PHY 712 Electrodynamics
10-10:50 AM MWF Olin 107

Plan for Lecture 29:

Start reading Chap. 15 –

Radiation from collisions of charged particles

1. **Overview**
2. **X-ray tube**

	Wed: 03/12/2014	*Spring Break*			
	Fri: 03/14/2014	*Spring Break*			
19	Mon: 03/17/2014	Chap. 8	Wave guides;Take-home exam due	#18	3/21/2014
20	Wed: 03/19/2014	Chap. 9	Sources of Electromagnetic Waves	#19	3/21/2014
21	Fri: 03/21/2014	Chap. 9	Sources of Electromagnetic Waves	#20	3/28/2014
22	Mon: 03/24/2014	Chap. 11	Special Theory of Relativity	#21	3/28/2014
23	Wed: 03/26/2014	Chap. 11	Special Theory of Relativity	#22	3/28/2014
24	Fri: 03/28/2014	Chap. 11	Special Theory of Relativity	#23	4/04/2014
25	Mon: 03/31/2014	Chap. 14	Radiation from moving charges	#24	4/04/2014
26	Wed: 04/02/2014	Chap. 14	Radiation from moving charges	#25	4/04/2014
27	Fri: 04/04/2014	Chap. 14	Radiation from moving charges	#26	4/11/2014
28	Mon: 04/07/2014	Chap. 14	Radiation from moving charges	#27	4/11/2014
29	Wed: 04/09/2014	Chap. 15	Radiation due to collision prosesses	#28	4/11/2014
30	Fri: 04/11/2014				
31	Mon: 04/14/2014				
32	Wed: 04/16/2014				
	Fri: 04/18/2014	Good Friday			
33	Mon: 04/21/2014				
34	Wed: 04/23/2014				
35	Fri: 04/23/2014				
	Mon: 04/28/2014		Presentations Part I		
	Wed: 04/30/2014		Presentations Part II		

WFU Physics Colloquium

TITLE: Crystal Structure, Electronic Structure and Physicochemical Characterization of Multi-Cation Diamond-Like Semiconductors

SPEAKER: Professor Jennifer Aitken,

Department of Chemistry and Biochemistry, Duquesne University

TIME: Wednesday April 9, 2014 at 4:00 PM

PLACE: Room 101 Olin Physical Laboratory

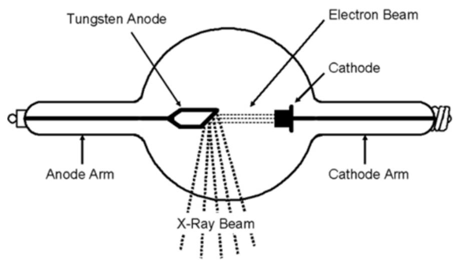
Refreshments will be served at 3:30 PM in the Olin Lounge. All interested persons are cordially invited to attend.

ABSTRACT

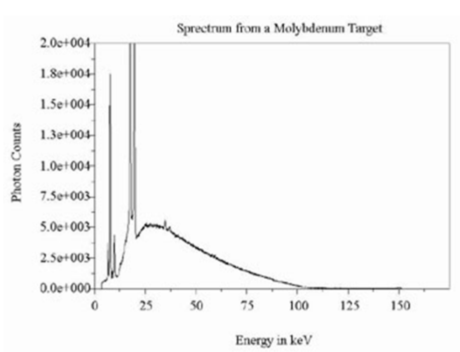
Diamond-like semiconductors (DLSs) possess crystal structures that can be considered as derivatives of cubic or hexagonal diamond. DLSs are one of the few classes of solid-state compounds, for which all chemical compositions can be calculated and a set of possible structures postulated. The use of the many elements that can adopt tetrahedral coordination leads to a multitude of possible compounds, and solid solutions thereof, that can be exploited for physical property tuning. The compositional flexibility and structural simplicity of these materials provides an avenue to develop an intimate understanding of composition-property and structure-property correlations. This seminar will cover our recent progress toward this goal made during our search for DLSs with applications in non-linear optics and thermoelectrics, and as lithium ion conductors.

Generation of X-rays in a Coolidge tube

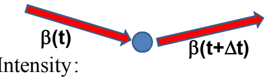
https://www.orau.org/ptp/collection/xraytubescoolidge/coolidgeinformation.htm



http://www.ndt-ed.org/EducationResources/CommunityCollege/Radiography/Physics/xrays.htm



Radiation during collisions



Intensity:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left| \int dt\, e^{i\omega(t - \hat{\mathbf{r}}\cdot\mathbf{R}_q(r)/c)} \frac{d}{dt}\left[\frac{\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\boldsymbol{\beta})}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}} \right] \right|^2$$

Note that $\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\boldsymbol{\beta}) = (\boldsymbol{\varepsilon}_1\cdot\boldsymbol{\beta})\boldsymbol{\varepsilon}_1 + (\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\beta})\boldsymbol{\varepsilon}_2$

For a collision of duration τ emitting radiation with polarization $\boldsymbol{\varepsilon}$ and frequency $\omega \to 0$:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left| \boldsymbol{\varepsilon}\cdot\left(\frac{\boldsymbol{\beta}(t+\tau)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t+\tau)} - \frac{\boldsymbol{\beta}(t)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t)} \right) \right|^2$$

Radiation during collisions -- continued

For a collision of duration τ emitting radiation with polarization $\boldsymbol{\varepsilon}$ and frequency $\omega \to 0$:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left| \boldsymbol{\varepsilon}\cdot\left(\frac{\boldsymbol{\beta}(t+\tau)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t+\tau)} - \frac{\boldsymbol{\beta}(t)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t)} \right) \right|^2$$

Non-relativistic limit:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left| \boldsymbol{\varepsilon}\cdot(\Delta\boldsymbol{\beta}) \right|^2 \qquad \Delta\boldsymbol{\beta} \equiv \boldsymbol{\beta}(t+\tau) - \boldsymbol{\beta}(t)$$

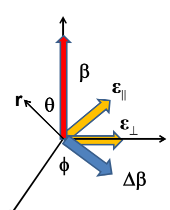
Relativistic collision with small $|\Delta\boldsymbol{\beta}|$:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left| \boldsymbol{\varepsilon}\cdot\left(\frac{\Delta\boldsymbol{\beta} + \hat{\mathbf{r}}\times(\boldsymbol{\beta}\times\Delta\boldsymbol{\beta})}{(1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta})^2} \right) \right|^2$$

Radiation during collisions -- continued

Relativistic collision with small $|\Delta\boldsymbol{\beta}|$:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left| \boldsymbol{\varepsilon}\cdot\left(\frac{\Delta\boldsymbol{\beta} + \hat{\mathbf{r}}\times(\boldsymbol{\beta}\times\Delta\boldsymbol{\beta})}{(1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta})^2} \right) \right|^2$$

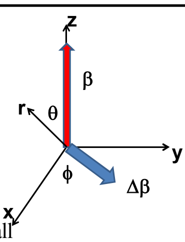


Expressions (averaging over ϕ) for $\parallel$ or $\perp$ polarization:

$$\frac{d^2 I_{\parallel}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c}|\Delta\boldsymbol{\beta}|^2 \frac{(\beta - \cos\theta)^2}{(1-\beta\cos\theta)^4}$$ polarization in ***r*** and β plane

$$\frac{d^2 I_{\perp}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c}|\Delta\boldsymbol{\beta}|^2 \frac{1}{(1-\beta\cos\theta)^2}$$ polarization perpendicular to ***r*** and β plane

Radiation during collisions -- continued

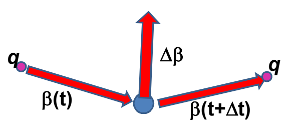


Relativistic collision at low ω and with small $|\Delta\boldsymbol{\beta}|$ and $\Delta\boldsymbol{\beta}$ perpendicular to plane of $\hat{\mathbf{r}}$ and $\boldsymbol{\beta}$, as a function of θ where $\hat{\mathbf{r}}\cdot\boldsymbol{\beta}=\beta\cos\theta$;

Integrating over solid angle:

$$\frac{dI}{d\omega}=\int d\Omega\left(\frac{d^2I_{\parallel}}{d\omega d\Omega}+\frac{d^2I_{\perp}}{d\omega d\Omega}\right)=\frac{2}{3\pi}\frac{q^2}{c}\gamma^2\ |\Delta\boldsymbol{\beta}|^2$$

Estimation of Δβ



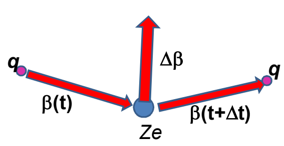
Momentum transfer:

$$Q\equiv|\mathbf{p}(t+\tau)-\mathbf{p}(t)|\approx\gamma Mc^2|\Delta\boldsymbol{\beta}|$$

mass of particle having charge *q*

$$\frac{dI}{d\omega}=\frac{2}{3\pi}\frac{q^2}{c}\gamma^2\,|\Delta\boldsymbol{\beta}|^2\approx\frac{2}{3\pi}\frac{q^2}{M^2c^3}Q^2$$

Estimation of Δβ -- for the case of Rutherford scattering



Assume that target nucleus (charge Ze) has mass >>M;

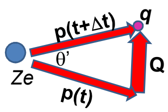
Rutherford scattering cross-section:

$$\frac{d\sigma}{d\Omega}=\left(\frac{2Zeq}{pv}\right)^2\frac{1}{\left(2\sin(\theta'/2)\right)^4}$$

Assuming elastic scattering:

$$Q^2=\left(2p\sin(\theta'/2)\right)^2=2p^2(1-\cos\theta')$$

Case of Rutherford scattering -- continued



Rutherford scattering cross-section :

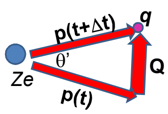
$$\frac{d\sigma}{d\Omega} = \left(\frac{2Zeq}{pv}\right)^2 \frac{1}{\left(2\sin(\theta'/2)\right)^4}$$

$$\frac{d\sigma}{dQ} = \int d\phi' \frac{d\sigma}{d\Omega}\frac{d\Omega}{dQ}$$

$$Q^2 = \left(2p\sin\left(\theta'/2\right)\right)^2 = 2p^2\left(1-\cos\theta'\right)$$

$$\Rightarrow \quad \frac{d\sigma}{dQ} = 8\pi\left(\frac{Zeq}{\beta c}\right)^2 \frac{1}{Q^3}$$

Case of Rutherford scattering -- continued



Differential radiation cross section :

$$\frac{d^2\chi}{d\omega dQ} = \frac{dI}{d\omega}\frac{d\sigma}{dQ} = \left(\frac{2}{3\pi}\frac{q^2}{M^2c^3}Q^2\right)\left(8\pi\left(\frac{Zeq}{\beta c}\right)^2\frac{1}{Q^3}\right)$$

$$= \frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2\frac{1}{\beta^2}\frac{1}{Q}$$

Differential radiation cross section -- continued

Integrating over momentum transfer

$$\frac{d\chi}{d\omega} = \int_{Q_{\min}}^{Q_{\max}} dQ \frac{d^2\chi}{d\omega dQ} = \frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2\frac{1}{\beta^2}\ln\left(\frac{Q_{\max}}{Q_{\min}}\right)$$

Comment on frequency dependence --

Original expression for radiation intensity :

$$\frac{d^2I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\int dt\, e^{i\omega\left(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c\right)}\frac{d}{dt}\left[\frac{\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}\right)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}}\right]\right|^2$$

In the previous derivations, we have assumed that

$\omega\left(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c\right) << 1.$

$$\omega\left(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c\right) = \omega\left(t-\hat{\mathbf{r}}\cdot\int_0^t dt'\,\boldsymbol{\beta}(t')\right) \approx \omega\tau\left(1-\hat{\mathbf{r}}\cdot\langle\boldsymbol{\beta}\rangle\right)$$

Differential radiation cross section -- continued

Radiation cross section in terms of momentum transfer

$$\frac{d\chi}{d\omega} = \int_{Q_{\min}}^{Q_{\max}} dQ \frac{d^2\chi}{d\omega dQ} = \frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2 \frac{1}{\beta^2}\ln\left(\frac{Q_{\max}}{Q_{\min}}\right)$$

Note that : $Q^2 = 2p^2(1-\cos\theta') \quad \Rightarrow Q_{\max} = 2p$

In general, $Q_{\min}$ is determined by the collision time condition $\omega\tau < 1 \quad \Rightarrow Q_{\min} \approx \frac{2Zeq\omega}{v^2}$

Radiation cross section for classical non-relativistic process

$$\frac{d\chi}{d\omega} = \frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2 \frac{1}{\beta^2}\ln\left(\frac{\lambda M v^3}{Zeq\omega}\right)$$

λ= "fudge factor" of order unity