PHY 712 Electrodynamics
11-11:50 AM MWF Olin 107

Plan for Lecture 35:

Read about antennas in Chap. 9

1. **Power distribution of linear antenna**
2. **Interference effects**

	03-18(Mon)	*APS Meeting*	(no class)	Exam
	03-20(Wed)	*APS Meeting*	(no class)	Exam
	03-22(Fri)	*APS Meeting*	(no class)	Exam
25	03-25(Mon)	Chap. 11	Lorentz transformations	#17
26	03-27(Wed)	Chap. 11	Transformations between electromagnetic fields	#18
27	03-28(Thur)	Chap. 11	Liénard-Wiechert potentials revisited	
	03-29(Fri)	*Good Friday*	(no class)	
28	04-01(Mon)	Chap. 14	Radiation by accelerated charges	#19
29	04-03(Wed)	Chap. 14	Radiation by accelerated charges	#20
30	04-05(Fri)	Chap. 14	Synchrotron radiation spectrum	#21
31	04-08(Mon)	Chap. 14	Synchrotron and other radiation sources	#22
32	04-10(Wed)	Chap. 15	Radiation due to collisions of charged particles	
33	04-12(Fri)	Chap. 15	Radiation due to energy loss processes	#23
	04-15(Mon)		(no class -- presentation preparation)	
	04-17(Wed)		(no class -- presentation preparation)	
	04-19(Fri)		(no class -- presentation preparation)	
34	04-22(Mon)	Chap. 15	Radiation due to energy loss processes	#24
35	04-24(Wed)	Review	Radiation from antennas	#25
36	04-26(Fri)	Review	Comprehensive review	
	04-29(Mon)		Student presentations I	
	05-01(Wed)		Student presentations II	
	05-02(Thurs)		Student presentations III	
	05-03(Fri)	Final exam	Take-home exam available -- due 05/10/2013	

Signup for presentations:

Schedule for PHY712 presentations -- please enter your own name replacing "presenter" and also list your presentation title replacing "title". Of course, please do not change other student entries without their permission.

Monday April 29, 2013

11:00 - 11:15 -- Pete Diemer Magnetrons and Microwaves

11:15 - 11:30 -- (Jiajie Xiao) (title)

11:30 - 11:45 -- (Chaochao Dun) (Antireflection Thin Film)

Wednesday May 1, 2013

11:00 - 11:15 -- Katelyn Goetz Space Charge Limited Current Measurements

11:15 - 11:30 -- Zach Lamport Dielectric properties of monolayer

11:30 - 11:45 -- Xiaohua Liu Electron paramagnetic resonance

Thursday May 2, 2013

9:00 - 9:15 -- (David Montgomery) (Auroras)

9:15 - 9:30 -- Ryan Godwin Drude Oscillator Model

9:30 - 9:45 -- David Harrison Charges on a sphere

9:45 - 10:00 -- (Evan Welchman) (Ewald Summation)

WFU Physics Colloquium

TITLE: Physics Ceremonies and Honors Theses Presentations I

TIME: Wednesday April 24, 2013 at 3:30* PM

PLACE: Room 101 Olin Physical Laboratory

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*Note early start time

At 3:00 PM in the Olin Lounge, Physics T-Shirts will be distributed and refreshments will be served.

PROGRAM

- ΣΠΣ (Physics Honor's Society) Ceremony
- Welcoming of New Physics Majors
- Honoring Our Graduating Students
- Presentation of Awards
- Physics T-shirt Design
- Honor's Thesis Presentations for 2013 -- Part I:
  - Sean Cusano
  - Sam Flynn
  - Brad Shugoll

Review from Lecture 23:

Maxwell's equations in terms of vector and scalar potentials (Lorentz gauge)

$$\text{Lorentz condition: } \nabla \cdot \mathbf{A}_L + \frac{1}{c^2}\frac{\partial \Phi_L}{\partial t} = 0$$

$$-\nabla^2 \Phi_L + \frac{1}{c^2}\frac{\partial^2 \Phi_L}{\partial t^2} = \rho / \varepsilon_0$$

$$-\nabla^2 \mathbf{A}_L + \frac{1}{c^2}\frac{\partial^2 \mathbf{A}_L}{\partial t^2} = \mu_0 \mathbf{J}$$

General equation form:

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Psi = -4\pi f$$

Solution of Maxwell's equations in the Lorentz gauge -- continued

$$G(\mathbf{r},t;\mathbf{r}',t') = \frac{1}{|\mathbf{r}-\mathbf{r}'|}\delta\left(t' - \left(t - |\mathbf{r}-\mathbf{r}'|/c\right)\right)$$

Solution for field $\Psi(\mathbf{r},t)$:

$$\Psi(\mathbf{r},t) = \Psi_{f=0}(\mathbf{r},t) +$$

$$\int d^3r' \int dt' \frac{1}{|\mathbf{r}-\mathbf{r}'|}\delta\left(t' - \left(t - \frac{1}{c}|\mathbf{r}-\mathbf{r}'|\right)\right) f(\mathbf{r}',t')$$

Electromagnetic waves from time harmonic sources

Charge density: $\rho(\mathbf{r},t) = \Re\left(\tilde{\rho}(\mathbf{r},\omega)e^{-i\omega t}\right)$

Current density: $\mathbf{J}(\mathbf{r},t) = \Re\left(\tilde{\mathbf{J}}(\mathbf{r},\omega)e^{-i\omega t}\right)$

Note that the continuity condition:

$$\frac{\partial \rho(\mathbf{r},t)}{\partial t} + \nabla \cdot \mathbf{J}(\mathbf{r},t) = 0 \quad \Rightarrow -i\omega\tilde{\rho}(\mathbf{r},\omega) + \nabla \cdot \tilde{\mathbf{J}}(\mathbf{r},\omega) = 0$$

General source: $f(\mathbf{r},t) = \Re\left(\tilde{f}(\mathbf{r},\omega)e^{-i\omega t}\right)$

For $$\tilde{f}(\mathbf{r},\omega) = \frac{1}{4\pi\varepsilon_0}\tilde{\rho}(\mathbf{r},\omega)$$

or $$\tilde{f}(\mathbf{r},\omega) = \frac{\mu_0}{4\pi}\tilde{J}_i(\mathbf{r},\omega)$$

Electromagnetic waves from time harmonic sources – continued:

$$\Psi(\mathbf{r},t) = \Psi_{f=0}(\mathbf{r},t) + \int d^3r' \int dt' \frac{1}{|\mathbf{r}-\mathbf{r}'|}\delta\left(t' - \left(t - \frac{1}{c}|\mathbf{r}-\mathbf{r}'|\right)\right) f(\mathbf{r}',t')$$

$$\widetilde{\Psi}(\mathbf{r},\omega)e^{-i\omega t} = \widetilde{\Psi}_{f=0}(\mathbf{r},\omega)e^{-i\omega t} + \int d^3r' \int dt' \frac{1}{|\mathbf{r}-\mathbf{r}'|}\delta\left(t' - \left(t - \frac{1}{c}|\mathbf{r}-\mathbf{r}'|\right)\right) \widetilde{f}(\mathbf{r}',\omega)e^{-i\omega t'}$$

$$= \widetilde{\Psi}_{f=0}(\mathbf{r},\omega)e^{-i\omega t} + \int d^3r' \frac{e^{i\frac{\omega}{c}|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \widetilde{f}(\mathbf{r}',\omega)e^{-i\omega t}$$

Electromagnetic waves from time harmonic sources – continued:

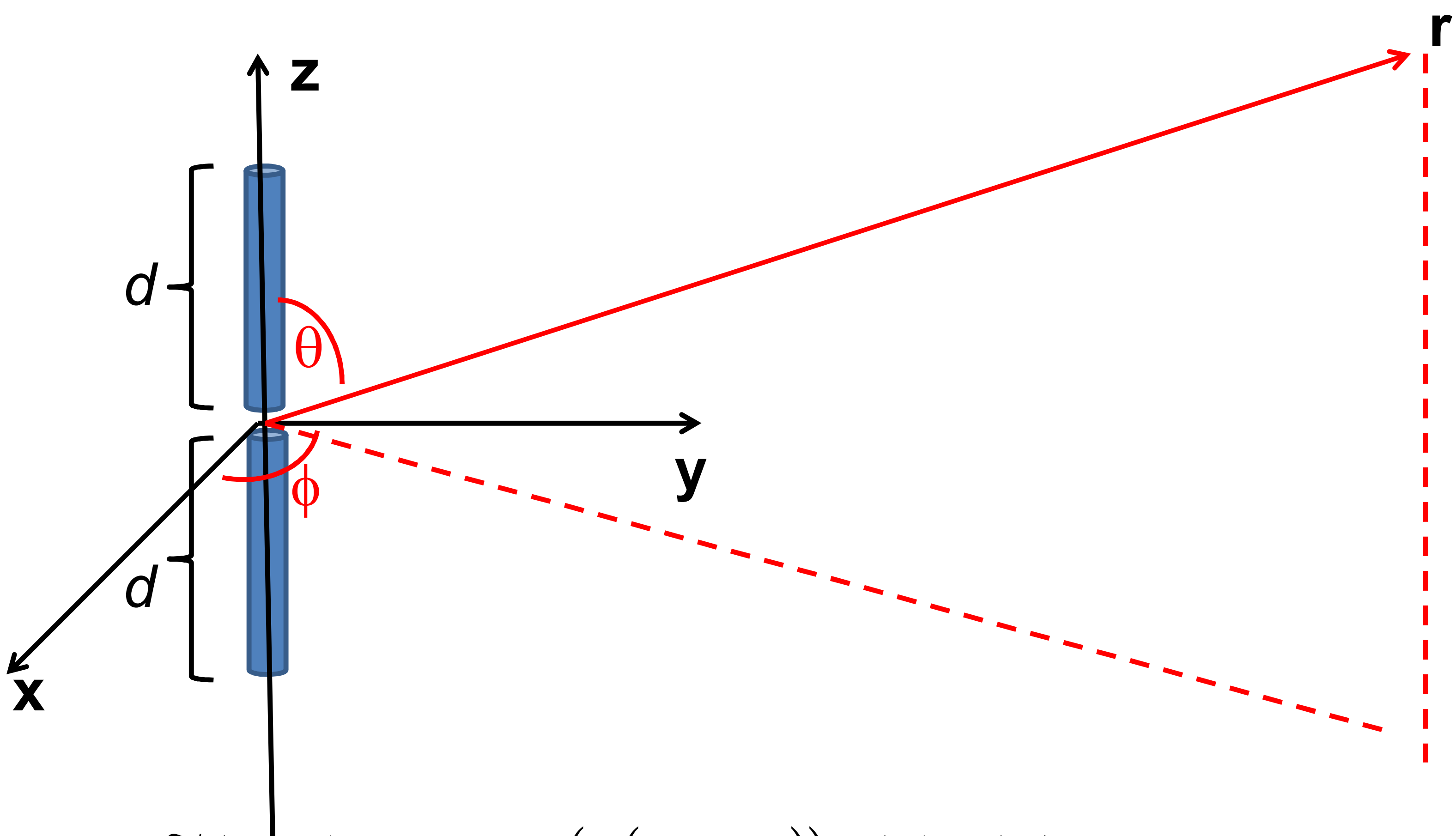
For scalar potential (Lorentz gauge, $k \equiv \frac{\omega}{c}$)

$$\tilde{\Phi}(\mathbf{r},\omega) = \tilde{\Phi}_0(\mathbf{r},\omega) + \frac{1}{4\pi\varepsilon_0}\int d^3r' \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\tilde{\rho}(\mathbf{r}',\omega)$$

For vector potential (Lorentz gauge, $k \equiv \frac{\omega}{c}$)

$$\tilde{\mathbf{A}}(\mathbf{r},\omega) = \tilde{\mathbf{A}}_0(\mathbf{r},\omega) + \frac{\mu_0}{4\pi}\int d^3r' \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\tilde{\mathbf{J}}(\mathbf{r}',\omega)$$

Consider antenna source



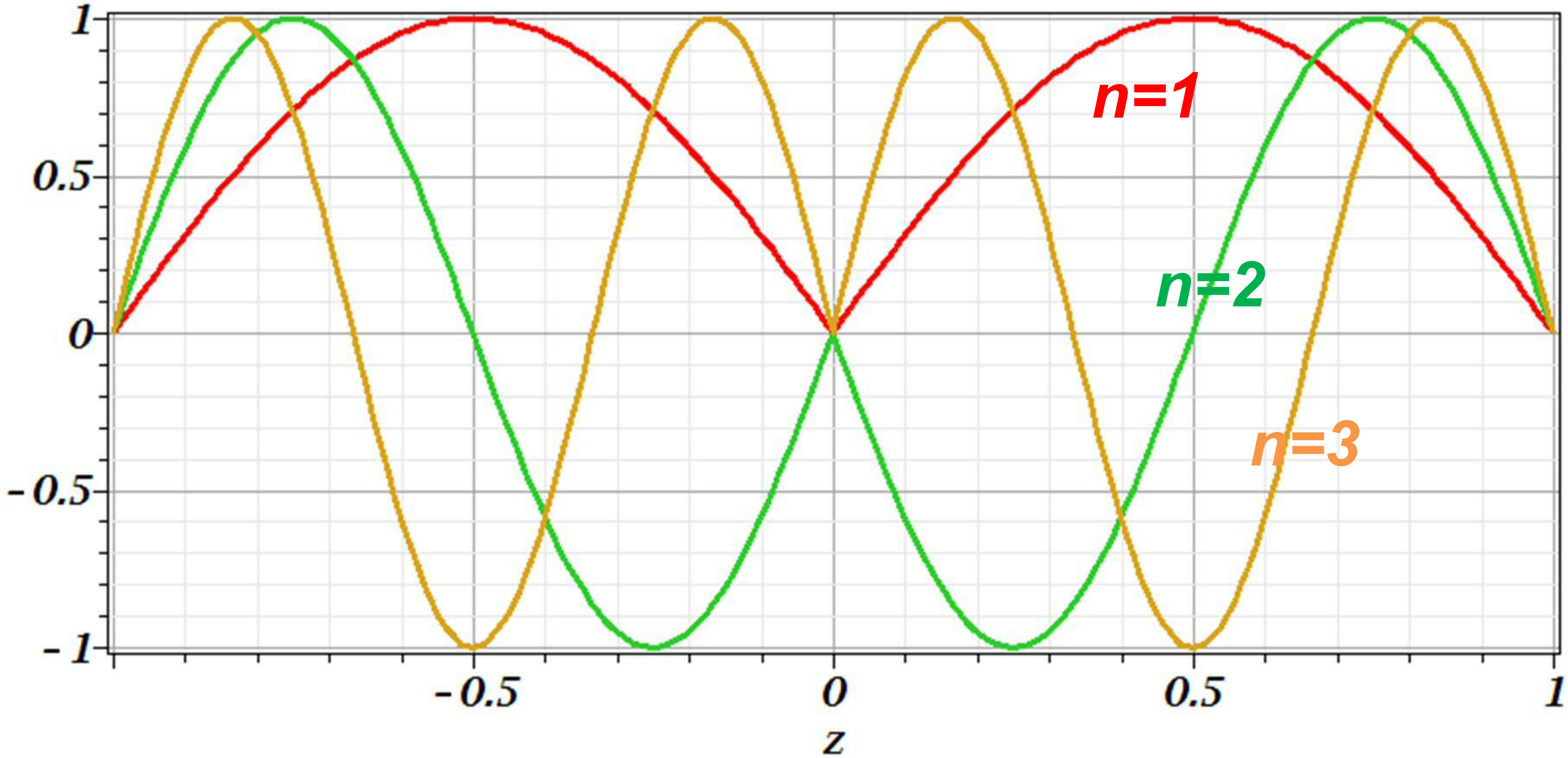
$$\tilde{\mathbf{J}}(\mathbf{r},\omega) = \hat{\mathbf{z}} I \sin\left(k\left(d - |z|\right)\right)\delta(x)\delta(y) \quad \text{for } -d \le z \le d$$

$$k \equiv \frac{\omega}{c}$$

Consider antenna source -- continued

$$\tilde{\mathbf{J}}(\mathbf{r},\omega) = \hat{\mathbf{z}} I \sin\left(k\left(d-|z|\right)\right)\delta(x)\delta(y) \quad \text{for } -d \le z \le d$$

$$k \equiv \frac{\omega}{c} = \frac{n\pi}{d}; \qquad n = 1,2,3....$$



Consider antenna source -- continued

$$\tilde{\mathbf{J}}(\mathbf{r},\omega) = \hat{\mathbf{z}} I \sin\big(k(d-|z|)\big)\delta(x)\delta(y) \quad \text{for } -d \le z \le d$$

$$k \equiv \frac{\omega}{c} = \frac{n\pi}{d}; \qquad n = 1,2,3....$$

Vector potential from source:

$$\tilde{\mathbf{A}}(\mathbf{r},\omega) = \frac{\mu_0}{4\pi}\int d^3r' \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\tilde{\mathbf{J}}(\mathbf{r}',\omega)$$

For $r >> d$ $\qquad \tilde{\mathbf{A}}(\mathbf{r},\omega) \approx \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int d^3r' e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}'}\tilde{\mathbf{J}}(\mathbf{r}',\omega)$

$$\tilde{\mathbf{A}}(\mathbf{r},\omega) \approx \hat{\mathbf{z}}\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r} I \int_{-d}^{d} dz\, e^{-ikz\cos\theta} \ \sin\big(k(d-|z|)\big)$$

Consider antenna source -- continued

$$\widetilde{\mathbf{A}}(\mathbf{r},\omega) \approx \hat{\mathbf{z}}\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r} I \int_{-d}^{d} dz\, e^{-ikz\cos\theta} \quad \sin\left(k\left(d-|z|\right)\right)$$

$$= \hat{\mathbf{z}}\frac{\mu_0}{4\pi}\frac{e^{ikr}}{kr} 2I\left[\frac{\cos(kd\cos\theta)-\cos(kd)}{\sin^2\theta}\right]$$

In the radiation zone:

$$\widetilde{\mathbf{B}}(\mathbf{r},\omega) = \nabla\times\widetilde{\mathbf{A}}(\mathbf{r},\omega) \approx ik\hat{\mathbf{r}}\times\widetilde{\mathbf{A}}(\mathbf{r},\omega)$$

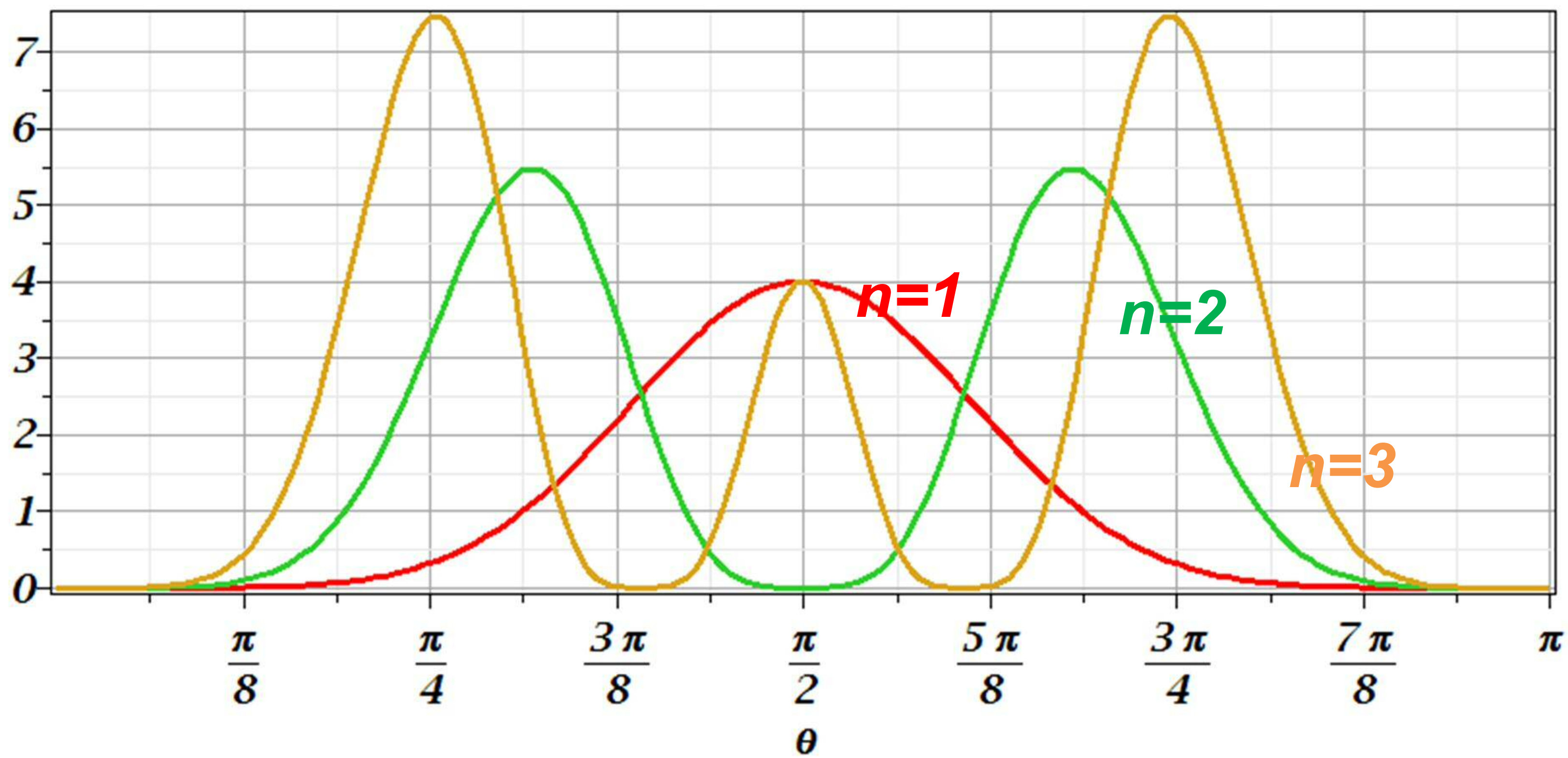
$$\widetilde{\mathbf{E}}(\mathbf{r},\omega) \approx -ikc\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\widetilde{\mathbf{A}}(\mathbf{r},\omega)\right)$$

$$\frac{dP}{d\Omega} = \frac{1}{2\mu_0} r^2\hat{\mathbf{r}}\cdot\Re\left(\widetilde{\mathbf{E}}(\mathbf{r},\omega)\times\widetilde{\mathbf{B}}^*(\mathbf{r},\omega)\right) = \frac{k^2c}{2\mu_0} r^2\left(\left|\widetilde{\mathbf{A}}(\mathbf{r},\omega)\right|^2 - \left|\hat{\mathbf{r}}\cdot\widetilde{\mathbf{A}}(\mathbf{r},\omega)\right|^2\right)$$

$$\frac{dP}{d\Omega} = \frac{\mu_0 c}{8\pi^2} I^2\left[\frac{\cos(kd\cos\theta)-\cos(kd)}{\sin\theta}\right]^2$$

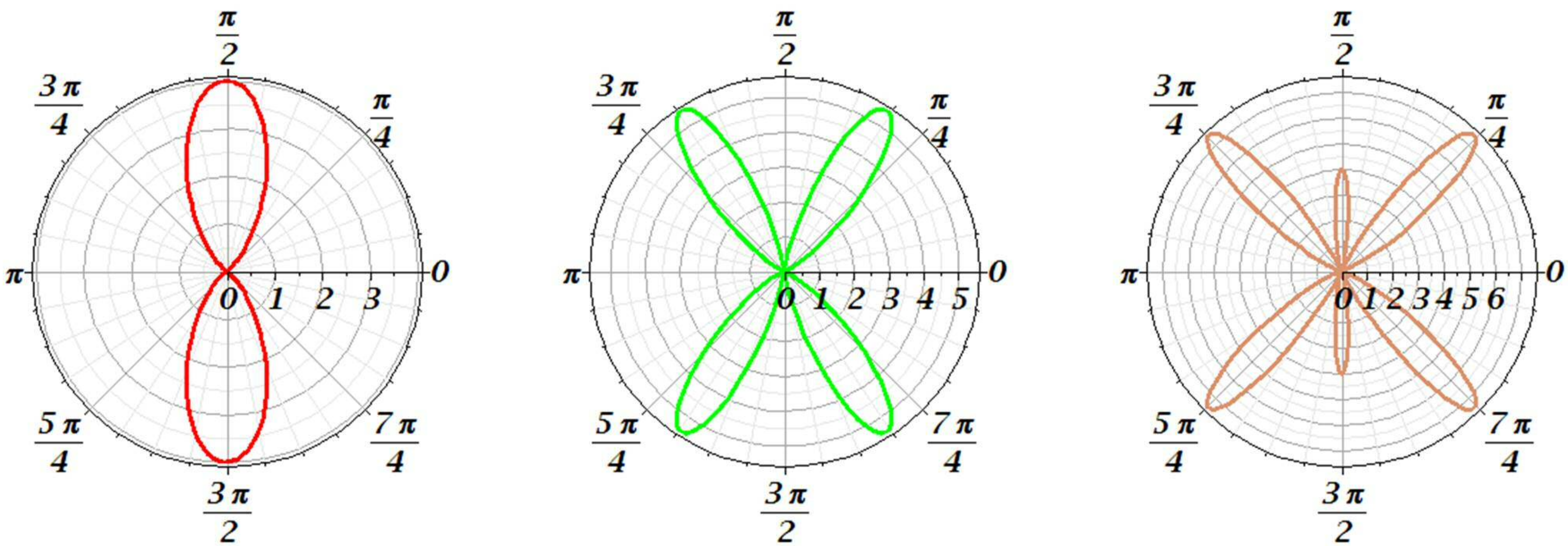
Consider antenna source -- continued

$$\frac{dP}{d\Omega} = \frac{\mu_0 c}{8\pi^2} I^2 \left[\frac{\cos(kd\cos\theta) - \cos(kd)}{\sin\theta} \right]^2$$



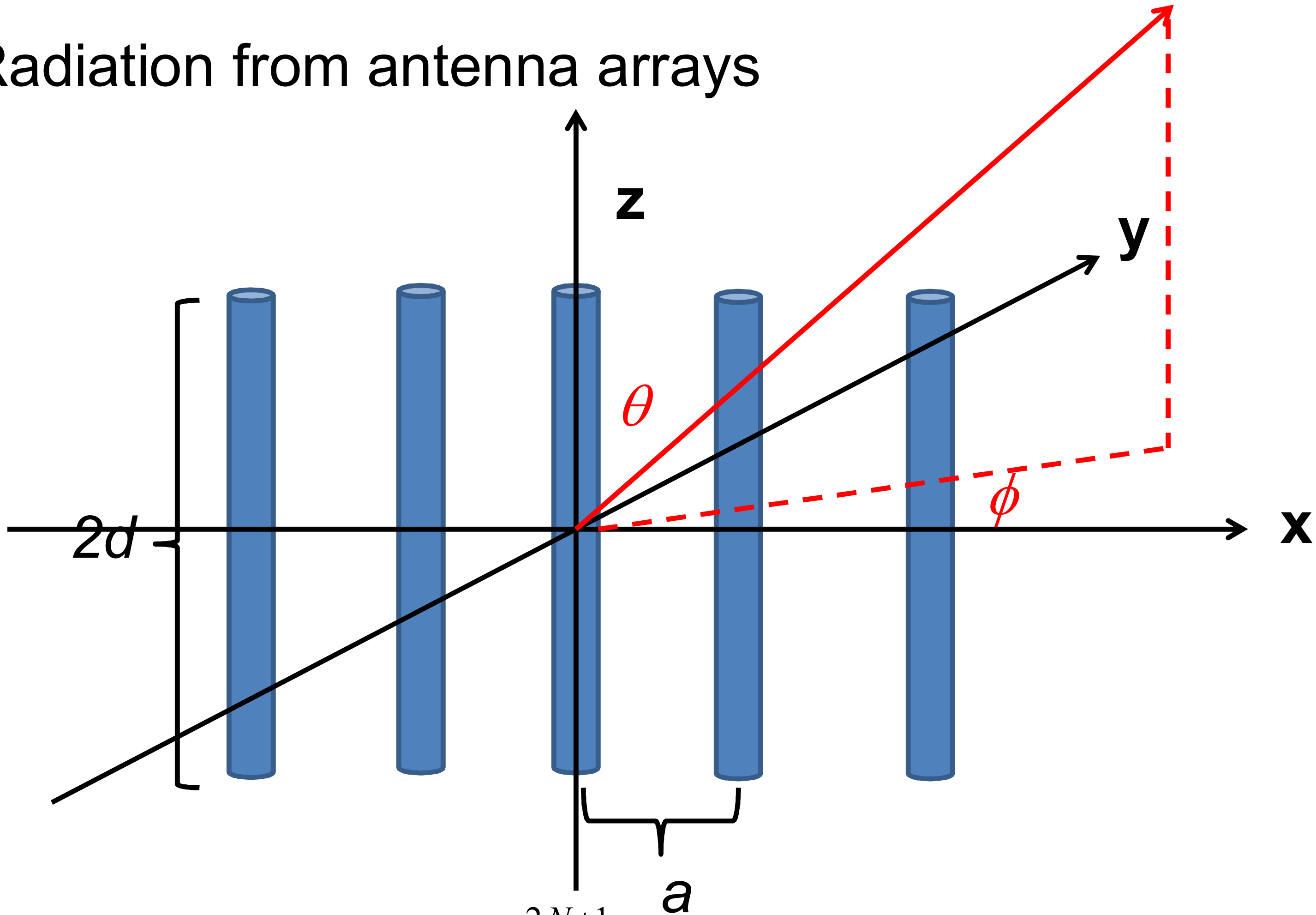
Consider antenna source -- continued

$$\frac{dP}{d\Omega} = \frac{\mu_0 c}{8\pi^2} I^2 \left[\frac{\cos(kd\cos\theta) - \cos(kd)}{\sin\theta} \right]^2$$



n=1 ***n=2*** ***n=3***

Radiation from antenna arrays



$$\tilde{\mathbf{J}}(\mathbf{r},\omega)=\hat{\mathbf{z}}I\sin\left(k\left(d-|z|\right)\right)\sum_{j=1}^{2N+1}\delta\left(x-\left(N+1-j\right)a\right)\delta(y)\quad \text{for } -d\le z\le d$$

$$k\equiv\frac{\omega}{c}=\frac{n\pi}{d};\qquad n=1,2,3....$$

Radiation from antenna arrays -- continued

Vector potential from array source:

$$\tilde{\mathbf{A}}(\mathbf{r},\omega)=\frac{\mu_0}{4\pi}\int d^3r'\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\tilde{\mathbf{J}}(\mathbf{r}',\omega)\approx\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int d^3r' e^{-ik\hat{\mathbf{r}}\cdot\mathbf{r}'}\tilde{\mathbf{J}}(\mathbf{r}',\omega)$$

$$\tilde{\mathbf{J}}(\mathbf{r},\omega)=\hat{\mathbf{z}}I\sin\left(k\left(d-|z|\right)\right)\sum_{j=1}^{2N+1}\delta\left(x-\left(N+1-j\right)a\right)\delta(y)\quad\text{for }-d\leq z\leq d$$

$$\tilde{\mathbf{A}}(\mathbf{r},\omega)\approx\hat{\mathbf{z}}\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\left(\sum_{j=-N}^{N}e^{-ikaj\sin\theta\cos\phi}\right)I\int_{-d}^{d}dz\,e^{-ikz\cos\theta}\;\sin\left(k\left(d-|z|\right)\right)$$

$$\sum_{j=-N}^{N}e^{-ikaj\sin\theta\cos\phi}=\frac{\sin\left(\frac{1}{2}ka(2N+1)\sin\theta\cos\phi\right)}{\sin\left(\frac{1}{2}ka\sin\theta\cos\phi\right)}$$

Radiation from antenna arrays -- continued

In the radiation zone:

$$\tilde{\mathbf{B}}(\mathbf{r},\omega) = \nabla \times \tilde{\mathbf{A}}(\mathbf{r},\omega) \approx ik\hat{\mathbf{r}} \times \tilde{\mathbf{A}}(\mathbf{r},\omega)$$

$$\tilde{\mathbf{E}}(\mathbf{r},\omega) \approx -ikc\hat{\mathbf{r}} \times \left(\hat{\mathbf{r}} \times \tilde{\mathbf{A}}(\mathbf{r},\omega)\right)$$

$$\frac{dP}{d\Omega} = \frac{1}{2\mu_0} r^2 \hat{\mathbf{r}} \cdot \Re\left(\tilde{\mathbf{E}}(\mathbf{r},\omega) \times \tilde{\mathbf{B}}^*(\mathbf{r},\omega)\right) = \frac{k^2 c r^2}{2\mu_0}\left(\left|\tilde{\mathbf{A}}(\mathbf{r},\omega)\right|^2 - \left|\hat{\mathbf{r}} \cdot \tilde{\mathbf{A}}(\mathbf{r},\omega)\right|^2\right)$$

$$\frac{dP}{d\Omega} = \frac{\mu_0 c}{8\pi^2} I^2 \left[\frac{\cos(kd\cos\theta) - \cos(kd)}{\sin\theta}\right]^2 \left[\frac{\sin\left(\frac{1}{2}ka(2N+1)\sin\theta\cos\phi\right)}{\sin\left(\frac{1}{2}ka\sin\theta\cos\phi\right)}\right]^2$$