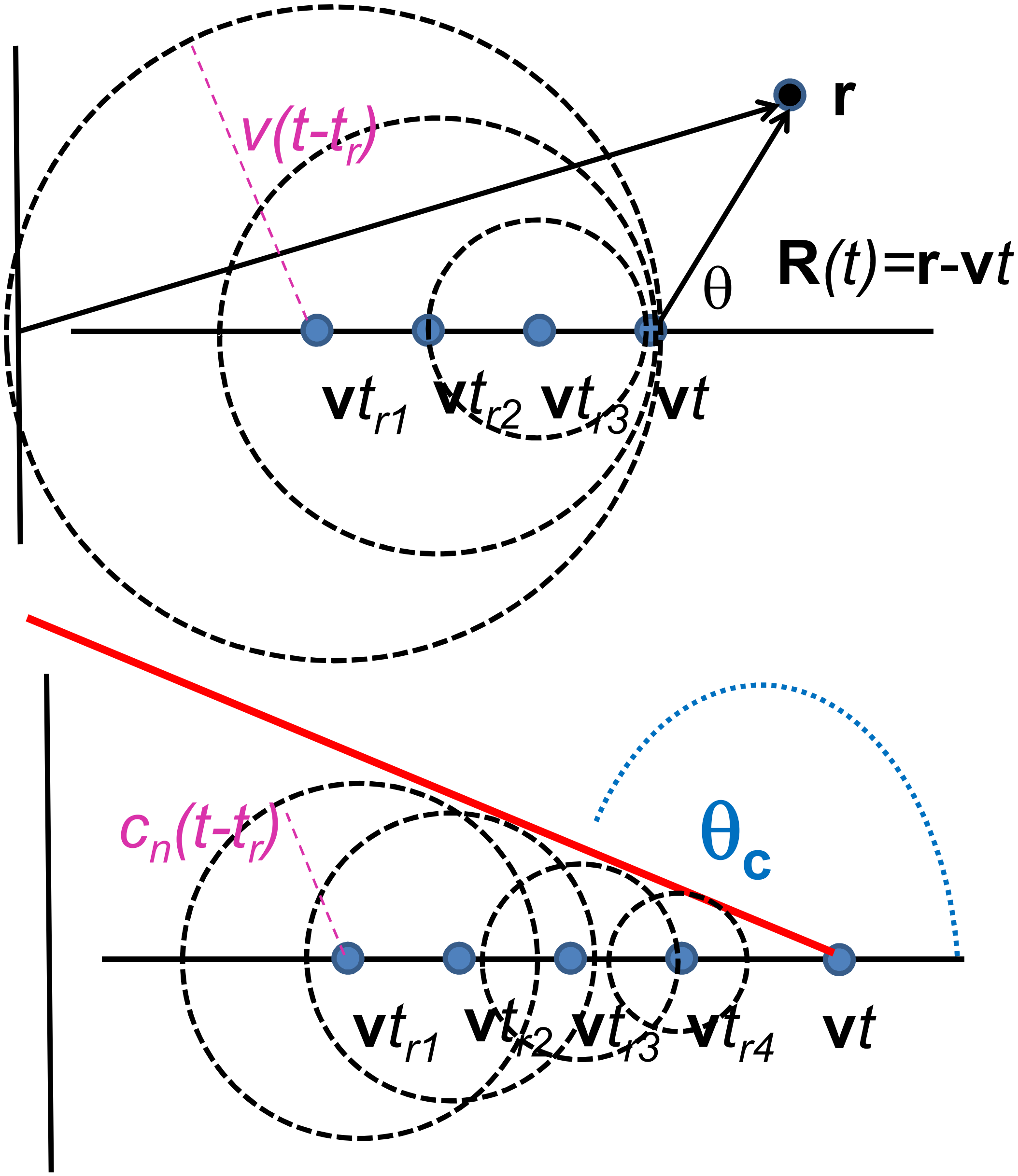
PHY 712 Electrodynamics
11-11:50 AM MWF Olin 107

Plan for Lecture 33:

Read material from Chap. 13 & 15

1. **Cherenkov radiation**
2. **Bremsstrahlung**

	03-13(Wed)	*Spring Break*		
	03-15(Fri)	*Spring Break*		
	03-18(Mon)	*APS Meeting*	(no class)	Exam
	03-20(Wed)	*APS Meeting*	(no class)	Exam
	03-22(Fri)	*APS Meeting*	(no class)	Exam
25	03-25(Mon)	Chap. 11	Lorentz transformations	#17
26	03-27(Wed)	Chap. 11	Transformations between electromagnetic fields	#18
27	03-28(Thur)	Chap. 11	Liénard-Wiechert potentials revisited	
	03-29(Fri)	*Good Friday*	(no class)	
28	04-01(Mon)	Chap. 14	Radiation by accelerated charges	#19
29	04-03(Wed)	Chap. 14	Radiation by accelerated charges	#20
30	04-05(Fri)	Chap. 14	Synchrotron radiation spectrum	#21
31	04-08(Mon)	Chap. 14	Synchrotron and other radiation sources	#22
32	04-10(Wed)	Chap. 15	Radiation due to collisions of charged particles	
33	04-12(Fri)	Chap. 15	Radiation due to energy loss processes	#23
	04-15(Mon)		(no class -- presentation preparation)	
	04-17(Wed)		(no class -- presentation preparation)	
	04-19(Fri)		(no class -- presentation preparation)	
34	04-22(Mon)	Chap. 15	Radiation due to energy loss processes	
35	04-24(Wed)			
36	04-26(Fri)			
	04-29(Mon)		Student presentations I	
	05-01(Wed)		Student presentations II	
	05-02(Thurs)		Student presentations III	

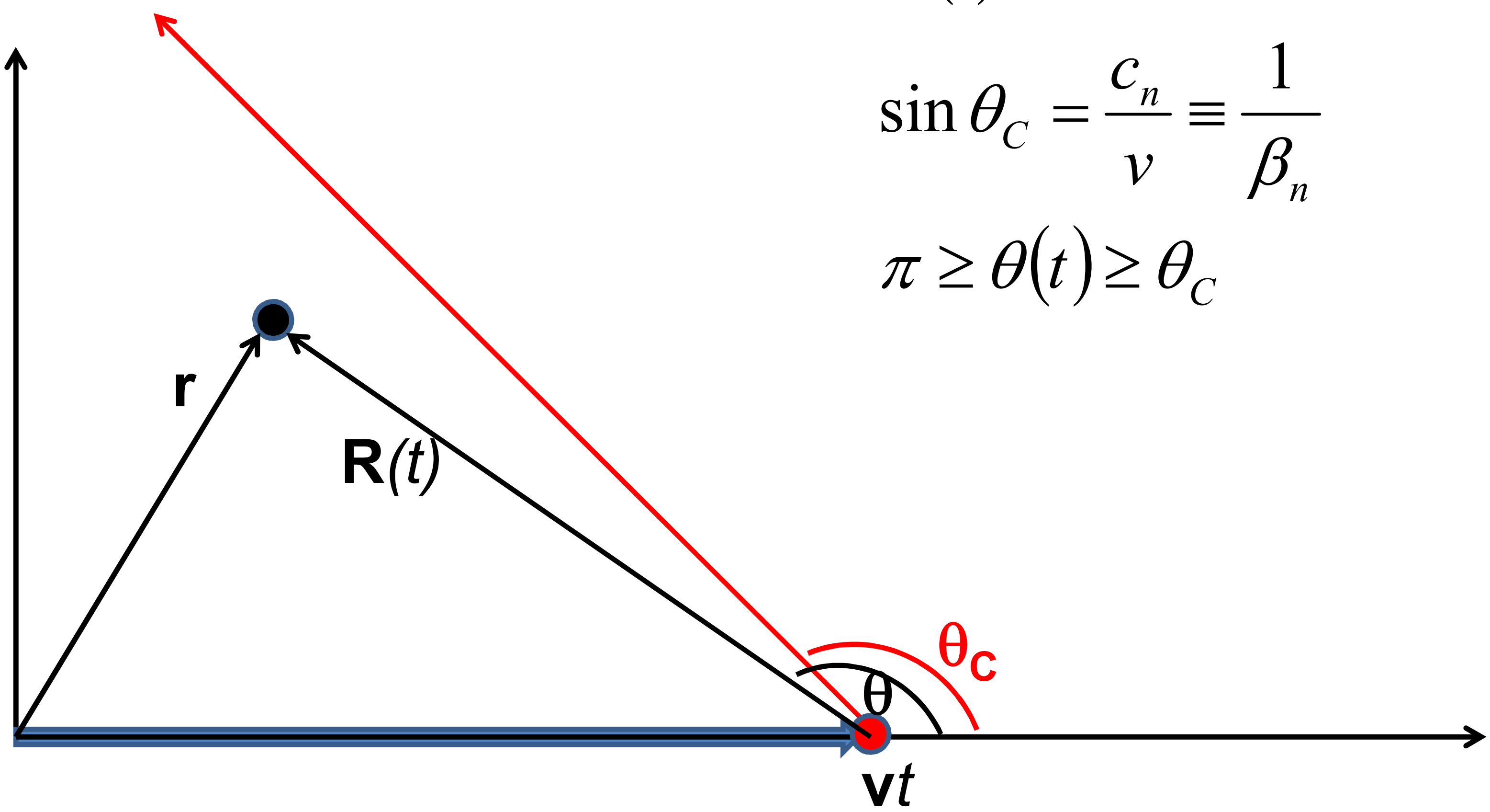

r
v(t-t_r)
R(t)=r-vt
θ
vt_r1 vt_r2 vt_r3 vt
c_n(t-t_r)
θ_c
vt_r1 vt_r2 vt_r3 vt_r4 vt

Cherenkov radiation

$$\mathbf{R}(t) = \mathbf{r} - \mathbf{v}t$$

$$\sin\theta_C = \frac{c_n}{v} \equiv \frac{1}{\beta_n}$$

$$\pi \geq \theta(t) \geq \theta_C$$

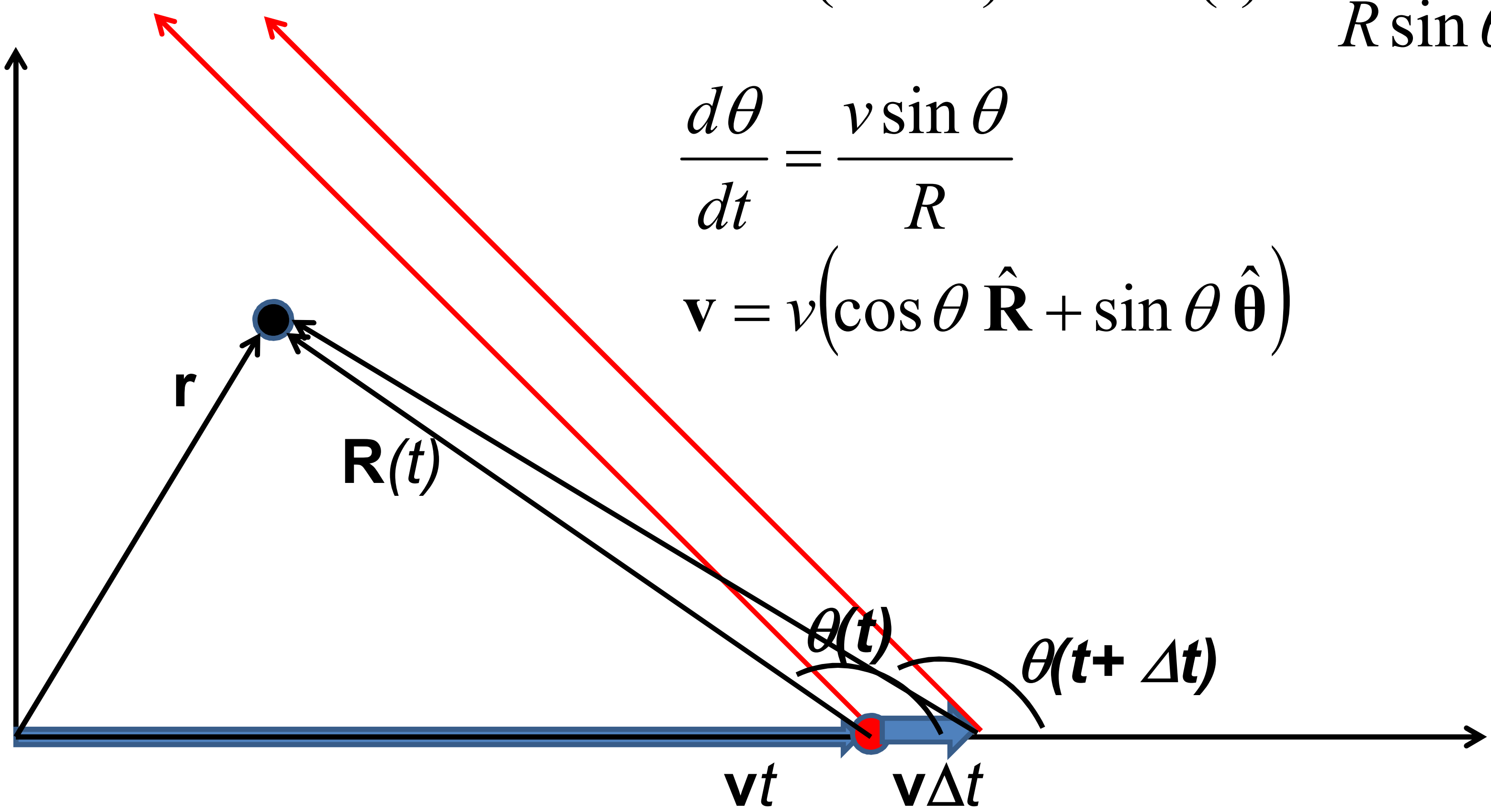


Cherenkov radiation

$$\cot\theta(t+\Delta t) - \cot\theta(t) = \frac{v\Delta t}{R\sin\theta}$$

$$\frac{d\theta}{dt} = \frac{v\sin\theta}{R}$$

$$\mathbf{v} = v\left(\cos\theta\,\hat{\mathbf{R}} + \sin\theta\,\hat{\boldsymbol{\theta}}\right)$$



Liénard-Wiechert potential solutions found previously:

$$\Phi(\mathbf{r},t) = \frac{2q}{\varepsilon} \frac{1}{R(t)\sqrt{1-\beta_n{}^2 \sin^2\theta(t)}} \Theta(\cos\theta_C - \cos\theta(t))$$

$$\mathbf{A}(\mathbf{r},t) = \frac{2q}{\varepsilon}\sqrt{\mu\varepsilon} \frac{\boldsymbol{\beta}_n}{R(t)\sqrt{1-\beta_n{}^2 \sin^2\theta(t)}} \Theta(\cos\theta_C - \cos\theta(t))$$

Electric and magnetic fields:

$$\mathbf{E}(\mathbf{r},t) = -\nabla\Phi(\mathbf{r},t) - \frac{1}{c}\frac{\partial \mathbf{A}(\mathbf{r},t)}{\partial t}$$

$$\mathbf{B}(\mathbf{r},t) = \nabla \times \mathbf{A}(\mathbf{r},t)$$

Intermediate steps:

$$\text{Using instantaneous polar coordinates}: \quad \nabla \equiv \hat{\mathbf{R}}\frac{\partial}{\partial R} + \hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial}{\partial \theta}$$

$$\nabla\Theta\left(\cos\theta_C - \cos\theta(t)\right) = \delta\left(\cos\theta_C - \cos\theta(t)\right)\frac{\sin\theta(t)}{R(t)}\hat{\boldsymbol{\theta}}$$

$$\frac{\partial\Theta\left(\cos\theta_C - \cos\theta(t)\right)}{\partial t} = \delta\left(\cos\theta_C - \cos\theta(t)\right)\frac{v\sin^2\theta(t)}{R(t)}$$

When the dust clears – apparently you should get the following (although I don't … yet …)

$$\mathbf{E}(\mathbf{r},t) = -\frac{2q}{\varepsilon}\frac{\hat{\mathbf{R}}(t)\left(\beta_n{}^2 - 1\right)}{\left(R(t)\right)^2\left(1-\beta_n{}^2\sin^2\theta(t)\right)^{3/2}}\Theta\left(\cos\theta_C - \cos\theta(t)\right)$$

$$+\frac{2q}{\varepsilon}\frac{\hat{\mathbf{R}}(t)\left(\beta_n{}^2 - 1\right)^{1/2}/\beta_n}{\left(R(t)\right)^2\left(1-\beta_n{}^2\sin^2\theta(t)\right)^{1/2}}\delta\left(\cos\theta_C - \cos\theta(t)\right)$$

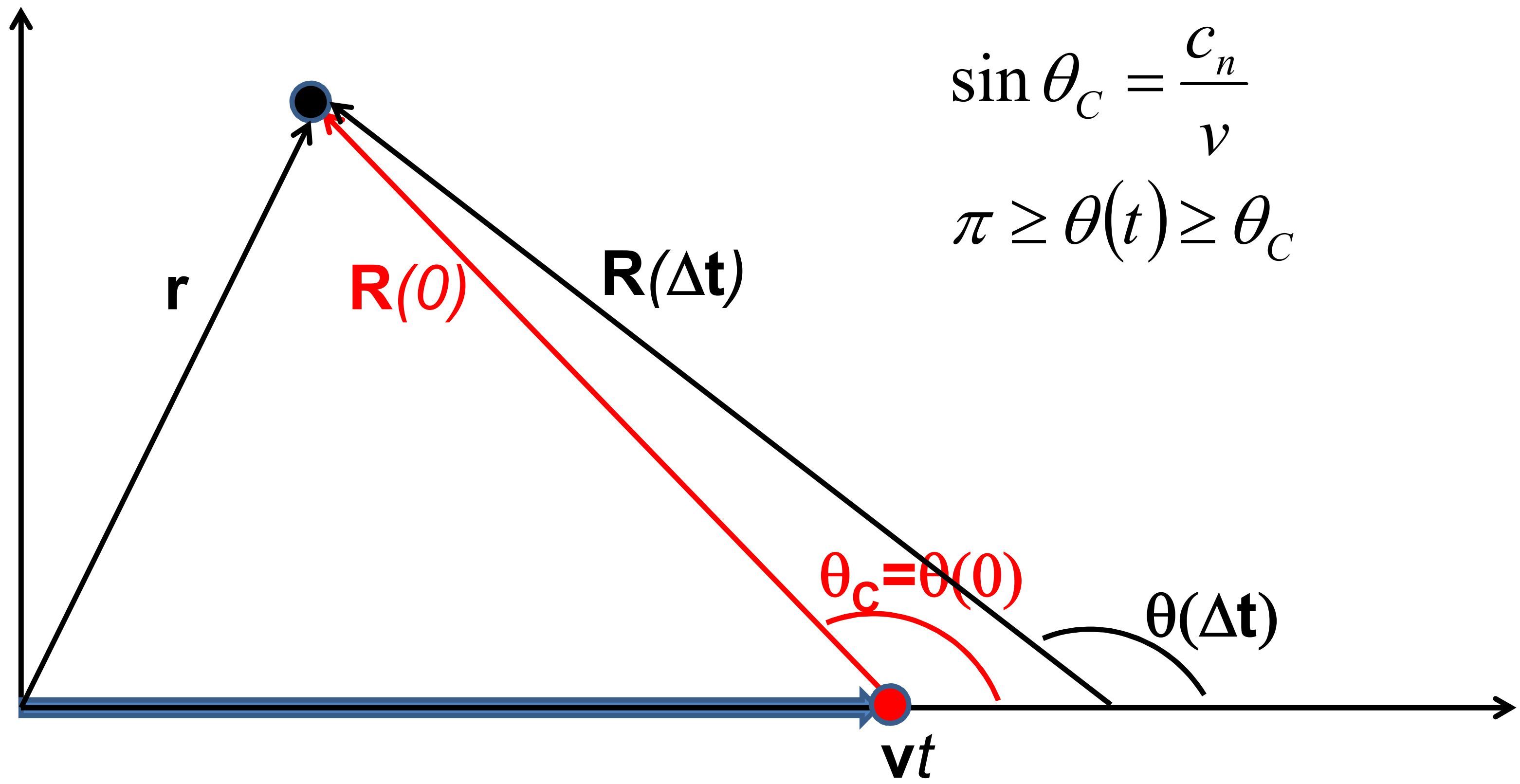
$$\mathbf{B}(\mathbf{r},t) = -\beta_n \sin\theta(t)\left(\hat{\boldsymbol{\theta}}(t)\times\mathbf{E}(\mathbf{r},t)\right)$$

Cherenkov radiation observed near the angle θ_c

$$\mathbf{R}(t) = \mathbf{r} - \mathbf{v}t$$

$$\sin\theta_C = \frac{c_n}{v}$$

$$\pi \geq \theta(t) \geq \theta_C$$

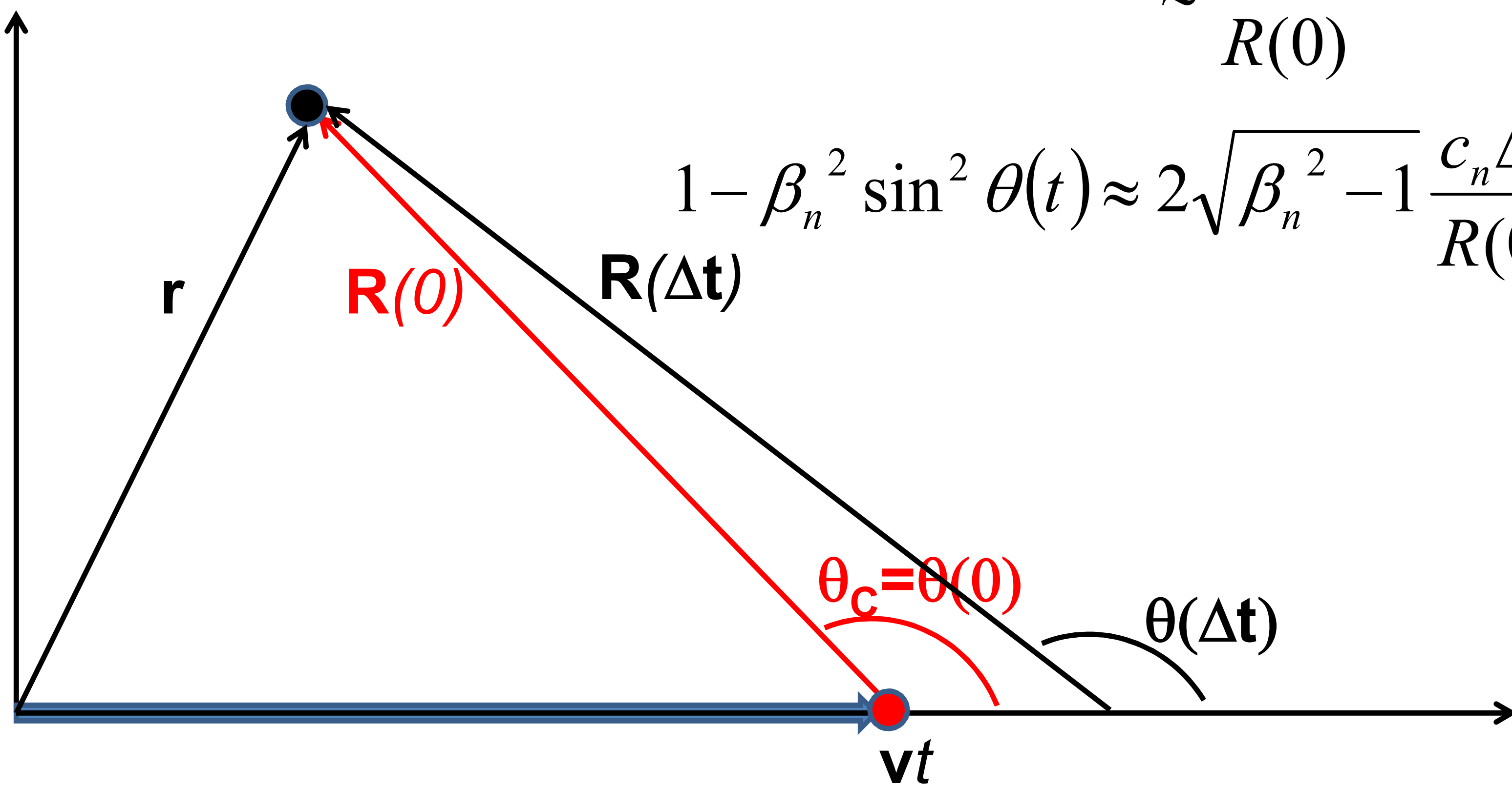


Cherenkov radiation observed near the angle θ_c -- continued

$$\cos\theta_C - \cos\theta(\Delta t) \approx \sin\theta_C \; \theta(\Delta t)$$

$$\approx \frac{c_n \Delta t}{R(0)}$$

$$1-\beta_n^{\,2}\sin^2\theta(t) \approx 2\sqrt{\beta_n^{\,2}-1}\,\frac{c_n\Delta t}{R(0)}$$



Cherenkov radiation observed near the angle θ_c -- continued

$$\mathbf{E}(\mathbf{r},t) = -\frac{2q}{\varepsilon}\frac{\hat{\mathbf{R}}(t)\left(\beta_n^{\ 2}-1\right)}{\left(R(t)\right)^2\left(1-\beta_n^{\ 2}\sin^2\theta(t)\right)^{3/2}}\Theta\left(\cos\theta_C - \cos\theta(t)\right)$$

$$+\frac{2q}{\varepsilon}\frac{\hat{\mathbf{R}}(t)\left(\beta_n^{\ 2}-1\right)^{1/2}/\beta_n}{\left(R(t)\right)^2\left(1-\beta_n^{\ 2}\sin^2\theta(t)\right)^{1/2}}\delta\left(\cos\theta_C - \cos\theta(t)\right)$$

$$\mathbf{B}(\mathbf{r},t) = -\beta_n \sin\theta(t)\left(\hat{\boldsymbol{\theta}}(t)\times\mathbf{E}(\mathbf{r},t)\right)$$

$$\Delta t \rightarrow t$$

$$\mathbf{E}(\mathbf{r},t) \approx -\frac{2q}{\varepsilon}\hat{\mathbf{R}}(0)\frac{\left(\beta_n^{\ 2}-1\right)^{1/4}}{\left(2c_n^3 R(0)\right)^{1/2}}\left[t^{-1/2}\delta(t) - \frac{1}{2}t^{-3/2}\Theta(t)\right]$$

$$\mathbf{B}(\mathbf{r},t) = -\left(\hat{\boldsymbol{\theta}}(0)\times\mathbf{E}(\mathbf{r},t)\right)$$

Cherenkov radiation observed near the angle θ_c -- continued

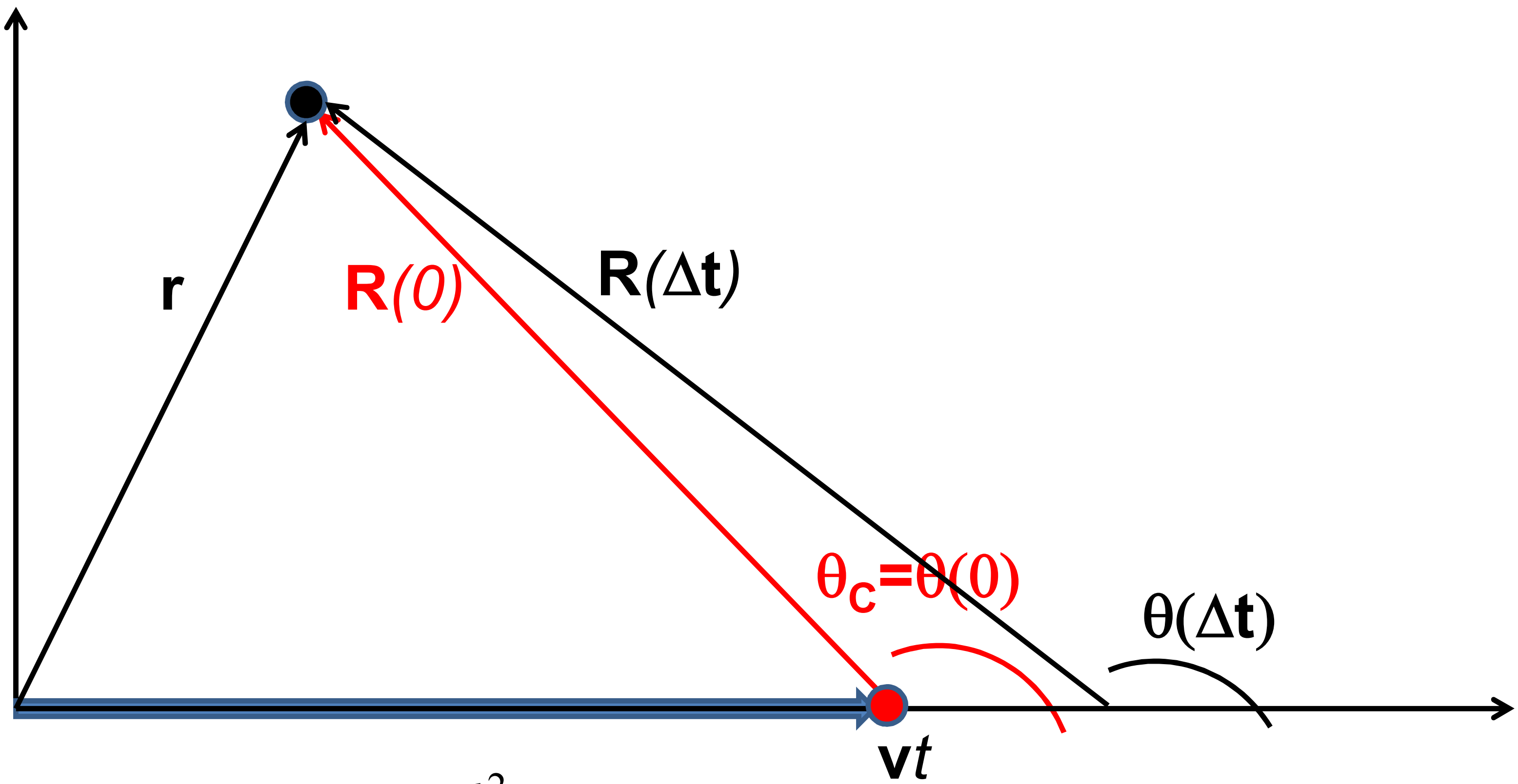
Spectralanalysis:

$$\tilde{\mathbf{E}}(\omega) = -\frac{2q}{\varepsilon}\hat{\mathbf{R}}(0)\frac{\left(\beta_n{}^2-1\right)^{1/4}}{\left(2c_n^3 R(0)\right)^{1/2}}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dt\left[t^{-1/2}\delta(t) - \frac{1}{2}t^{-3/2}\Theta(t)\right]e^{i\omega t}$$

$$= -i\omega\frac{2q}{\varepsilon}\hat{\mathbf{R}}(0)\frac{\left(\beta_n{}^2-1\right)^{1/4}}{\left(2c_n^3 R(0)\right)^{1/2}}\frac{1}{\sqrt{2\pi}}\int_{0}^{\infty} dt\;\; t^{-1/2}e^{i\omega t}$$

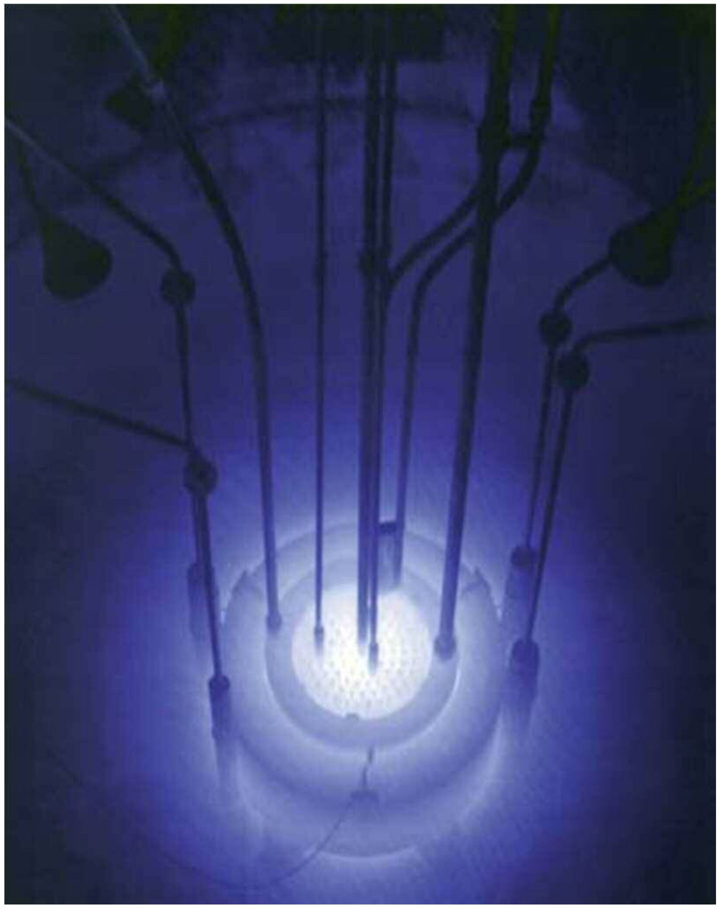
$$= \frac{q}{\varepsilon}\hat{\mathbf{R}}(0)\frac{\left(\beta_n{}^2-1\right)^{1/4}}{\left(2c_n^3 R(0)\right)^{1/2}}(1-i)\sqrt{\omega}$$

Cherenkov radiation observed near the angle θ_c -- continued



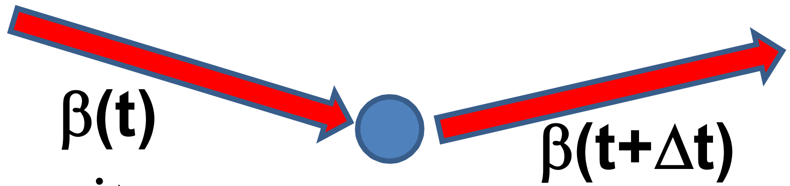
$$\text{Intensity:} \quad \frac{d^2 I}{d\omega d\ell} = 2\pi R(0) \sin\theta_C \cos\theta_C \, \frac{c_n}{4\pi} \left|\mathbf{E}(\omega)\right|^2$$

$$\propto K \frac{\left(\beta_n{}^2 - 1\right)}{\beta_n{}^2} \omega = K \left(1 - \left(\frac{v n(\omega)}{c} \right)^2 \right) \omega$$



Cherenkov radiation emitted by the core of the Reed Research Reactor located at Reed College in Portland, Oregon, U.S. *Cherenkov radiation*. Photograph. *Encyclopædia Britannica Online*. Web. 12 Apr. 2013.
http://www.britannica.com/EBchecked/media/174732

Radiation during collisions



Intensity:

$$\frac{d^2I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\int dt\, e^{i\omega\left(t-\hat{\mathbf{r}}\cdot\mathbf{R}_q(r)/c\right)}\frac{d}{dt}\left[\frac{\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}\right)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}}\right]\right|^2$$

Note that $\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}\right) = \left(\boldsymbol{\varepsilon}_1\cdot\boldsymbol{\beta}\right)\boldsymbol{\varepsilon}_1 + \left(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\beta}\right)\boldsymbol{\varepsilon}_2$

For a collision of duration τ emitting radiation with polarization $\boldsymbol{\varepsilon}$ and frequency $\omega \to 0$:

$$\frac{d^2I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot\left(\frac{\boldsymbol{\beta}(t+\tau)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t+\tau)} - \frac{\boldsymbol{\beta}(t)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t)}\right)\right|^2$$

Radiation during collisions -- continued

For a collision of duration τ emitting radiation with polarization $\boldsymbol{\varepsilon}$ and frequency $\omega \rightarrow 0$:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot\left(\frac{\boldsymbol{\beta}(t+\tau)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t+\tau)} - \frac{\boldsymbol{\beta}(t)}{1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta}(t)}\right)\right|^2$$

Non-relativistic limit:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot(\Delta\boldsymbol{\beta})\right|^2 \qquad \Delta\boldsymbol{\beta} \equiv \boldsymbol{\beta}(t+\tau)-\boldsymbol{\beta}(t)$$

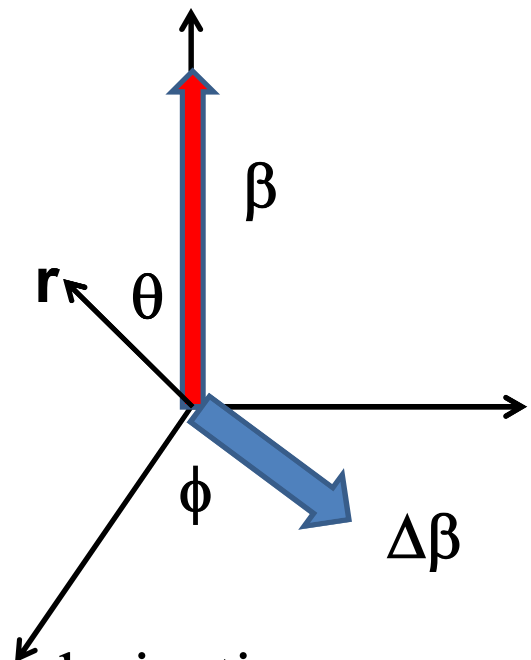
Relativistic collision with small $|\Delta\boldsymbol{\beta}|$:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot\left(\frac{\Delta\boldsymbol{\beta}+\hat{\mathbf{r}}\times(\boldsymbol{\beta}\times\Delta\boldsymbol{\beta})}{(1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta})^2}\right)\right|^2$$

Radiation during collisions -- continued

Relativistic collision with small $\left|\Delta\boldsymbol{\beta}\right|$:

$$\frac{d^2I}{d\omega d\Omega} = \frac{q^2}{4\pi^2 c}\left|\boldsymbol{\varepsilon}\cdot\left(\frac{\Delta\boldsymbol{\beta} + \hat{\mathbf{r}}\times(\boldsymbol{\beta}\times\Delta\boldsymbol{\beta})}{(1-\hat{\mathbf{r}}\cdot\boldsymbol{\beta})^2}\right)\right|^2$$



Expressions (averaging over ϕ) for $\|$ or $\perp$ polarization :

$$\frac{d^2I_{\|}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c}\left|\Delta\boldsymbol{\beta}\right|^2 \frac{(\beta - \cos\theta)^2}{(1-\beta\cos\theta)^4}$$

$$\frac{d^2I_{\perp}}{d\omega d\Omega} = \frac{q^2}{8\pi^2 c}\left|\Delta\boldsymbol{\beta}\right|^2 \frac{1}{(1-\beta\cos\theta)^2}$$