PHY 712 Electrodynamics
11-11:50 AM MWF Olin 107

Plan for Lecture 31:

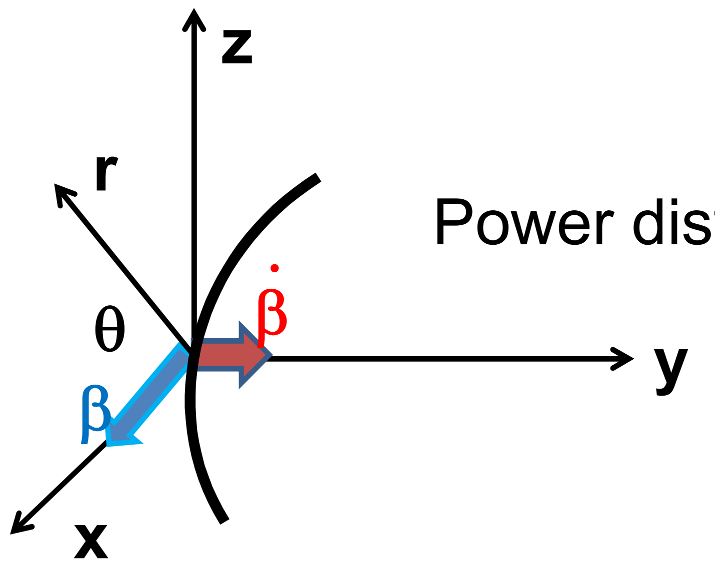
Finish reading sections of Chap. 14

1. Radiation from electron synchrotron and other accelerator devices

2. Thompson scattering

	03-18(Mon)	*APS Meeting*	(no class)	Exam
	03-20(Wed)	*APS Meeting*	(no class)	Exam
	03-22(Fri)	*APS Meeting*	(no class)	Exam
25	03-25(Mon)	Chap. 11	Lorentz transformations	#17
26	03-27(Wed)	Chap. 11	Transformations between electromagnetic fields	#18
27	03-28(Thur)	Chap. 11	Liénard-Wiechert potentials revisited	
	03-29(Fri)	*Good Friday*	(no class)	
28	04-01(Mon)	Chap. 14	Radiation by accelerated charges	#19
29	04-03(Wed)	Chap. 14	Radiation by accelerated charges	#20
30	04-05(Fri)	Chap. 14	Synchrotron radiation spectrum	#21
31	04-08(Mon)	Chap. 14	Synchrotron and other radiation sources	#22
32	04-10(Wed)	Chap. 15	Radiation due to collisions of charged particles	
33	04-12(Fri)	Chap. 15	Radiation due to energy loss processes	
	04-15(Mon)		(no class -- presentation preparation)	
	04-17(Wed)		(no class -- presentation preparation)	
	04-19(Fri)		(no class -- presentation preparation)	
34	04-22(Mon)			
35	04-24(Wed)			
36	04-26(Fri)			
	04-29(Mon)		Student presentations I	
	05-01(Wed)		Student presentations II	
	05-02(Thurs)		Student presentations III	

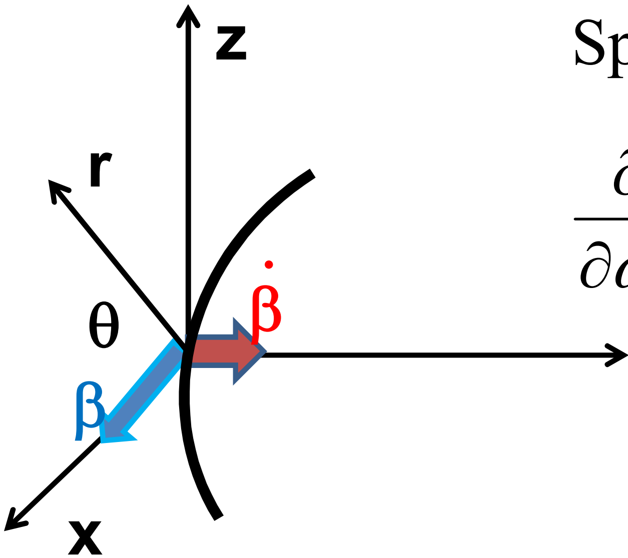
Radiation from charged particle in circular path



Power distribution for circular acceleration

The spectral intensity depends on the following integral:

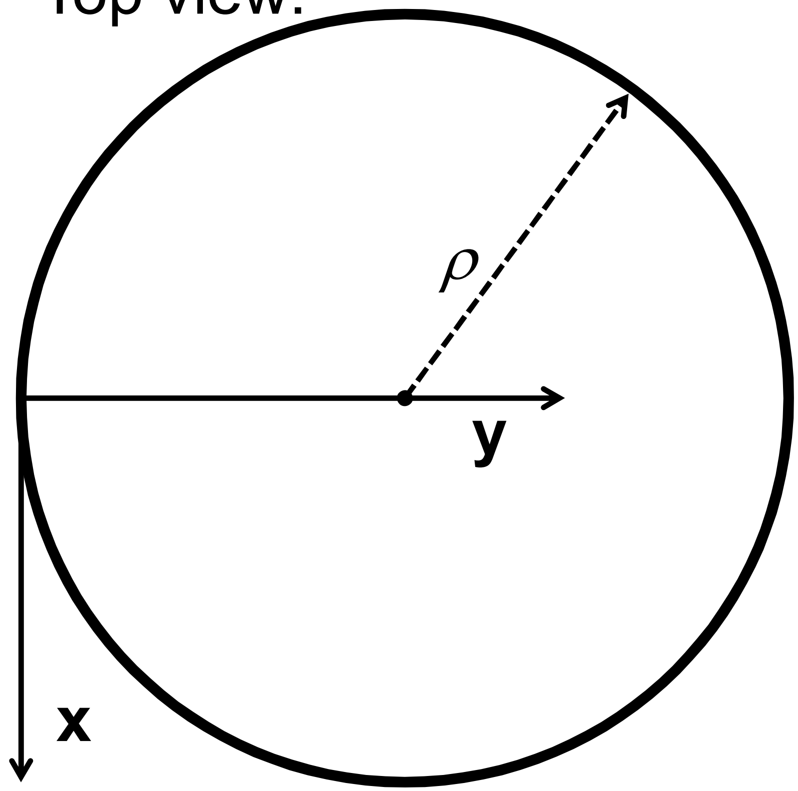
$$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{q^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} dt_r \left[\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}(t_r)\right)\right] e^{i\omega\left(t_r - \hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)/c\right)}\right|^2$$



Spectral intensity relationship:

$$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{q^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} dt_r \left[\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}(t_r)\right)\right] e^{i\omega\left(t_r - \hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)/c\right)}\right|^2$$

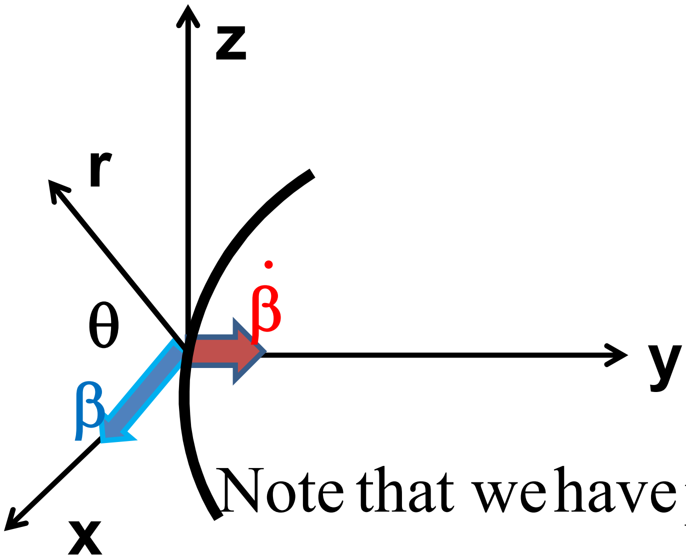
Top view:



$$\mathbf{R}_q(t_r) = \rho\hat{\mathbf{x}}\sin(vt_r/\rho)$$
$$+\ \rho\hat{\mathbf{y}}\left(1-\cos(vt_r/\rho)\right)$$
$$\boldsymbol{\beta}(t_r) = \beta\left(\hat{\mathbf{x}}\cos(vt_r/\rho) + \hat{\mathbf{y}}\sin(vt_r/\rho)\right)$$

For convenience, choose:

$$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\theta + \hat{\mathbf{z}}\sin\theta$$



$$\mathbf{R}_q(t_r) = \rho\hat{\mathbf{x}}\sin(vt_r/\rho) + \rho\hat{\mathbf{y}}(1-\cos(vt_r/\rho))$$

$$\boldsymbol{\beta}(t_r) = \beta(\hat{\mathbf{x}}\cos(vt_r/\rho) + \hat{\mathbf{y}}\sin(vt_r/\rho))$$

For convenience, choose:

$$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\theta + \hat{\mathbf{z}}\sin\theta$$

Note that we have previously shown that in the radiation zone, the Poynting vector is in the $\hat{\mathbf{r}}$ direction; we can then choose to analyze two orthogonal polarization directions:

$$\boldsymbol{\varepsilon}_{\parallel} = \hat{\mathbf{y}} \qquad \boldsymbol{\varepsilon}_{\perp} = -\hat{\mathbf{x}}\sin\theta + \hat{\mathbf{z}}\cos\theta$$

$$\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \boldsymbol{\beta}) = \beta\left(-\boldsymbol{\varepsilon}_{\parallel}\sin(vt_r/\rho) + \boldsymbol{\varepsilon}_{\perp}\sin\theta\cos(vt_r/\rho)\right)$$

$$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{q^2\omega^2\beta^2}{4\pi^2 c}\left(|C_{\parallel}|^2 + |C_{\perp}|^2\right)$$

$$C_{\parallel}(\omega) = \int_{-\infty}^{\infty} dt \sin(vt/\rho) e^{i\omega\left(t - \frac{\rho}{c}\cos\theta\sin(vt/\rho)\right)}$$

$$C_{\perp}(\omega) = \sin\theta \int_{-\infty}^{\infty} dt \cos(vt/\rho) e^{i\omega\left(t - \frac{\rho}{c}\cos\theta\sin(vt/\rho)\right)}$$

Modified Bessel functions

$$K_{1/3}(\xi)=\sqrt{3}\int_0^\infty dx\cos\left[\tfrac{3}{2}\xi\left(x+\tfrac{1}{3}x^3\right)\right] \qquad K_{2/3}(\xi)=\sqrt{3}\int_0^\infty dx\,x\sin\left[\tfrac{3}{2}\xi\left(x+\tfrac{1}{3}x^3\right)\right]$$

Exponential factor

$$\omega\left(t_r-\frac{\hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)}{c}\right)=\omega\left(t_r-\frac{\rho}{c}\cos\theta\sin(vt_r/\rho)\right)$$

$$\text{In the limit of}\quad t_r\approx 0,\quad \theta\approx 0,\ v\approx c\left(1-\frac{1}{2\gamma^2}\right)$$

$$\omega\left(t_r-\frac{\hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)}{c}\right)\approx\frac{\omega t_r}{2\gamma^2}\left(1+\gamma^2\theta^2\right)+\frac{\omega c^2 t_r^{\;3}}{6\rho^2}=\frac{3}{2}\xi\left(x+\frac{1}{3}x^3\right)$$

$$\text{where}\quad \xi=\frac{\omega\rho}{3c\gamma^3}\left(1+\gamma^2\theta^2\right)^{3/2}\ \text{ and }\ x=\frac{c\gamma t_r}{\rho\left(1+\gamma^2\theta^2\right)^{1/2}}$$

Spectral form of synchrotron radiation in this case:

$$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{3q^2\gamma^2}{4\pi^2 c}\left(\frac{\omega}{\omega_c}\right)^2 \left(1+\gamma^2\theta^2\right)^2 \left\{\left[K_{2/3}\left(\frac{\omega}{2\omega_c}\left(1+\gamma^2\theta^2\right)^{3/2}\right)\right]^2 + \frac{\gamma^2\theta^2}{1+\gamma^2\theta^2}\left[K_{1/3}\left(\frac{\omega}{2\omega_c}\left(1+\gamma^2\theta^2\right)^{3/2}\right)\right]^2\right\}$$

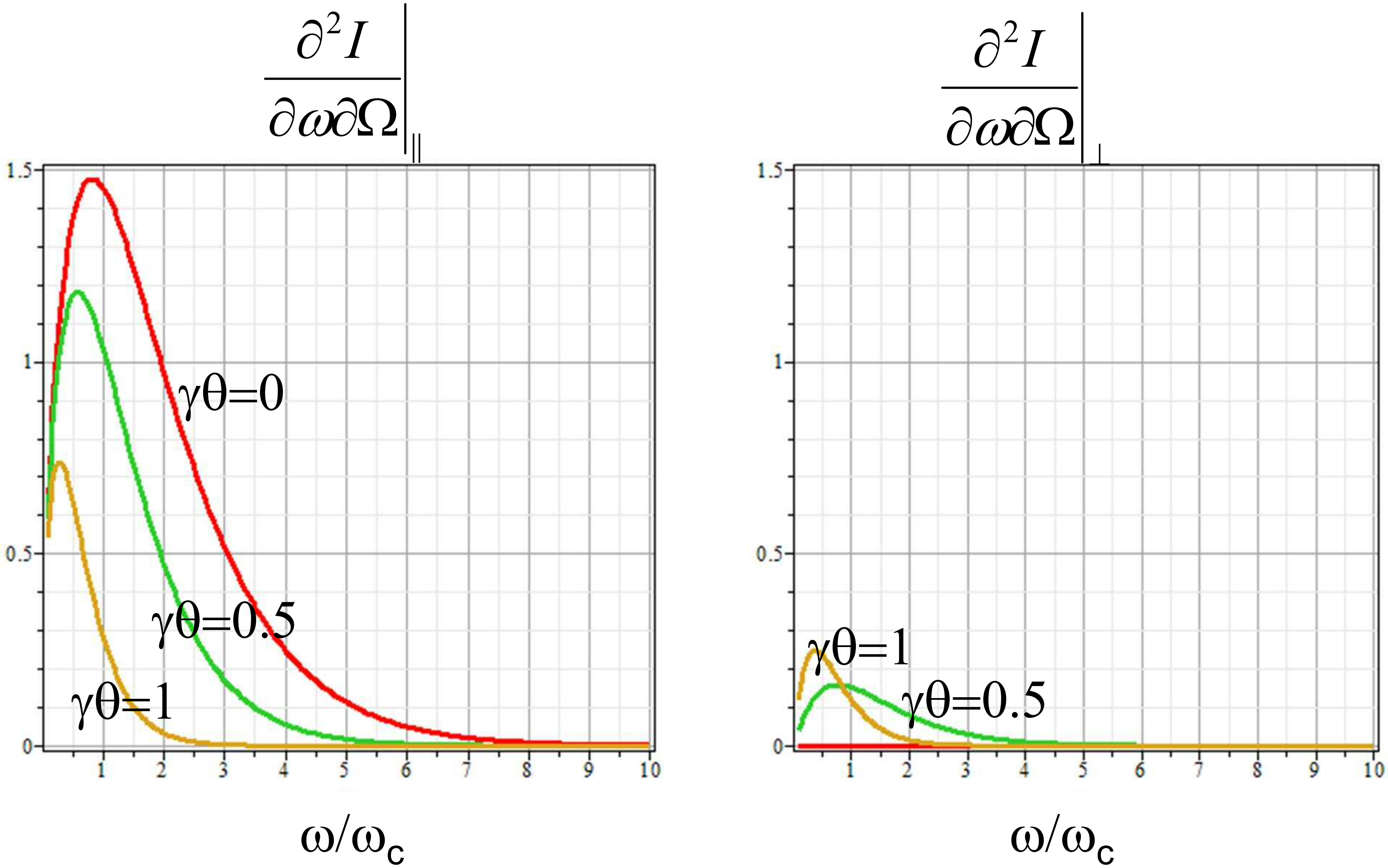
$$\omega_c \equiv \frac{3c\gamma^3}{2\rho}$$

At $\theta = 0$:

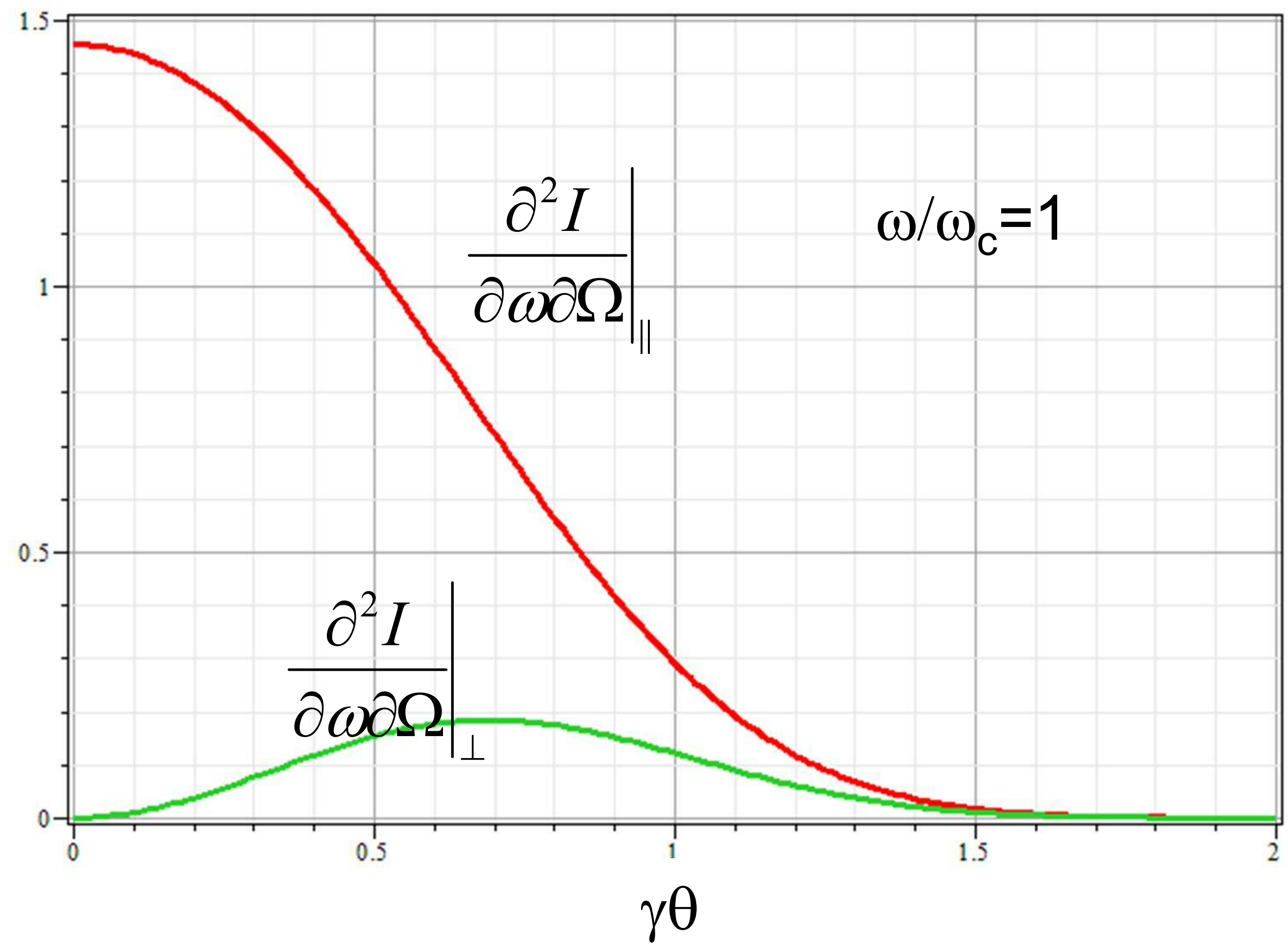
note that for $\omega << \omega_c \quad \Rightarrow \dfrac{\partial^2 I}{\partial\omega\partial\Omega} \approx \dfrac{q^2}{\pi^2 c}\left(\Gamma\left(\tfrac{2}{3}\right)\right)^2 \left(\dfrac{3\omega^2\rho^2}{4c^2}\right)^{1/3}$

and for $\omega >> \omega_c \quad \Rightarrow \dfrac{\partial^2 I}{\partial\omega\partial\Omega} \approx \dfrac{3q^2}{4\pi c}\gamma^2\left(\dfrac{\omega}{\omega_c}\right)^2 e^{-\omega/\omega_c}$

Plots of the intensity function for synchrotron radiation



Plots of the intensity function for synchrotron radiation



Undulators and Free Electron Lasers

http://cas.web.cern.ch/cas/ZEUTHEN/Afternoon-courses/Linac/Schmueser-FEL1.pdf

Thompson scattering – non relativistic approximation

Power radiated in direction $\hat{\mathbf{r}}$ by charged particle with acceleration $\dot{\mathbf{v}}$:

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3}\left|\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\dot{\mathbf{v}}\right)\right|^2$$

Suppose that the acceleration $\dot{\mathbf{v}}$ of a particle (charge q and mass m_q)

is caused by an electric field: $\mathbf{E}(\mathbf{r},t) = \Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k_0}\cdot\mathbf{r}-i\omega t}\right)$

$$\dot{\mathbf{v}} = \frac{q}{m_q}\Re\left(\boldsymbol{\varepsilon}_0 E_0 e^{i\mathbf{k_0}\cdot\mathbf{r}-i\omega t}\right)$$

Time averaged power: $\left\langle\frac{dP}{d\Omega}\right\rangle = \frac{c}{8\pi}\left(\frac{q^2}{m_q c^2}\right)^2 \left|E_0\right|^2 \left|\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\varepsilon}_0\right)\right|^2$

Thompson scattering – non relativistic approximation -- continued

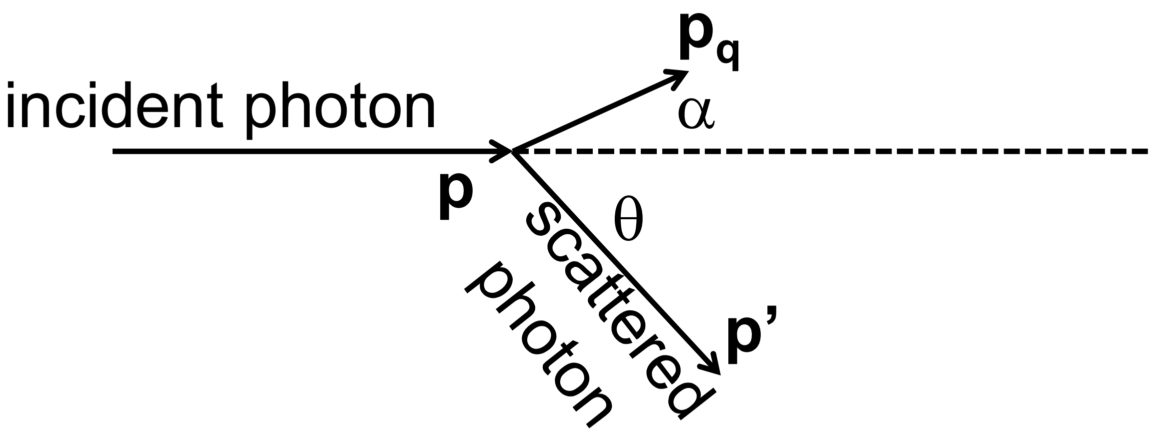
$$\text{Time averaged power:}\quad \left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi}\left(\frac{q^2}{m_q c^2}\right)^2 |E_0|^2 \left|\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\boldsymbol{\varepsilon}_0)\right|^2$$

Scattered light may be polarized parallel to incident field or polarized with an angle θ so that the time and polarization averaged cross section is given by:

$$\text{Averaged cross section:}\quad \left\langle \frac{d\sigma}{d\Omega} \right\rangle = \left(\frac{q^2}{m_q c^2}\right)^2 \frac{1}{2}\left(1+\cos^2\theta\right)$$

This formula is appropriate in the X-ray scattering of electrons or soft γ-ray scattering of protons

Thompson scattering – relativistic and quantum modifications



Conservation of momentum and energy :

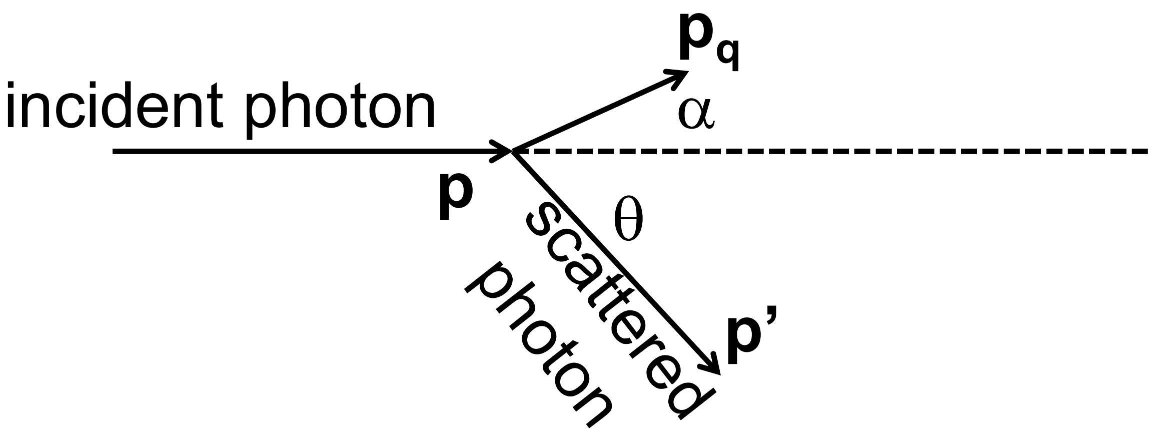
$$p = p'\cos\theta + p_q \cos\alpha \qquad\qquad pc = \hbar\omega$$

$$0 = p'\sin\theta - p_q \sin\alpha \qquad\qquad p'c = \hbar\omega'$$

$$\hbar\omega + m_q c^2 = \hbar\omega' + \sqrt{p_q{}^2 c^2 + \left(m_q c^2\right)^2}$$

$$\frac{p'}{p} = \frac{1}{1 + \dfrac{\hbar\omega}{m_q c^2}\left(1 - \cos\theta\right)}$$

Thompson scattering – relativistic and quantum modifications



Relativistic and quantum modifications to averaged cross section:

$$\left\langle \frac{d\sigma}{d\Omega} \right\rangle = \left(\frac{q^2}{m_q c^2} \right)^2 \left(\frac{p'}{p} \right)^2 \frac{1}{2} \left(1 + \cos^2 \theta \right)$$