0.77$$

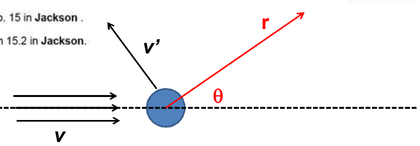
Comment on HW

PHY 712 -- Assignment #28

April 9, 2014

Start reading Chap. 15 in **Jackson** .

1. Work problem 15.2 in **Jackson**.



Suppose that

$\mathbf{v}=v\hat{\mathbf{z}}$

$\mathbf{v}'=v(\sin a\cos b\hat{\mathbf{x}}+\sin a\sin b\hat{\mathbf{y}}+\cos a\hat{\mathbf{z}})$

$\mathbf{r}=\sin\theta\hat{\mathbf{x}}+\cos\theta\hat{\mathbf{z}}$

$\boldsymbol{\varepsilon}_1=\hat{\mathbf{y}}$

$\boldsymbol{\varepsilon}_2=-\cos\theta\hat{\mathbf{x}}+\sin\theta\hat{\mathbf{z}}$

Cross section depends on $\left\langle\left|\boldsymbol{\varepsilon}_i\cdot(\mathbf{v}'-\mathbf{v})\right|^2\right\rangle$

Special topic: Electromagnetic properties of superconductors

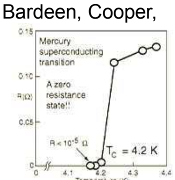
Ref:D. Teplitz, editor, Electromagnetism – paths to research, Plenum Press (1982); Chapter 1 written by Brian Schwartz and Sonia Frota-Pessoa

History:

1908 H. Kamerlingh Onnes successfully liquified He

1911 H. Kamerlingh Onnes discovered that Hg at 4.2 K has vanishing resistance

1957 Theory of superconductivity by Bardeen, Cooper, and Schrieffer



Some phenomenological theories < 1957

Drude model of conductivity in "normal" materials

$$m\frac{d\mathbf{v}}{dt} = -e\mathbf{E} - m\frac{\mathbf{v}}{\tau}$$

$$\mathbf{v}(t) = \mathbf{v}_0 e^{-t/\tau} - \frac{e\mathbf{E}\tau}{m}$$

$$\mathbf{J} = -ne\mathbf{v} \qquad \text{for } t >> \tau \qquad \mathbf{J} = \frac{ne^2\tau}{m}\mathbf{E} \equiv \sigma\mathbf{E}$$

London model of conductivity in superconducting materials

$$m\frac{d\mathbf{v}}{dt} = -e\mathbf{E}$$

$$\frac{d\mathbf{v}}{dt} = -\frac{e\mathbf{E}}{m} \qquad \frac{d\mathbf{J}}{dt} = -ne\frac{d\mathbf{v}}{dt} = \frac{ne^2\mathbf{E}}{m}$$

From Maxwell's equations:

$$\nabla\times\mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}$$

$$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}$$

Some phenomenological theories < 1957

London model of conductivity in superconducting materials

$$\frac{d\mathbf{J}}{dt} = -ne\frac{d\mathbf{v}}{dt} = \frac{ne^2\mathbf{E}}{m}$$

From Maxwell's equations:

$$\nabla\times\mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} \qquad \nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\times(\nabla\times\mathbf{B}) = -\nabla^2\mathbf{B} = \frac{4\pi}{c}\nabla\times\mathbf{J} - \frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2}$$

$$-\nabla^2\frac{\partial\mathbf{B}}{\partial t} = \frac{4\pi}{c}\nabla\times\frac{\partial\mathbf{J}}{\partial t} - \frac{1}{c^2}\frac{\partial^3\mathbf{B}}{\partial t^3}$$

$$-\nabla^2\frac{\partial\mathbf{B}}{\partial t} = \frac{4\pi ne^2}{mc}\nabla\times\mathbf{E} - \frac{1}{c^2}\frac{\partial^3\mathbf{B}}{\partial t^3}$$

$$-\nabla^2\frac{\partial\mathbf{B}}{\partial t} = -\frac{4\pi ne^2}{mc^2}\frac{\partial\mathbf{B}}{\partial t} - \frac{1}{c^2}\frac{\partial^3\mathbf{B}}{\partial t^3}$$

$$\frac{\partial}{\partial t}\left(\nabla^2 - \frac{1}{\lambda_L^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{B} = 0 \qquad \text{with } \lambda_L^2 \equiv \frac{mc^2}{4\pi ne^2}$$

London model – continued

London model of conductivity in superconducting materials

$$\frac{d\mathbf{J}}{dt} = -ne\frac{d\mathbf{v}}{dt} = \frac{ne^2\mathbf{E}}{m}$$

$$\frac{\partial}{\partial t}\left(\nabla^2 - \frac{1}{\lambda_L^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{B} = 0 \qquad \text{with } \lambda_L^2 \equiv \frac{mc^2}{4\pi ne^2}$$

For slowly varying solution:

$$\frac{\partial}{\partial t}\left(\nabla^2 - \frac{1}{\lambda_L^2}\right)\mathbf{B} = 0 \qquad \text{for } \frac{\partial\mathbf{B}}{\partial t} = \hat{\mathbf{z}}\frac{\partial B_z(x,t)}{\partial t}:$$

$$\Rightarrow \frac{\partial B_z(x,t)}{\partial t} = \frac{\partial B_z(0,t)}{\partial t}e^{-x/\lambda_L}$$

London leap: $B_z(x,t) = B_z(0,t)e^{-x/\lambda_L}$

Consistent results for current density:

$$\frac{4\pi}{c}\nabla\times\mathbf{J} = -\nabla^2\mathbf{B} = -\frac{1}{\lambda_L^2}\mathbf{B} \qquad \mathbf{J} = \hat{\mathbf{y}}J_y(x) \quad \Rightarrow \quad J_y(x) = \lambda_L\frac{ne^2}{mc}\mathrm{B}_z(0)\mathrm{e}^{-x/\lambda_L}$$

London model – continued

Penetration length for superconductor: $\lambda_L^2 \equiv \frac{mc^2}{4\pi ne^2}$

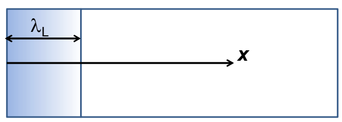
$B_z(x,t) = B_z(0,t)e^{-x/\lambda_L}$

Vector potential for $\nabla \cdot \mathbf{A} = 0$:

$\mathbf{A} = \hat{\mathbf{y}} A_y(x) \qquad A_y(x) = -\lambda_L B_z(0) e^{-x/\lambda_L}$

Recall form for current density: $J_y(x) = \lambda_L \frac{ne^2}{mc} \mathrm{B}_z(0)\,\mathrm{e}^{-x/\lambda_L}$

$\Rightarrow \mathbf{J} + \frac{ne^2}{mc}\mathbf{A} = 0 \quad \text{or} \quad \frac{ne}{m}\left(m\mathbf{v} + \frac{e}{c}\mathbf{A}\right) = 0$



Magnetization field

Treating London current in terms of corresponding magnetization field **M**:

$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$

$\Rightarrow$ For $x >> \lambda_L$, $\quad \mathbf{H} = -4\pi\mathbf{M}$

Gibbs free energy associated with magnetization for superconductor:

$$G_S(H_a) = G_S(H=0) - \int_0^{H_a} dH M(H) = G_S(0) + \frac{1}{8\pi} H_a^2$$

Gibbs free energy associated with magnetization for normal conductor:

$G_N(H_a) \approx G_N(H=0)$

Condition at phase boundary between normal and superconducting states:

$$G_N(H_C) \approx G_N(0) = G_S(H_C) = G_S(0) + \frac{1}{8\pi} H_C^2$$

$$\Rightarrow G_S(0) - G_N(0) = -\frac{1}{8\pi} H_C^2$$

$$G_S(H_a) - G_N(H_a) = \begin{cases} -\frac{1}{8\pi}\left(H_C^2 - H_a^2\right) & \text{for } H_a < H_C \\ 0 & \text{otherwise} \end{cases}$$

Magnetization field