

PHY 712 Electrodynamics
10-10:50 AM MWF Olin 107

Plan for Lecture 21:

Continue reading Chap. 9

A. Electromagnetic waves due to specific sources

B. Dipole radiation examples

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10	Fri	02/07/2014	Chap. 4	Dipoles and dielectrics	#10	02/10/2014
11	Mon	02/10/2014	Chap. 5	Magnetostatics	#11	02/12/2014
12	Wed	02/12/2014	Chap. 5	Magnetostatics	#12	02/14/2014
	Fri	02/14/2014		Class cancelled because of weather		
13	Mon	02/17/2014	Chap. 5	Magnetostatics	#13	02/28/2014
14	Mon	02/17/2014	Chap. 6	Maxwell's equations	#14	02/28/2014
15	Wed	02/19/2014	Chap. 6	Electromagnetic energy and force	#15	02/28/2014
16	Fri	02/21/2014	Chap. 7	Electromagnetic plane waves	#16	02/28/2014
17	Fri	02/21/2014	Chap. 7	Dynamic dielectric media and their effects	#17	02/28/2014
	Mon	02/24/2014		No class -- NAWH out of town		
	Wed	02/26/2014		No class -- NAWH out of town		
18	Fri	02/28/2014	Chap. 7	Complex dielectrics; TEM modes		
	Mon	03/03/2014	APS Meeting	Take-home exam (no class meeting)		
	Wed	03/05/2014	APS Meeting	Take-home exam (no class meeting)		
	Fri	03/07/2014	APS Meeting	Take-home exam (no class meeting)		
	Mon	03/10/2014	Spring Break			
	Wed	03/12/2014	Spring Break			
	Fri	03/14/2014	Spring Break			
19	Mon	03/17/2014	Chap. 8	Wave guides; Take-home exam due	#18	3/21/2014
20	Wed	03/19/2014	Chap. 9	Sources of Electromagnetic Waves	#19	3/21/2014
21	Fri	03/21/2014	Chap. 9	Sources of Electromagnetic Waves	#20	3/28/2014

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Maxwell's equations

Microscopic or vacuum form ($\mathbf{P} = 0$; $\mathbf{M} = 0$):

Coulomb's law : $\nabla \cdot \mathbf{E} = \rho / \epsilon_0$

Ampere - Maxwell's law : $\nabla \times \mathbf{B} - \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}$

Faraday's law : $\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$

No magnetic monopoles : $\nabla \cdot \mathbf{B} = 0$

$$\Rightarrow c^2 = \frac{1}{\epsilon_0 \mu_0}$$

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Formulation of Maxwell's equations in terms of vector and scalar potentials:

Lorentz gauge form -- require : $\nabla \cdot \mathbf{A}_L + \frac{1}{c^2} \frac{\partial \Phi_L}{\partial t} = 0$

$-\nabla^2 \Phi_L + \frac{1}{c^2} \frac{\partial^2 \Phi_L}{\partial t^2} = \rho / \epsilon_0$

$-\nabla^2 \mathbf{A}_L + \frac{1}{c^2} \frac{\partial^2 \mathbf{A}_L}{\partial t^2} = \mu_0 \mathbf{J}$

$\mathbf{B} = \nabla \times \mathbf{A} \quad \mathbf{E} = -\nabla \Phi - \frac{\partial \mathbf{A}}{\partial t}$

Note that the Lorentz gauge is consistent with the source continuity condition : $\frac{\partial \rho(\mathbf{r}, t)}{\partial t} + \nabla \cdot \mathbf{J}(\mathbf{r}, t) = 0$

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Electromagnetic waves from time harmonic sources

Charge density : $\rho(\mathbf{r}, t) = \Re(\tilde{\rho}(\mathbf{r}, \omega)e^{-i\omega t})$

Current density : $\mathbf{J}(\mathbf{r}, t) = \Re(\tilde{\mathbf{J}}(\mathbf{r}, \omega)e^{-i\omega t})$

\Rightarrow Scalar potential : $\Phi(\mathbf{r}, t) = \Re(\tilde{\Phi}(\mathbf{r}, \omega)e^{-i\omega t})$

\Rightarrow Vector potential : $\mathbf{A}(\mathbf{r}, t) = \Re(\tilde{\mathbf{A}}(\mathbf{r}, \omega)e^{-i\omega t})$

For $k \equiv \frac{\omega}{c}$:

$\tilde{\Phi}(\mathbf{r}, \omega) = \tilde{\Phi}_0(\mathbf{r}, \omega) + \frac{1}{4\pi\epsilon_0} \int d^3r' \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \tilde{\rho}(\mathbf{r}', \omega)$

$\tilde{\mathbf{A}}(\mathbf{r}, \omega) = \tilde{\mathbf{A}}_0(\mathbf{r}, \omega) + \frac{\mu_0}{4\pi} \int d^3r' \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \tilde{\mathbf{J}}(\mathbf{r}', \omega)$

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Electromagnetic waves from time harmonic sources -- continued:

Useful expansion :

$\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} = ik \sum_{lm} j_l(kr_<) h_l(kr_>) Y_{lm}(\hat{\mathbf{r}}) Y_{lm}^*(\hat{\mathbf{r}}')$

$\tilde{\Phi}(\mathbf{r}, \omega) = \tilde{\Phi}_0(\mathbf{r}, \omega) + \sum_{lm} \tilde{\phi}_{lm}(r, \omega) Y_{lm}(\hat{\mathbf{r}})$

$\tilde{\phi}_{lm}(r, \omega) = \frac{ik}{\epsilon_0} \int d^3r' \tilde{\rho}(\mathbf{r}', \omega) j_l(kr_<) h_l(kr_>) Y_{lm}^*(\hat{\mathbf{r}}')$

$\tilde{\mathbf{A}}(\mathbf{r}, \omega) = \tilde{\mathbf{A}}_0(\mathbf{r}, \omega) + \sum_{lm} \tilde{\mathbf{a}}_{lm}(r, \omega) Y_{lm}(\hat{\mathbf{r}})$

$\tilde{\mathbf{a}}_{lm}(r, \omega) = ik\mu_0 \int d^3r' \tilde{\mathbf{J}}(\mathbf{r}', \omega) j_l(kr_<) h_l(kr_>) Y_{lm}^*(\hat{\mathbf{r}}')$

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Forms of spherical Bessel and Hankel functions:

$$j_0(x) = \frac{\sin(x)}{x} \qquad h_0(x) = \frac{e^{ix}}{ix}$$

$$j_1(x) = \frac{\sin(x)}{x^2} - \frac{\cos(x)}{x} \qquad h_1(x) = -\left(1 + \frac{i}{x}\right) \frac{e^{ix}}{x}$$

$$j_2(x) = \left(\frac{3}{x^3} - \frac{1}{x}\right) \sin(x) - \frac{3\cos(x)}{x^2} \qquad h_2(x) = i\left(1 + \frac{3i}{x} - \frac{3}{x^2}\right) \frac{e^{ix}}{x}$$

Asymptotic behavior :

$$x \ll 1 \Rightarrow j_l(x) \approx \frac{(x)^l}{(2l+1)!}$$

$$x \gg 1 \Rightarrow h_l(x) \approx (-i)^{l+1} \frac{e^{ix}}{x}$$

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Electromagnetic waves from time harmonic sources – continued:

Dipole radiation case :

Define dipole moment at frequency ω :

$$\mathbf{p}(\omega) \equiv \int d^3r \mathbf{r} \tilde{\rho}(\mathbf{r}, \omega) = -\frac{1}{i\omega} \int d^3r \tilde{\mathbf{J}}(\mathbf{r}, \omega)$$

For fields outside extent of source and $kr' \ll 1$ within the source :

$$\tilde{\mathbf{A}}(\mathbf{r}, \omega) = -\frac{i\mu_0\omega}{4\pi} \mathbf{p}(\omega) \frac{e^{ikr}}{r}$$

$$\tilde{\Phi}(\mathbf{r}, \omega) = -\frac{i}{4\pi\omega\epsilon_0} \mathbf{p}(\omega) \cdot \hat{\mathbf{r}} \left(1 + \frac{i}{kr}\right) \frac{e^{ikr}}{r}$$

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Electromagnetic waves from time harmonic sources – continued:

$$\tilde{\mathbf{E}}(\mathbf{r}, \omega) = -\nabla \tilde{\Phi}(\mathbf{r}, \omega) + i\omega \tilde{\mathbf{A}}(\mathbf{r}, \omega)$$

$$= \frac{1}{4\pi\epsilon_0} \frac{e^{ikr}}{r} \left(k^2 ((\hat{\mathbf{r}} \times \mathbf{p}(\omega)) \times \hat{\mathbf{r}}) + \left(\frac{3\hat{\mathbf{r}}(\hat{\mathbf{r}} \cdot \mathbf{p}(\omega)) - \mathbf{p}(\omega)}{r^2} \right) (1 - ikr) \right)$$

$$\tilde{\mathbf{B}}(\mathbf{r}, \omega) = \nabla \times \tilde{\mathbf{A}}(\mathbf{r}, \omega)$$

$$= \frac{1}{4\pi\epsilon_0 c^2} \frac{e^{ikr}}{r} k^2 (\hat{\mathbf{r}} \times \mathbf{p}(\omega)) \left(1 - \frac{1}{ikr}\right)$$

Power radiated for $kr \gg 1$:

$$\frac{dP}{d\Omega} = r^2 \hat{\mathbf{r}} \cdot \langle \mathbf{S} \rangle_{\text{avg}} = \frac{r^2}{2\mu_0} \hat{\mathbf{r}} \cdot \Re(\tilde{\mathbf{E}}(\mathbf{r}, \omega) \times \tilde{\mathbf{B}}^*(\mathbf{r}, \omega))$$

$$= \frac{c^2 k^4}{32\pi^2} \sqrt{\frac{\mu_0}{\epsilon_0}} |(\hat{\mathbf{r}} \times \mathbf{p}(\omega)) \times \hat{\mathbf{r}}|^2$$

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Properties of dipole radiation field for $kr \gg 1$:

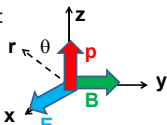
$$\tilde{\mathbf{E}}(\mathbf{r}, \omega) = \frac{1}{4\pi\epsilon_0} \frac{e^{ikr}}{r} \left(k^2 (\hat{\mathbf{r}} \times \mathbf{p}(\omega)) \times \hat{\mathbf{r}} \right)$$

$$\tilde{\mathbf{B}}(\mathbf{r}, \omega) = \frac{1}{4\pi\epsilon_0 c^2} \frac{e^{ikr}}{r} k^2 (\hat{\mathbf{r}} \times \mathbf{p}(\omega))$$

Power radiated for $kr \gg 1$:

$$\frac{dP}{d\Omega} = r^2 \hat{\mathbf{r}} \cdot \langle \mathbf{S} \rangle_{\text{avg}} = \frac{c^2 k^4}{32\pi^2} \sqrt{\frac{\mu_0}{\epsilon_0}} \left| (\hat{\mathbf{r}} \times \mathbf{p}(\omega)) \times \hat{\mathbf{r}} \right|^2$$

Example:



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Example of dipole radiation source

$$\tilde{\mathbf{J}}(\mathbf{r}, \omega) = \hat{\mathbf{z}} J_0 e^{-r/R} \quad \tilde{\rho}(\mathbf{r}, \omega) = -\frac{J_0}{i\omega R} \cos \theta e^{-r/R}$$

$$\tilde{\mathbf{A}}(\mathbf{r}, \omega) = \hat{\mathbf{z}} J_0 (ik\mu_0) \int_0^\infty r'^2 dr' e^{-r'/R} h_0(kr'_z) j_0(kr'_\perp)$$

$$\tilde{\Phi}(\mathbf{r}, \omega) = -\frac{J_0 k}{\epsilon_0 \omega R} \cos \theta \int_0^\infty r'^2 dr' e^{-r'/R} h_1(kr'_z) j_1(kr'_\perp)$$

Evaluation for $r \gg R$:

$$\tilde{\mathbf{A}}(\mathbf{r}, \omega) = \hat{\mathbf{z}} J_0 \mu_0 \frac{e^{ikr}}{r} \frac{2R^3}{(1+k^2R^2)^2}$$

$$\tilde{\Phi}(\mathbf{r}, \omega) = \frac{J_0 k}{\epsilon_0 \omega} \cos \theta \frac{e^{ikr}}{r} \left(1 + \frac{i}{kr} \right) \frac{2R^3}{(1+k^2R^2)^2}$$

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Example of dipole radiation source -- continued

Evaluation for $r \gg R$:

$$\tilde{\mathbf{A}}(\mathbf{r}, \omega) = \hat{\mathbf{z}} J_0 \mu_0 \frac{e^{ikr}}{r} \frac{2R^3}{(1+k^2R^2)^2}$$

$$\tilde{\Phi}(\mathbf{r}, \omega) = \frac{J_0 k}{\epsilon_0 \omega} \cos \theta \frac{e^{ikr}}{r} \left(1 + \frac{i}{kr} \right) \frac{2R^3}{(1+k^2R^2)^2}$$

Relationship to pure dipole approximation (exact when $kR \rightarrow 0$)

$$\mathbf{p}(\omega) \equiv \int d^3r \mathbf{r} \tilde{\rho}(\mathbf{r}, \omega) = -\frac{1}{i\omega} \int d^3r \tilde{\mathbf{J}}(\mathbf{r}, \omega) = -\frac{8\pi R^3 J_0}{i\omega} \hat{\mathbf{z}}$$

Corresponding dipole fields: $\tilde{\mathbf{A}}(\mathbf{r}, \omega) = -\frac{i\mu_0 \omega}{4\pi} \mathbf{p}(\omega) \frac{e^{ikr}}{r}$

$$\tilde{\Phi}(\mathbf{r}, \omega) = -\frac{i}{4\pi\omega\epsilon_0} \mathbf{p}(\omega) \cdot \hat{\mathbf{r}} \left(1 + \frac{i}{kr} \right) \frac{e^{ikr}}{r}$$

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Alternative approach

Fields from time harmonic source:

$$\tilde{\Phi}(\mathbf{r}, \omega) = \frac{1}{4\pi\epsilon_0} \int d^3r' \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \tilde{\rho}(\mathbf{r}', \omega)$$

$$\tilde{\mathbf{A}}(\mathbf{r}, \omega) = \frac{\mu_0}{4\pi} \int d^3r' \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \tilde{\mathbf{J}}(\mathbf{r}', \omega)$$

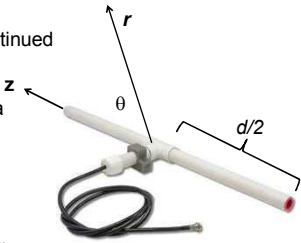
For $r \gg r'$: $|\mathbf{r}-\mathbf{r}'| \approx r(1 - \hat{\mathbf{r}} \cdot \mathbf{r}'/r + \dots)$

$$\tilde{\mathbf{A}}(\mathbf{r}, \omega) \approx \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \int d^3r' e^{-ik\hat{\mathbf{r}} \cdot \mathbf{r}'} \tilde{\mathbf{J}}(\mathbf{r}', \omega)$$

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Alternative approach -- continued

Linear center-fed antenna



$$\tilde{\mathbf{A}}(\mathbf{r}, \omega) \approx \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \int d^3r' e^{-ik\hat{\mathbf{r}} \cdot \mathbf{r}'} \tilde{\mathbf{J}}(\mathbf{r}', \omega)$$

$$\tilde{\mathbf{J}}(\mathbf{r}', \omega) = I_0 \sin\left(\frac{kd}{2} - k|z'|\right) \delta(x)\delta(y)\hat{\mathbf{z}}$$

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Alternative approach – linear center-fed antenna continued

$$\tilde{\mathbf{A}}(\mathbf{r}, \omega) \approx \hat{\mathbf{z}} \frac{\mu_0 I_0}{4\pi} \frac{e^{ikr}}{r} \int_{-d/2}^{d/2} dz' e^{-ik\cos(\theta)z'} \sin\left(\frac{kd}{2} - k|z'|\right)$$

$$= \hat{\mathbf{z}} \frac{\mu_0 I_0}{2\pi} \frac{e^{ikr}}{r} \left(\frac{\cos\left(\frac{kd}{2}\cos\theta\right) - \cos\left(\frac{kd}{2}\right)}{\sin^2\theta} \right)$$

Time averaged power:

$$\frac{dP}{d\Omega} = I_0^2 \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{1}{8\pi^2} \left| \frac{\cos\left(\frac{kd}{2}\cos\theta\right) - \cos\left(\frac{kd}{2}\right)}{\sin\theta} \right|^2$$

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Alternative approach – linear center-fed antenna continued

Time averaged power:

$$\frac{dP}{d\Omega} = I_0^2 \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{1}{8\pi^2} \left| \frac{\cos\left(\frac{kd}{2}\cos\theta\right) - \cos\left(\frac{kd}{2}\right)}{\sin\theta} \right|^2$$

for $kd = \pi$:
$$\frac{dP}{d\Omega} = I_0^2 \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{1}{8\pi^2} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$

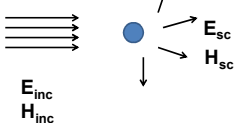
for $kd = 2\pi$:
$$\frac{dP}{d\Omega} = I_0^2 \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{4}{8\pi^2} \frac{\cos^4\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$

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Dipole radiation in light scattering by small (dielectric) particles



$$\mathbf{E}_{inc} = \hat{\mathbf{e}}_0 E_0 e^{i\mathbf{k}_0 \cdot \mathbf{r}} \quad \mathbf{H}_{inc} = \frac{1}{\mu_0 c} \hat{\mathbf{k}}_0 \times \mathbf{E}_{inc}$$

In electric dipole approximation :

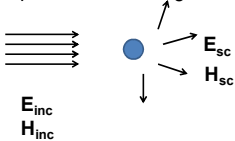
$$\mathbf{E}_{sc} = \frac{1}{4\pi\epsilon_0} k^2 \frac{e^{ikr}}{r} ((\hat{\mathbf{r}} \times \mathbf{p}) \times \hat{\mathbf{r}}) \quad \mathbf{H}_{sc} = \frac{1}{\mu_0 c} \hat{\mathbf{r}} \times \mathbf{E}_{sc}$$

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Dipole radiation in light scattering by small (dielectric) particles



Scattering cross section :

$$\begin{aligned} \frac{d\sigma}{d\Omega}(\hat{\mathbf{r}}, \hat{\mathbf{e}}; \hat{\mathbf{k}}_0, \hat{\mathbf{e}}_0) &= \frac{r^2 \hat{\mathbf{r}} \cdot \langle \mathbf{S}_{sc} \rangle_{avg}}{\hat{\mathbf{k}}_0 \cdot \langle \mathbf{S}_{inc} \rangle_{avg}} \\ &= \frac{r^2 |\hat{\mathbf{e}} \cdot \mathbf{E}_{sc}|^2}{|\hat{\mathbf{e}}_0 \cdot \mathbf{E}_{inc}|^2} = \frac{k^4}{(4\pi\epsilon_0 E_0)^2} |\hat{\mathbf{e}} \cdot \mathbf{p}|^2 \end{aligned}$$

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Estimation of scattering dipole moment:

Suppose the scattering particle is a dielectric sphere with permittivity ϵ and radius a :

$$\mathbf{p} = 4\pi a^3 \epsilon_0 \left(\frac{\epsilon / \epsilon_0 - 1}{\epsilon / \epsilon_0 + 2} \right) \mathbf{E}_{inc}$$

Scattering cross section :

$$\begin{aligned} \frac{d\sigma}{d\Omega}(\hat{\mathbf{r}}, \hat{\boldsymbol{\epsilon}}; \hat{\mathbf{k}}_0, \hat{\boldsymbol{\epsilon}}_0) &= \frac{r^2 |\hat{\boldsymbol{\epsilon}} \cdot \mathbf{E}_{sc}|^2}{|\hat{\boldsymbol{\epsilon}}_0 \cdot \mathbf{E}_{inc}|^2} = \frac{k^4}{(4\pi\epsilon_0 E_0)^2} |\hat{\boldsymbol{\epsilon}} \cdot \mathbf{p}|^2 \\ &= k^4 a^6 \left| \frac{\epsilon / \epsilon_0 - 1}{\epsilon / \epsilon_0 + 2} \right|^2 |\hat{\boldsymbol{\epsilon}} \cdot \hat{\boldsymbol{\epsilon}}_0|^2 \end{aligned}$$

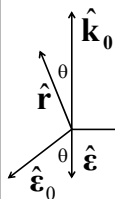
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Scattering by dielectric sphere with permittivity ϵ and radius a :

For \mathbf{E}_{inc} polarized in scattering plane:



$$\begin{aligned} \frac{d\sigma}{d\Omega}(\hat{\mathbf{r}}, \hat{\boldsymbol{\epsilon}}; \hat{\mathbf{k}}_0, \hat{\boldsymbol{\epsilon}}_0) &= k^4 a^6 \left| \frac{\epsilon / \epsilon_0 - 1}{\epsilon / \epsilon_0 + 2} \right|^2 |\hat{\boldsymbol{\epsilon}} \cdot \hat{\boldsymbol{\epsilon}}_0|^2 \\ &= k^4 a^6 \left| \frac{\epsilon / \epsilon_0 - 1}{\epsilon / \epsilon_0 + 2} \right|^2 \cos^2 \theta \end{aligned}$$

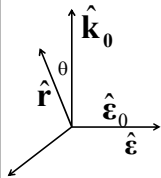
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Scattering by dielectric sphere with permittivity ϵ and radius a :

For \mathbf{E}_{inc} polarized perpendicular to scattering plane:



$$\begin{aligned} \frac{d\sigma}{d\Omega}(\hat{\mathbf{r}}, \hat{\boldsymbol{\epsilon}}; \hat{\mathbf{k}}_0, \hat{\boldsymbol{\epsilon}}_0) &= k^4 a^6 \left| \frac{\epsilon / \epsilon_0 - 1}{\epsilon / \epsilon_0 + 2} \right|^2 |\hat{\boldsymbol{\epsilon}} \cdot \hat{\boldsymbol{\epsilon}}_0|^2 \\ &= k^4 a^6 \left| \frac{\epsilon / \epsilon_0 - 1}{\epsilon / \epsilon_0 + 2} \right|^2 \end{aligned}$$

Assuming both polarizations are equally likely, average cross section is given by :

$$\frac{d\sigma}{d\Omega}(\hat{\mathbf{r}}, \hat{\boldsymbol{\epsilon}}; \hat{\mathbf{k}}_0, \hat{\boldsymbol{\epsilon}}_0) = \frac{k^4 a^6}{2} \left| \frac{\epsilon / \epsilon_0 - 1}{\epsilon / \epsilon_0 + 2} \right|^2 (\cos^2 \theta + 1)$$

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Scattering by dielectric sphere with permittivity ϵ and radius a :

