PHY 712 Electrodynamics
11-11:50 AM MWF Olin 107

Plan for Lecture 34:

Read material from Chap. 15

1. **Radiation due collisions of charged particles**
2. **Braking radiation – "Bremsstrahlung"**

	03-18(Mon)	*APS Meeting*	(no class)	Exam
	03-20(Wed)	*APS Meeting*	(no class)	Exam
	03-22(Fri)	*APS Meeting*	(no class)	Exam
25	03-25(Mon)	Chap. 11	Lorentz transformations	#17
26	03-27(Wed)	Chap. 11	Transformations between electromagnetic fields	#18
27	03-28(Thur)	Chap. 11	Liénard-Wiechert potentials revisited	
	03-29(Fri)	*Good Friday*	(no class)	
28	04-01(Mon)	Chap. 14	Radiation by accelerated charges	#19
29	04-03(Wed)	Chap. 14	Radiation by accelerated charges	#20
30	04-05(Fri)	Chap. 14	Synchrotron radiation spectrum	#21
31	04-08(Mon)	Chap. 14	Synchrotron and other radiation sources	#22
32	04-10(Wed)	Chap. 15	Radiation due to collisions of charged particles	
33	04-12(Fri)	Chap. 15	Radiation due to energy loss processes	#23
	04-15(Mon)		(no class -- presentation preparation)	
	04-17(Wed)		(no class -- presentation preparation)	
	04-19(Fri)		(no class -- presentation preparation)	
34	04-22(Mon)	Chap. 15	Radiation due to energy loss processes	#24
35	04-24(Wed)	Review	Radiation from antennas	#25
36	04-26(Fri)	Review	Comprehensive review	
	04-29(Mon)		Student presentations I	
	05-01(Wed)		Student presentations II	
	05-02(Thurs)		Student presentations III	
	05-03(Fri)	Final exam	Take-home exam available -- due 05/10/2013	

Signup for presentations:

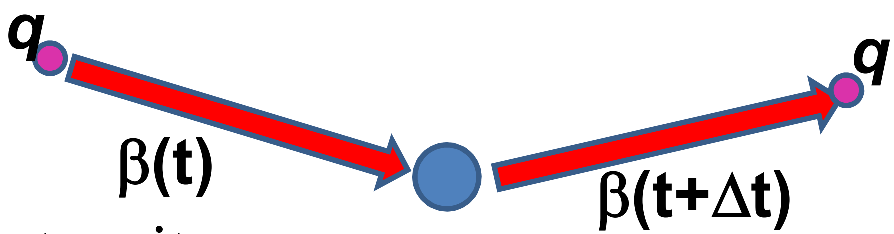
Schedule for PHY712 presentations -- please enter your own name replacing "presenter" and also list your presentation title replacing "title". Of course, please do not change other student entries without their permission.

Monday April 29, 2013
11:00 - 11:15 -- (presenter) (title)
11:15 - 11:30 -- (presenter) (title)
11:30 - 11:45 -- (presenter) (title)

Wednesday May 1, 2013
11:00 - 11:15 -- Katelyn Goetz Space Charge Limited Current Measurements
11:15 - 11:30 -- (presenter) (title)
11:30 - 11:45 -- Xiaohua Liu Electron paramagnetic resonance

Thursday May 2, 2013
9:00 - 9:15 -- (David Montgomery) (Auroras)
9:15 - 9:30 -- Ryan Godwin Drude Oscillator Model
9:30 - 9:45 -- David Harrison Charges on a sphere
9:45 - 10:00 -- (presenter) (title)

Radiation due to collisions of charged particles



Intensity:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left| \int dt\, e^{i\omega\left(t - \hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c\right)} \frac{d}{dt}\left[\frac{\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}\right)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}}\right]\right|^2$$

Note that $\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}\right) = \left(\hat{\boldsymbol{\varepsilon}}_1\cdot\boldsymbol{\beta}\right)\hat{\boldsymbol{\varepsilon}}_1 + \left(\hat{\boldsymbol{\varepsilon}}_2\cdot\boldsymbol{\beta}\right)\hat{\boldsymbol{\varepsilon}}_2$

For a collision of duration τ emitting radiation with polarization $\hat{\boldsymbol{\varepsilon}}$ and frequency $\omega \to 0$:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left| \hat{\boldsymbol{\varepsilon}}\cdot\left(\frac{\boldsymbol{\beta}(t+\tau)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t+\tau)} - \frac{\boldsymbol{\beta}(t)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t)}\right)\right|^2$$

Radiation due to collisions -- continued

For a collision of duration τ emitting radiation with polarization $\hat{\boldsymbol{\varepsilon}}$ and frequency $\omega \to 0$:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\hat{\boldsymbol{\varepsilon}}\cdot\left(\frac{\boldsymbol{\beta}(t+\tau)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t+\tau)} - \frac{\boldsymbol{\beta}(t)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t)}\right)\right|^2$$

Non-relativistic limit:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\hat{\boldsymbol{\varepsilon}}\cdot(\Delta\boldsymbol{\beta})\right|^2 \qquad \Delta\boldsymbol{\beta} \equiv \boldsymbol{\beta}(t+\tau) - \boldsymbol{\beta}(t)$$

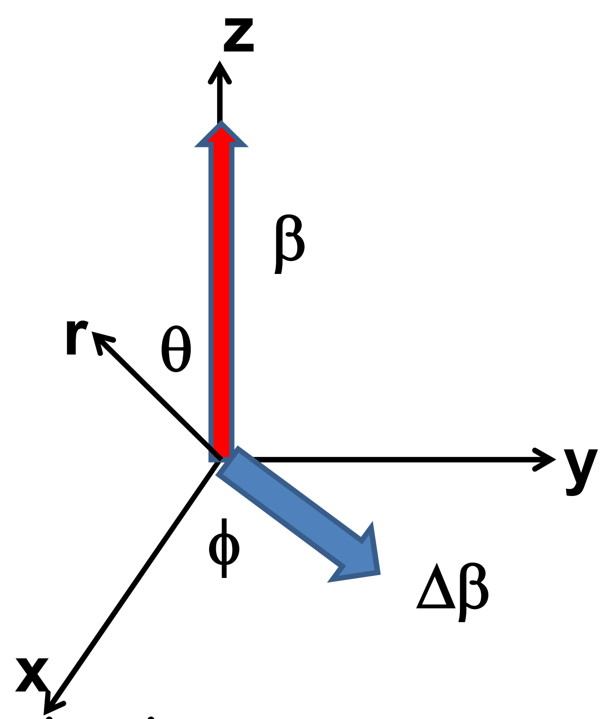
Relativistic collision with small $|\Delta\boldsymbol{\beta}|$:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\hat{\boldsymbol{\varepsilon}}\cdot\left(\frac{\Delta\boldsymbol{\beta} + \hat{\mathbf{r}}\times(\boldsymbol{\beta}\times\Delta\boldsymbol{\beta})}{(1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta})^2}\right)\right|^2$$

Radiation during collisions -- continued

Relativistic collision with small $|\Delta\boldsymbol{\beta}|$:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\hat{\boldsymbol{\varepsilon}}\cdot\left(\frac{\Delta\boldsymbol{\beta} + \hat{\mathbf{r}}\times(\boldsymbol{\beta}\times\Delta\boldsymbol{\beta})}{(1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta})^2}\right)\right|^2$$



Assume for simplicity that Δβ is perpendicular to ***r*** and *β* plane.

Expressions (averaging over ϕ) for $\|$ or $\perp$ polarization :

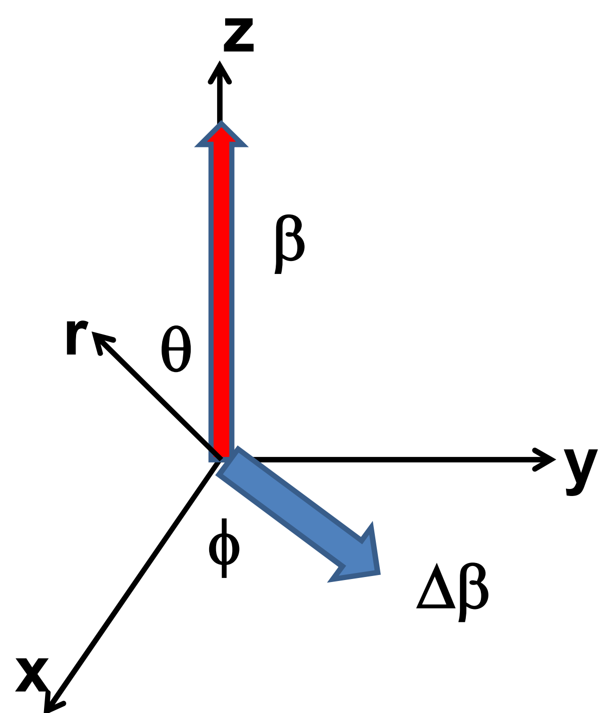
$$\frac{d^2 I_{\|}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c}|\Delta\boldsymbol{\beta}|^2 \frac{(\beta - \cos\theta)^2}{(1-\beta\cos\theta)^4}$$

polarization in ***r*** and *β* plane

$$\frac{d^2 I_{\perp}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c}|\Delta\boldsymbol{\beta}|^2 \frac{1}{(1-\beta\cos\theta)^2}$$

polarization perpendicular to ***r*** and *β* plane

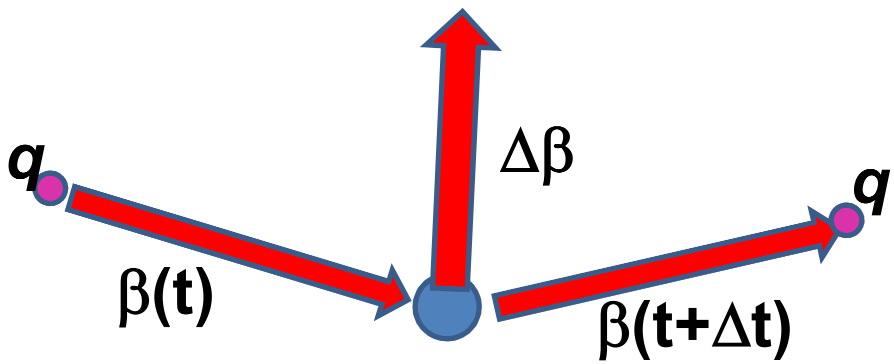
Radiation during collisions -- continued



Relativistic collision at low ω and with small $|\Delta\boldsymbol{\beta}|$ and $\Delta\boldsymbol{\beta}$ perpendicular to plane of $\hat{\mathbf{r}}$ and $\boldsymbol{\beta}$, as a function of θ where $\hat{\mathbf{r}}\cdot\boldsymbol{\beta} = \beta\cos\theta$ -- continued:

$$\frac{dI}{d\omega} = \int d\Omega \left(\frac{d^2 I_{\parallel}}{d\omega d\Omega} + \frac{d^2 I_{\perp}}{d\omega d\Omega} \right) = \frac{2}{3\pi} \; \frac{q^2}{c} \gamma^2 \, |\Delta\boldsymbol{\beta}|^2$$

Estimation of Δβ



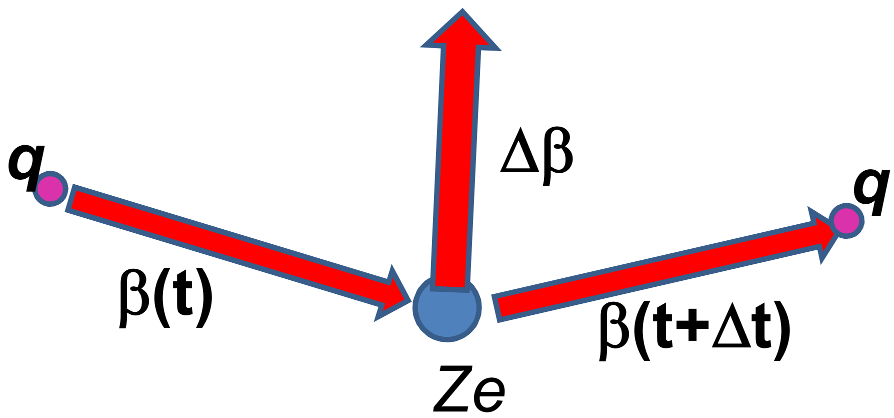
Momentum transfer:

$$Q \equiv \left|\mathbf{p}(t+\tau)-\mathbf{p}(t)\right| \approx \gamma M c^2 \left|\mathbf{\Delta\beta}\right|$$

mass of particle having charge *q*

$$\frac{dI}{d\omega} = \frac{2}{3\pi}\;\frac{q^2}{c}\gamma^2\left|\mathbf{\Delta\beta}\right|^2 \approx \frac{2}{3\pi}\;\frac{q^2}{M^2c^3}Q^2$$

Estimation of Δβ -- for the case of Rutherford scattering



Assume that target nucleus (charge Ze) has mass >>M;

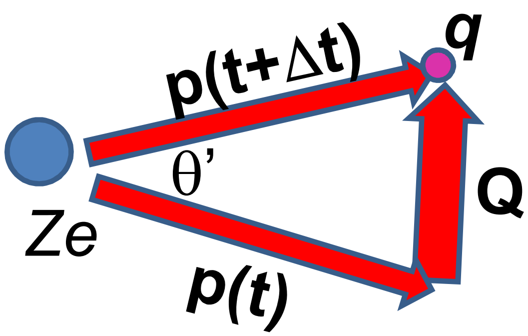
Rutherford scattering cross-section:

$$\frac{d\sigma}{d\Omega} = \left(\frac{2Zeq}{pv}\right)^2 \frac{1}{\left(2\sin(\theta'/2)\right)^4}$$

Assuming elastic scattering:

$$Q^2 = \left(2p\sin(\theta'/2)\right)^2 = 2p^2\left(1-\cos\theta'\right)$$

Case of Rutherford scattering -- continued



Rutherford scattering cross-section:

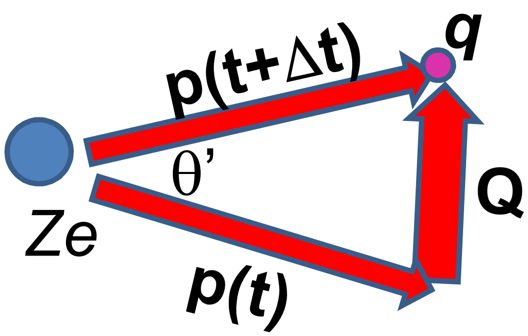
$$\frac{d\sigma}{d\Omega} = \left(\frac{2Zeq}{pv}\right)^2 \frac{1}{\left(2\sin(\theta'/2)\right)^4}$$

$$\frac{d\sigma}{dQ} = \int d\phi' \frac{d\sigma}{d\Omega}\frac{d\Omega}{dQ}$$

$$Q^2 = \left(2p\sin(\theta'/2)\right)^2 = 2p^2\left(1-\cos\theta'\right)$$

$$\Rightarrow \text{(Up to a factor of 2:)} \quad \frac{d\sigma}{dQ} = 8\pi\left(\frac{Zeq}{\beta c}\right)^2 \frac{1}{Q^3}$$

Case of Rutherford scattering -- continued



Differential radiation cross section:

$$\frac{d^2\chi}{d\omega dQ} = \frac{dI}{d\omega}\frac{d\sigma}{dQ} = \left(\frac{2}{3\pi}\frac{q^2}{M^2c^3}Q^2\right)\left(8\pi\left(\frac{Zeq}{\beta c}\right)^2\frac{1}{Q^3}\right)$$

$$= \frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2\frac{1}{\beta^2}\frac{1}{Q}$$

Differential radiation cross section -- continued

Integrating over momentum transfer

$$\frac{d\chi}{d\omega} = \int_{Q_{\min}}^{Q_{\max}} dQ \frac{d^2\chi}{d\omega dQ} = \frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2 \frac{1}{\beta^2}\ln\left(\frac{Q_{\max}}{Q_{\min}}\right)$$

Comment on frequency dependence --

Original expression for radiation intensity:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\int dt\, e^{i\omega\left(t - \hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c\right)} \frac{d}{dt}\left[\frac{\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}\right)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}}\right]\right|^2$$

In the previous derivations, we have assumed that

$$\omega\left(t - \hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c\right) << 1.$$

$$\omega\left(t - \hat{\mathbf{r}}\cdot\mathbf{R}_q(t)/c\right) = \omega\left(t - \hat{\mathbf{r}}\cdot\int_0^t dt'\boldsymbol{\beta}(t')\right) \approx \omega\tau\left(1 - \hat{\mathbf{r}}\cdot\langle\boldsymbol{\beta}\rangle\right)$$

Differential radiation cross section -- continued

Radiation cross section in terms of momentum transfer

$$\frac{d\chi}{d\omega}=\int_{Q_{\min}}^{Q_{\max}} dQ\,\frac{d^2\chi}{d\omega dQ}=\frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2\frac{1}{\beta^2}\ln\left(\frac{Q_{\max}}{Q_{\min}}\right)$$

Note that: $Q^2=2p^2(1-\cos\theta') \quad \Rightarrow Q_{\max}=2p$

In general, $Q_{\min}$ is determined by the collision time

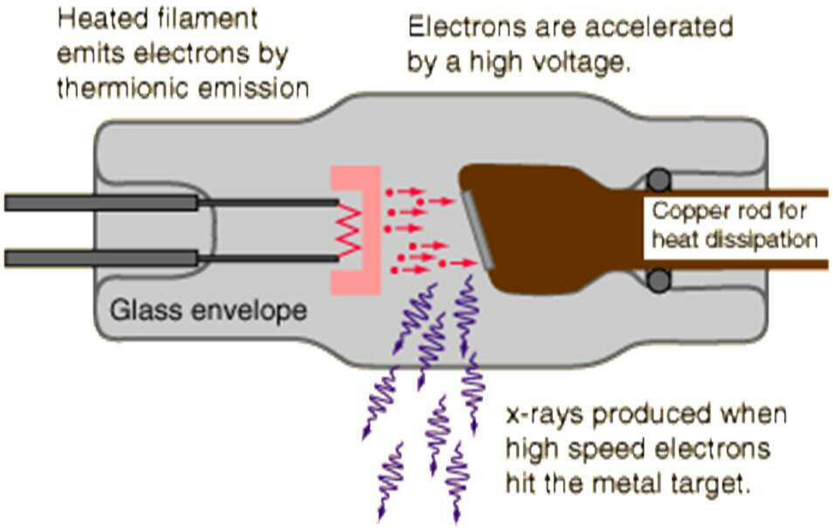
condition $\omega\tau<1 \quad \Rightarrow Q_{\min}\approx\dfrac{2Zeq\omega}{v^2}$

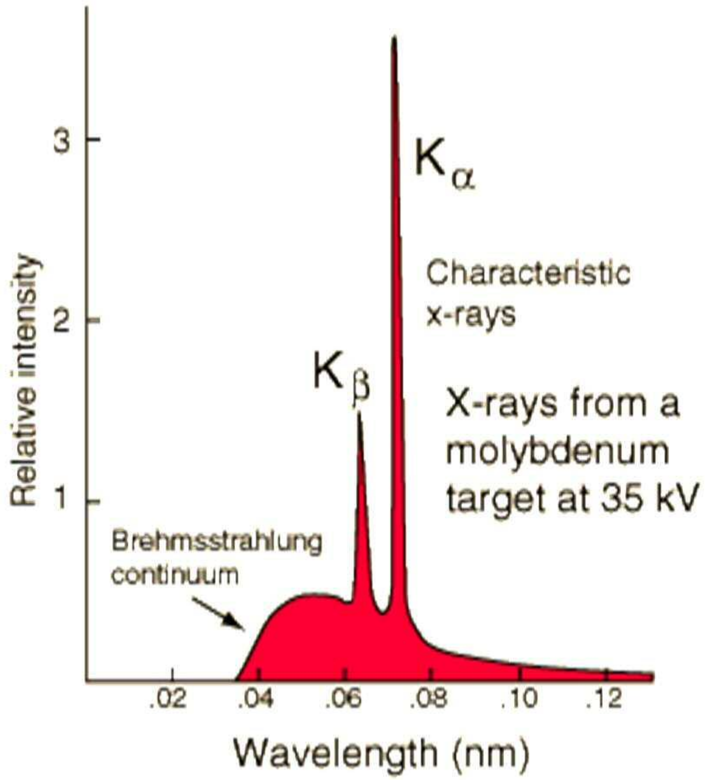
Radiation cross section for classical non-relativistic process

$$\frac{d\chi}{d\omega}=\frac{16}{3}\frac{(Ze)^2}{c}\left(\frac{q^2}{Mc^2}\right)^2\frac{1}{\beta^2}\ln\left(\frac{\lambda Mv^3}{Zeq\omega}\right)$$

X-ray tube

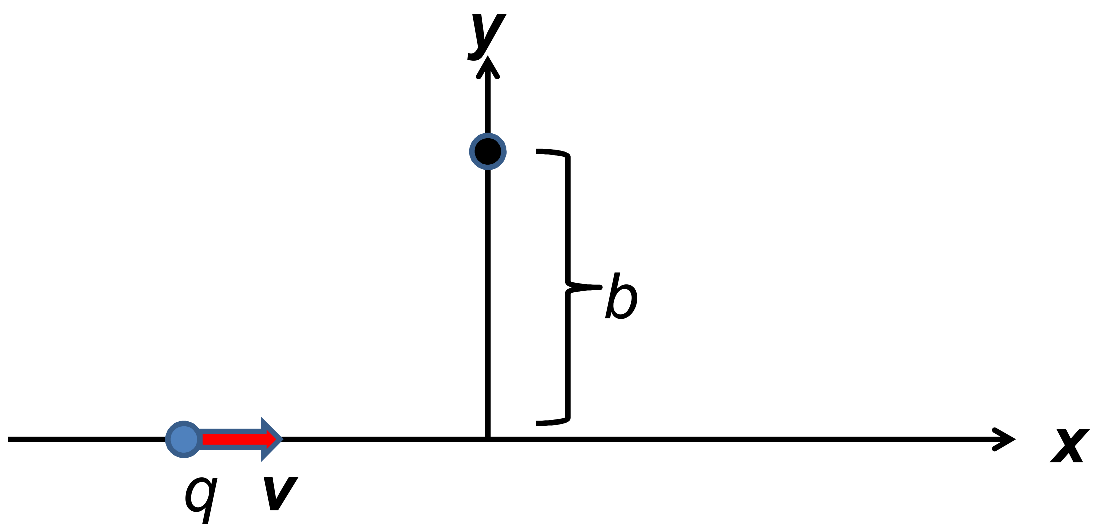
http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/xtube.html





http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/xrayc.html#c1

Virtual "quanta" method; Weizsäker-Williams treatment

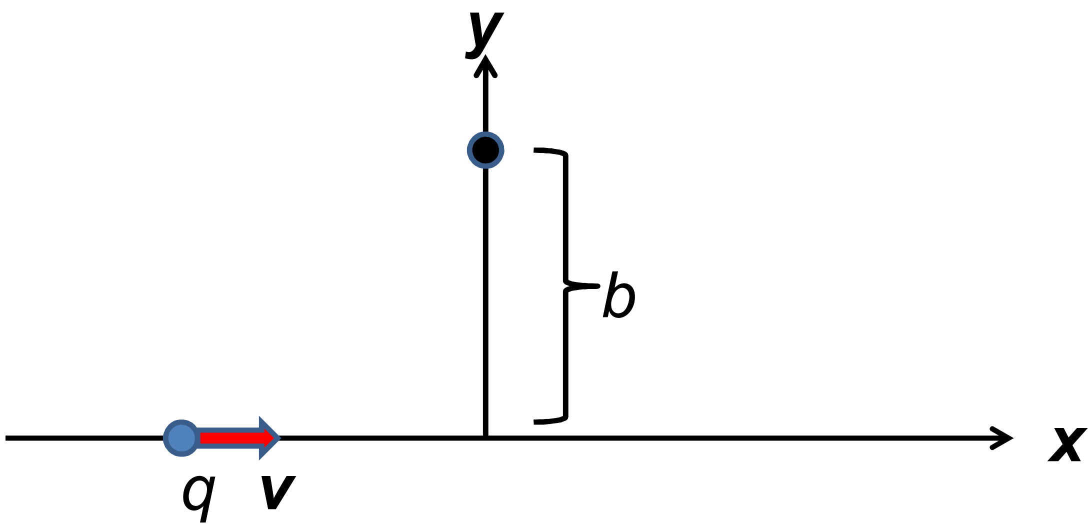


Electric and magnetic fields at impact parameter b

$$E_x(t) = q\frac{\gamma v t}{\left(b^2 + (\gamma v t)^2\right)^{3/2}} \qquad E_y(t) = q\frac{\gamma b}{\left(b^2 + (\gamma v t)^2\right)^{3/2}}$$

$$B_z(t) = q\frac{\gamma \beta b}{\left(b^2 + (\gamma v t)^2\right)^{3/2}}$$

Virtual "quanta" method; Weizsäker-Williams treatment



Intensity of radiation at impact parameter b

$$\frac{dI}{d\omega}(\omega,b)=\frac{c}{2\pi}\left(\left|\widetilde{E}_x(\omega,b)\right|^2+\left|\widetilde{E}_y(\omega,b)\right|^2\right)$$

where $\widetilde{E}_x(\omega,b)\equiv\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}dt\,e^{i\omega t}E_x(t,b)$

Desired intensity must be integrated over parameter b

$$\frac{dI}{d\omega}(\omega)=2\pi\int_{b_{\min}}^{\infty}b\,db\frac{dI}{d\omega}(\omega,b)=\frac{2q^2}{\pi c}\left(\frac{c}{v}\right)^2\left[xK_0(x)K_1(x)-\frac{v^2x^2}{2c^2}\left(K_1{}^2(x)-K_0{}^2(x)\right)\right]$$

$$x\equiv\frac{\omega b_{\min}}{\gamma v}$$