PHY 712 Electrodynamics
11-11:50 AM MWF Olin 107

Plan for Lecture 36:

General review

	03-18(Mon)	*APS Meeting*	(no class)	Exam
	03-20(Wed)	*APS Meeting*	(no class)	Exam
	03-22(Fri)	*APS Meeting*	(no class)	Exam
25	03-25(Mon)	Chap. 11	Lorentz transformations	#17
26	03-27(Wed)	Chap. 11	Transformations between electromagnetic fields	#18
27	03-28(Thur)	Chap. 11	Liénard-Wiechert potentials revisited	
	03-29(Fri)	*Good Friday*	(no class)	
28	04-01(Mon)	Chap. 14	Radiation by accelerated charges	#19
29	04-03(Wed)	Chap. 14	Radiation by accelerated charges	#20
30	04-05(Fri)	Chap. 14	Synchrotron radiation spectrum	#21
31	04-08(Mon)	Chap. 14	Synchrotron and other radiation sources	#22
32	04-10(Wed)	Chap. 15	Radiation due to collisions of charged particles	
33	04-12(Fri)	Chap. 15	Radiation due to energy loss processes	#23
	04-15(Mon)		(no class -- presentation preparation)	
	04-17(Wed)		(no class -- presentation preparation)	
	04-19(Fri)		(no class -- presentation preparation)	
34	04-22(Mon)	Chap. 15	Radiation due to energy loss processes	#24
35	04-24(Wed)	Review	Radiation from antennas	#25
36	04-26(Fri)	Review	Comprehensive review	
	04-29(Mon)		Student presentations I	
	05-01(Wed)		Student presentations II	
	05-02(Thurs)		Student presentations III	
	05-03(Fri)	Final exam	Take-home exam available -- due 05/10/2013	

Signup for presentations:

Schedule for PHY712 presentations -- please enter your own name replacing "presenter" and also list your presentation title replacing "title". Of course, please do not change other student entries without their permission.

Monday April 29, 2013
11:00 - 11:15 -- Pete Diemer Magnetrons and Microwaves
11:15 - 11:30 -- Jiajie Xiao Negative Refraction
11:30 - 11:45 -- (Chaochao Dun) (Antireflection Thin Film)

Wednesday May 1, 2013
11:00 - 11:15 -- Katelyn Goetz Space Charge Limited Current Measurements
11:15 - 11:30 -- Zach Lamport Dielectric properties of monolayer
11:30 - 11:45 -- Xiaohua Liu Electron paramagnetic resonance

Thursday May 2, 2013
9:00 - 9:15 -- (David Montgomery) (Auroras)
9:15 - 9:30 -- Ryan Godwin Drude Oscillator Model
9:30 - 9:45 -- David Harrison Charges on a sphere
9:45 - 10:00 -- (Evan Welchman) (Ewald Summation)

Basic equations of electrodynamics

CGS (Gaussian)	SI
$\nabla \cdot \mathbf{D} = 4\pi\rho$	$\nabla \cdot \mathbf{D} = \rho$
$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}$	$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$
$\mathbf{F} = q(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B})$	$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$
$u = \frac{1}{8\pi}(\mathbf{E}\cdot\mathbf{D} + \mathbf{B}\cdot\mathbf{H})$	$u = \frac{1}{2}(\mathbf{E}\cdot\mathbf{D} + \mathbf{B}\cdot\mathbf{H})$
$\mathbf{S} = \frac{c}{4\pi}(\mathbf{E} \times \mathbf{H})$	$\mathbf{S} = (\mathbf{E} \times \mathbf{H})$

Vector and scalar potentials in vacuum

CGS Gaussian units:

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{E} = -\nabla\Phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}$$

Source equations:

$$\nabla^2\Phi + \frac{1}{c}\frac{\partial(\nabla\cdot\mathbf{A})}{\partial t} = -4\pi\rho$$

$$\nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} - \left(\nabla\cdot\mathbf{A} + \frac{1}{c}\frac{\partial\Phi}{\partial t}\right) = -\frac{4\pi}{c}\mathbf{J}$$

SI units:

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{E} = -\nabla\Phi - \frac{\partial \mathbf{A}}{\partial t}$$

Source equations:

$$\nabla^2\Phi + \frac{\partial(\nabla\cdot\mathbf{A})}{\partial t} = -\rho/\varepsilon_0$$

$$\nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} - \left(\nabla\cdot\mathbf{A} + \frac{1}{c^2}\frac{\partial\Phi}{\partial t}\right) = -\mu_0\mathbf{J}$$

Polarization and Magnetization

CGS Gaussian units:

$$\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P} = \varepsilon\mathbf{E}$$

$$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M} = \mu\mathbf{H}$$

$$k = \sqrt{\mu\varepsilon}\,\frac{\omega}{c} \equiv n\frac{\omega}{c}$$

SI units:

$$\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P} = \varepsilon\mathbf{E}$$

$$\mathbf{B} = \mu_0\left(\mathbf{H} + \mathbf{M}\right) = \mu\mathbf{H}$$

$$k = \sqrt{\mu\varepsilon}\,\omega = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}\,\frac{\omega}{c} \equiv n\frac{\omega}{c}$$

Some identities useful for spherical geometries:

$$\frac{1}{|\mathbf{r}-\mathbf{r}'|} = \sum_{l=0}^{\infty} \frac{r_<^{\,l}}{r_>^{\,l+1}} P_l(\hat{\mathbf{r}}\cdot\hat{\mathbf{r}}') = \sum_{l=0}^{\infty} \frac{r_<^{\,l}}{r_>^{\,l+1}} \sum_{m=-l}^{l} Y_{lm}(\hat{\mathbf{r}}) Y_{lm}^{\;*}(\hat{\mathbf{r}}')$$

$$\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} = 4\pi ik \sum_{l=0}^{\infty} j_l(kr_<) h_l(kr_>) \sum_{m=-l}^{l} Y_{lm}(\hat{\mathbf{r}}) Y_{lm}^{\;*}(\hat{\mathbf{r}}')$$

Some significant results:

- Ewald summation formula

$$W = \frac{1}{8\pi\epsilon_0} \sum_{i,j;i\neq j} \frac{q_i q_j}{|\mathbf{r}_i - \mathbf{r}_j|}$$

Lecture 2 – Ewald summation methods –

- exact result for periodic system

$$\frac{W}{N} = \sum_{\alpha\beta} \frac{q_\alpha q_\beta}{8\pi\varepsilon_0} \left(\frac{4\pi}{\Omega} \sum_{\mathbf{G}\neq\mathbf{0}} \frac{e^{-i\mathbf{G}\cdot\tau_{\alpha\beta}}\ e^{-G^2/\eta}}{G^2} - \sqrt{\frac{\eta}{\pi}}\delta_{\alpha\beta} + \right.$$

$$\left. + \sum_{\mathbf{T}}{}' \frac{\mathrm{erfc}(\frac{1}{2}\sqrt{\eta}|\tau_{\alpha\beta} + \mathbf{T}|)}{|\tau_{\alpha\beta} + \mathbf{T}|} \right) - \frac{4\pi Q^2}{8\pi\varepsilon_0 \Omega \eta}.$$

Some significant results:

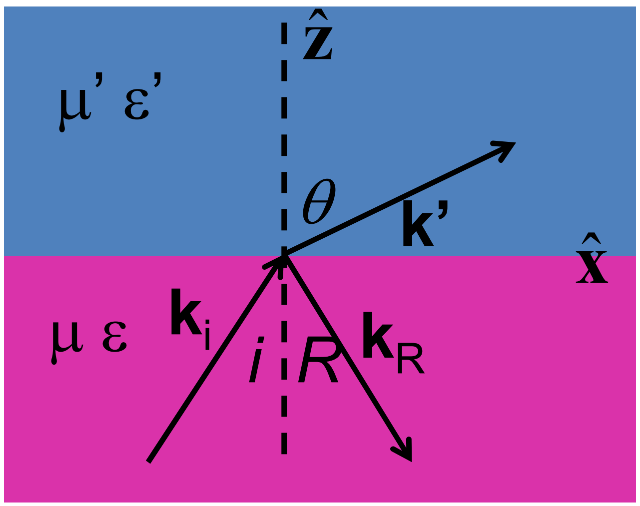
- Magnetic dipoles in atoms and hyperfine Hamiltonian

$$\mathcal{H}_{\mathrm{HF}} = -\frac{\mu_0}{4\pi}\left(\frac{3(\mu_{\mathbf{N}}\cdot\hat{\mathbf{r}})(\mu_{\mathbf{e}}\cdot\hat{\mathbf{r}}) - \mu_{\mathbf{N}}\cdot\mu_{\mathbf{e}}}{r^3} + \right.$$

$$\left. + \frac{8\pi}{3}\mu_{\mathbf{N}}\cdot\mu_{\mathbf{e}}\delta^3(\mathbf{r}) + \frac{e}{m_e}\left\langle\frac{\mathbf{L}\cdot\mu_{\mathbf{N}}}{r^3}\right\rangle\right)$$

Some significant results:

- Reflection and refraction of plane polarized electromagnetic waves



s-polarization – **E** field "polarized" perpendicular to plane of incidence

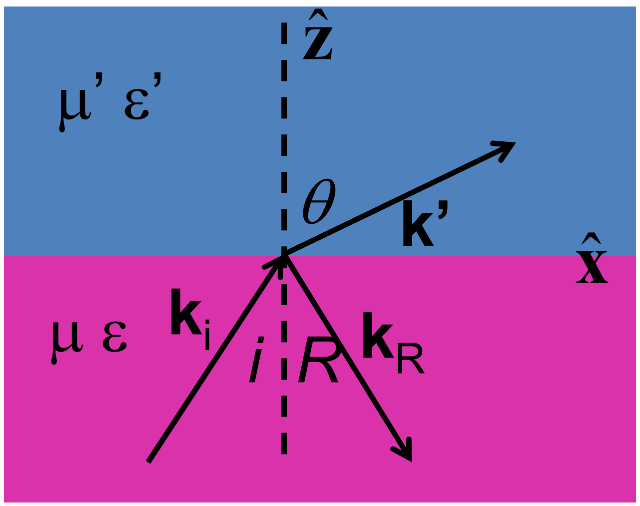
$$\frac{E_{0R}}{E_{0i}} = \frac{n\cos i - \frac{\mu}{\mu'} n'\cos\theta}{n\cos i + \frac{\mu}{\mu'} n'\cos\theta}$$

$$\frac{E'_0}{E_{0i}} = \frac{2n\cos i}{n\cos i + \frac{\mu}{\mu'} n'\cos\theta}$$

$$\text{Note that:} \quad n'\cos\theta = \sqrt{n'^2 - n^2\sin^2 i}$$

Some significant results:

- Reflection and refraction of plane polarized electromagnetic waves



p-polarization – **E** field "polarized" parallel to plane of incidence

$$\frac{E_{0R}}{E_{0i}} = \frac{\frac{\mu}{\mu'} n' \cos i - n \cos\theta}{\frac{\mu}{\mu'} n' \cos i + n \cos\theta}$$

$$\frac{E'_0}{E_{0i}} = \frac{2n \cos i}{\frac{\mu}{\mu'} n' \cos i + n \cos\theta}$$

$$\text{Note that: } \quad n' \cos\theta = \sqrt{n'^2 - n^2 \sin^2 i}$$

Some significant results:

- Drude model of dielectric function

$$\frac{\varepsilon(\omega)}{\varepsilon_0} = 1 + N\sum_i f_i \frac{q_i^2}{\varepsilon_0 m_i} \frac{1}{\omega_i^2 - \omega^2 - i\omega\gamma_i}$$

$$= \frac{\varepsilon_R(\omega)}{\varepsilon_0} + i\frac{\varepsilon_I(\omega)}{\varepsilon_0}$$

$$\frac{\varepsilon_R(\omega)}{\varepsilon_0} = 1 + N\sum_i f_i \frac{q_i^2}{\varepsilon_0 m_i} \frac{\omega_i^2 - \omega^2}{\left(\omega_i^2 - \omega^2\right)^2 + \omega^2\gamma_i^2}$$

$$\frac{\varepsilon_I(\omega)}{\varepsilon_0} = N\sum_i f_i \frac{q_i^2}{\varepsilon_0 m_i} \frac{\omega\gamma_i}{\left(\omega_i^2 - \omega^2\right)^2 + \omega^2\gamma_i^2}$$

Some significant results:

- Kramers-Kronig transform of dielectric function

$$\frac{\varepsilon_R(\omega)}{\varepsilon_0} - 1 = \frac{1}{\pi} P \int_{-\infty}^{\infty} d\omega' \frac{\varepsilon_I(\omega')}{\varepsilon_0} \frac{1}{\omega' - \omega}$$

$$\frac{\varepsilon_I(\omega)}{\varepsilon_0} = -\frac{1}{\pi} P \int_{-\infty}^{\infty} d\omega' \left(\frac{\varepsilon_R(\omega')}{\varepsilon_0} - 1 \right) \frac{1}{\omega' - \omega}$$

$$\text{with} \quad \varepsilon_R(-\omega) = \varepsilon_R(\omega); \quad \varepsilon_I(-\omega) = -\varepsilon_I(\omega)$$

Some significant results:

- Radiation from dipole source; example:

$$\tilde{\mathbf{J}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0 e^{-r/R} \qquad \tilde{\rho}(\mathbf{r},\omega)=\frac{J_0}{-i\omega R}\cos\theta e^{-r/R}$$

$$\tilde{\mathbf{A}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0\,(ik\mu_0)\int_0^\infty r'^2\,dr'\,e^{-r'/R}h_0(kr_>)j_0(kr_<)$$

$$\tilde{\Phi}(\mathbf{r},\omega)=-\frac{J_0 k}{\varepsilon_0\omega R}\cos\theta\int_0^\infty r'^2\,dr'\,e^{-r'/R}h_1(kr_>)j_1(kr_<)$$

Evaluation for $r >> R$:

$$\tilde{\mathbf{A}}(\mathbf{r},\omega)=\hat{\mathbf{z}}J_0\mu_0\frac{e^{ikr}}{r}\;\frac{2R^3}{\left(1+k^2R^2\right)^2}$$

$$\tilde{\Phi}(\mathbf{r},\omega)=\frac{J_0 k}{\varepsilon_0\omega}\cos\theta\,\frac{e^{ikr}}{r}\left(1+\frac{i}{kr}\right)\;\frac{2R^3}{\left(1+k^2R^2\right)^2}$$

Some significant results:

- Lorentz transformation of electromagnetic fields

Field strength tensor $F^{\alpha\beta} \equiv \left(\partial^\alpha A^\beta - \partial^\beta A^\alpha\right)$

$$F^{\alpha\beta} \equiv \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

$$\text{Transformation}: \; F^{\alpha\beta} = \mathcal{L}_v^{\alpha\gamma} F'^{\gamma\delta} \mathcal{L}_v^{\delta\beta} \qquad \mathcal{L}_v = \begin{pmatrix} \gamma_v & \gamma_v\beta_v & 0 & 0 \\ \gamma_v\beta_v & \gamma_v & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$F^{\alpha\beta} = \begin{pmatrix} 0 & -E'_x & -\gamma_v\left(E'_y + \beta_v B'_z\right) & -\gamma_v\left(E'_z - \beta_v B'_y\right) \\ E'_x & 0 & -\gamma_v\left(B'_z + \beta_v E'_y\right) & \gamma_v\left(B'_y - \beta_v E'_z\right) \\ \gamma_v\left(E'_y + \beta_v B'_z\right) & \gamma_v\left(B'_z + \beta_v E'_y\right) & 0 & -B'_x \\ \gamma_v\left(E'_z - \beta_v B'_y\right) & -\gamma_v\left(B'_y - \beta_v E'_z\right) & B'_x & 0 \end{pmatrix}$$

Some significant results:

- Liénard-Wiechert potentials

$$\Phi(\mathbf{r},t) = \frac{q}{4\pi\epsilon_0}\frac{1}{R - \frac{\mathbf{v}\cdot\mathbf{R}}{c}},$$

$$\mathbf{A}(\mathbf{r},t) = \frac{q}{4\pi\epsilon_0 c^2}\frac{\mathbf{v}}{R - \frac{\mathbf{v}\cdot\mathbf{R}}{c}},$$

$$\mathbf{R} \equiv \mathbf{r} - \mathbf{R}_q(t_r) \text{ and } \mathbf{v} \equiv \dot{\mathbf{R}}_q(t_r).$$

Some significant results:

- Power radiated from accelerating charge *q:*

$$\mathbf{S}(\mathbf{r},t) = \frac{c}{4\pi}\hat{\mathbf{R}}\left|\mathbf{E}(\mathbf{r},t)\right|^2 = \frac{q^2}{4\pi c R^2}\hat{\mathbf{R}}\frac{\left|\hat{\mathbf{R}}\times\left[\left(\hat{\mathbf{R}}-\boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right|^2}{\left(1-\boldsymbol{\beta}\cdot\hat{\mathbf{R}}\right)^6}$$

$$\frac{dP}{d\Omega} = \mathbf{S}\cdot\hat{\mathbf{R}}R^2 = \frac{q^2}{4\pi c}\frac{\left|\hat{\mathbf{R}}\times\left[\left(\hat{\mathbf{R}}-\boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right|^2}{\left(1-\boldsymbol{\beta}\cdot\hat{\mathbf{R}}\right)^6}$$

In the non-relativistic limit: $\beta << 1$

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c}\left|\hat{\mathbf{R}}\times\left[\hat{\mathbf{R}}\times\dot{\boldsymbol{\beta}}\right]\right|^2 = \frac{q^2}{4\pi c^3}\left|\dot{\mathbf{v}}\right|^2 \sin^2\Theta$$