PHY 712 Electrodynamics
11-11:50 AM MWF Olin 107

Plan for Lecture 29:

Continue reading Chap. 14 – Radiation by moving charges

   A. Radiation due to moving charges

   1. **Motion in a line**

   2. **Motion in a circle**

   3. **Spectral analysis**

				Exam
	03-22(Fri)	*APS Meeting*	(no class)	Exam
25	03-25(Mon)	Chap. 11	Lorentz transformations	#17
26	03-27(Wed)	Chap. 11	Transformations between electromagnetic fields	#18
27	03-28(Thur)	Chap. 11	Liénard-Wiechert potentials revisited	
	03-29(Fri)	*Good Friday*	(no class)	
28	04-01(Mon)	Chap. 14	Radiation by accelerated charges	#19
29	04-03(Wed)	Chap. 14	Radiation by accelerated charges	#20
30	04-05(Fri)	Chap. 14	Synchrotron radiation spectrum	#21
31	04-08(Mon)			
32	04-10(Wed)			
33	04-12(Fri)			
	04-15(Mon)		(no class -- presentation preparation)	
	04-17(Wed)		(no class -- presentation preparation)	
	04-19(Fri)		(no class -- presentation preparation)	
34	04-22(Mon)			
35	04-24(Wed)			
36	04-26(Fri)			
	04-29(Mon)		Student presentations I	
	05-01(Wed)		Student presentations II	
	05-02(Thurs)		Student presentations III	

WFU Physics Colloquium

TITLE: The search for the quirky quantum particle struck gold: The emergence of Majorana fermions in condensed matter physics and implications

SPEAKER: Professor Sumanta Tewari,

Department of Physics and Astronomy,
Clemson University, Clemson, South Carolina

TIME: Wednesday April 3, 2013 at 4:00 PM

PLACE: Room 101 Olin Physical Laboratory

Refreshments will be served at 3:30 PM in the Olin Lounge. All interested persons are cordially invited to attend.
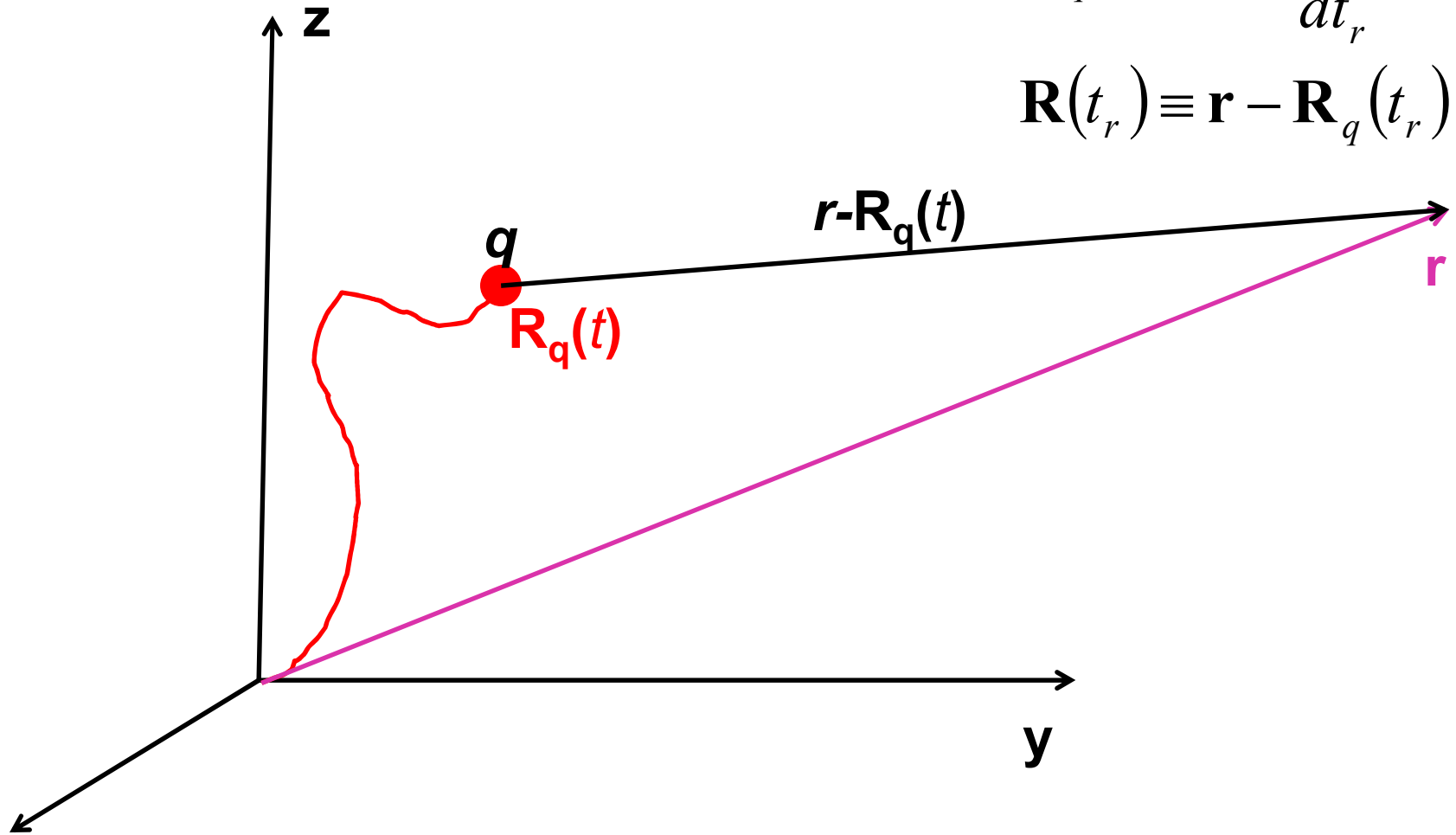
ABSTRACT

Majorana fermion (MF), the elusive quantum particle proposed more than seven decades ago by E. Majorana to describe neutrinos, is finally beginning to be experimentally realized in condensed matter systems. Following our recent proposal to realize them in

Radiation from a moving charged particle

Variables (notation):

$$\dot{\mathbf{R}}_q(t_r) \equiv \frac{d\mathbf{R}_q(t_r)}{dt_r} \equiv \mathbf{v}$$

$$\mathbf{R}(t_r) \equiv \mathbf{r} - \mathbf{R}_q(t_r) \equiv \mathbf{R}$$

Electric and magnetic fields far from source:

$$\mathbf{E}(\mathbf{r},t)=\frac{q}{\left(R-\dfrac{\mathbf{v}\cdot\mathbf{R}}{c}\right)^3}\left\{\mathbf{R}\times\left[\left(\mathbf{R}-\frac{\mathbf{v}R}{c}\right)\times\frac{\dot{\mathbf{v}}}{c^2}\right]\right\}$$

$$\mathbf{B}(\mathbf{r},t)=\frac{\mathbf{R}\times\mathbf{E}(\mathbf{r},t)}{R}$$

Let $\hat{\mathbf{R}}\equiv\dfrac{\mathbf{R}}{R}$ $\qquad$ $\boldsymbol{\beta}\equiv\dfrac{\mathbf{v}}{c}$ $\qquad$ $\dot{\boldsymbol{\beta}}\equiv\dfrac{\dot{\mathbf{v}}}{c}$

$$\mathbf{E}(\mathbf{r},t)=\frac{q}{cR\left(1-\boldsymbol{\beta}\cdot\hat{\mathbf{R}}\right)^3}\left\{\hat{\mathbf{R}}\times\left[\left(\hat{\mathbf{R}}-\boldsymbol{\beta}\right)\times\dot{\boldsymbol{\beta}}\right]\right\}$$

$$\mathbf{B}(\mathbf{r},t)=\hat{\mathbf{R}}\times\mathbf{E}(\mathbf{r},t)$$

Poynting vector:

$$\mathbf{S}(\mathbf{r},t) = \frac{c}{4\pi}\left(\mathbf{E} \times \mathbf{B}\right)$$

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{cR\left(1 - \boldsymbol{\beta}\cdot\hat{\mathbf{R}}\right)^3}\left\{\hat{\mathbf{R}} \times \left[\left(\hat{\mathbf{R}} - \boldsymbol{\beta}\right) \times \dot{\boldsymbol{\beta}}\right]\right\}$$

$$\mathbf{B}(\mathbf{r},t) = \hat{\mathbf{R}} \times \mathbf{E}(\mathbf{r},t)$$

$$\mathbf{S}(\mathbf{r},t) = \frac{c}{4\pi}\hat{\mathbf{R}}\left|\mathbf{E}(\mathbf{r},t)\right|^2 = \frac{q^2}{4\pi c R^2}\hat{\mathbf{R}}\frac{\left|\hat{\mathbf{R}} \times \left[\left(\hat{\mathbf{R}} - \boldsymbol{\beta}\right) \times \dot{\boldsymbol{\beta}}\right]\right|^2}{\left(1 - \boldsymbol{\beta}\cdot\hat{\mathbf{R}}\right)^6}$$

Radiation power distribution

$$\frac{dP}{d\Omega} = \mathbf{S} \cdot \hat{\mathbf{R}} R^2 = \frac{q^2}{4\pi c} \frac{\left| \hat{\mathbf{R}} \times \left[\left(\hat{\mathbf{R}} - \boldsymbol{\beta} \right) \times \dot{\boldsymbol{\beta}} \right] \right|^2}{\left(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}} \right)^6} \Bigg|_{t_r = t - R/c}$$

This expression gives us the energy per unit field time *t*. We are often interested in the power per unit retarded time $t_r = t - R/c$:

$$\frac{dP(t)}{d\Omega} = \frac{dP_r(t_r)}{d\Omega} \frac{dt_t}{dt} \qquad \frac{dt}{dt_r} = 1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}}$$

$$\frac{dP_r(t_r)}{d\Omega} = \frac{q^2}{4\pi c} \frac{\left| \hat{\mathbf{R}} \times \left[\left(\hat{\mathbf{R}} - \boldsymbol{\beta} \right) \times \dot{\boldsymbol{\beta}} \right] \right|^2}{\left(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}} \right)^5} \Bigg|_{t_r = t - R/c}$$

Radiation power distribution -- continued

$$\frac{dP_r(t_r)}{d\Omega} = \frac{q^2}{4\pi c} \left. \frac{\left|\hat{\mathbf{R}} \times \left[(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}\right]\right|^2}{\left(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}}\right)^5} \right|_{t_r = t - R/c}$$
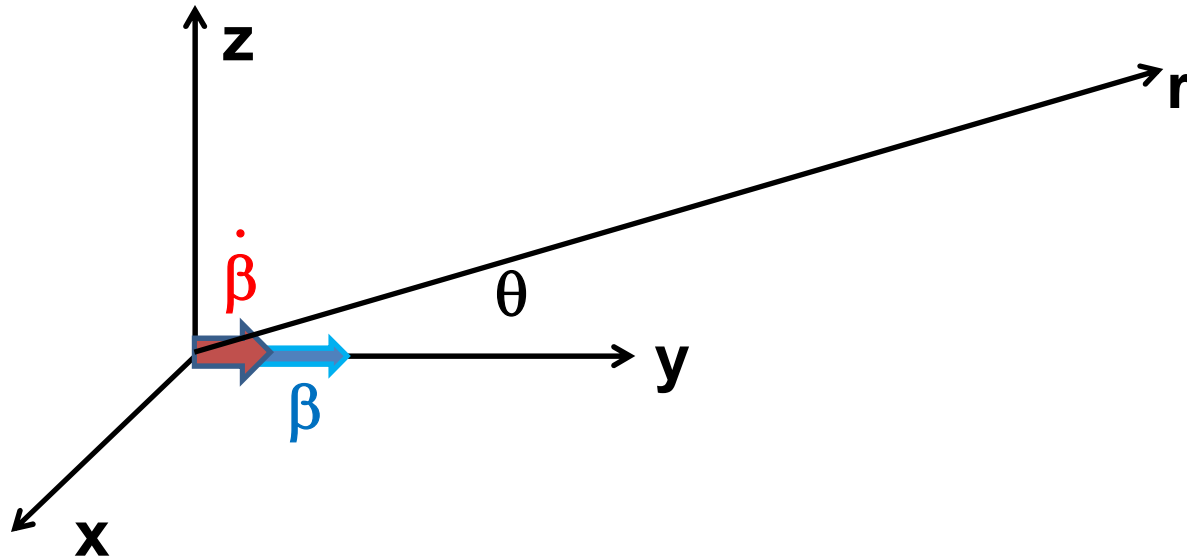
For linear acceleration:

$$\boldsymbol{\beta} \times \dot{\boldsymbol{\beta}} = 0$$

$$\frac{dP_r(t_r)}{d\Omega} = \frac{q^2}{4\pi c} \left. \frac{\left|\hat{\mathbf{R}} \times (\hat{\mathbf{R}} \times \dot{\boldsymbol{\beta}})\right|^2}{\left(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}}\right)^5} \right|_{t_r = t - R/c} = \frac{q^2}{4\pi c^3} |\dot{\mathbf{v}}|^2 \frac{\sin^2 \theta}{(1 - \beta \cos \theta)^5}$$

$$P_r(t_r) = \int \frac{dP_r(t_r)}{d\Omega} d\Omega = \frac{2}{3} \frac{q^2}{c^3} |\dot{\mathbf{v}}|^2 \gamma^6 \quad \text{where} \ \gamma \equiv \frac{1}{\sqrt{1 - \beta^2}}$$

Power distribution for linear acceleration -- continued

$$\frac{dP_r(t_r)}{d\Omega} = \frac{q^2}{4\pi c} \frac{\left|\hat{\mathbf{R}} \times \left(\hat{\mathbf{R}} \times \dot{\boldsymbol{\beta}}\right)\right|^2}{\left(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}}\right)^5}\Bigg|_{t_r = t - R/c} = \frac{q^2}{4\pi c^3} |\dot{\mathbf{v}}|^2 \frac{\sin^2 \theta}{\left(1 - \beta \cos \theta\right)^5}$$

$$P_r(t_r) = \int \frac{dP_r(t_r)}{d\Omega} d\Omega = \frac{2}{3} \frac{q^2}{c^3} |\dot{\mathbf{v}}|^2 \gamma^6 \quad \text{where} \quad \gamma \equiv \frac{1}{\sqrt{1 - \beta^2}}$$
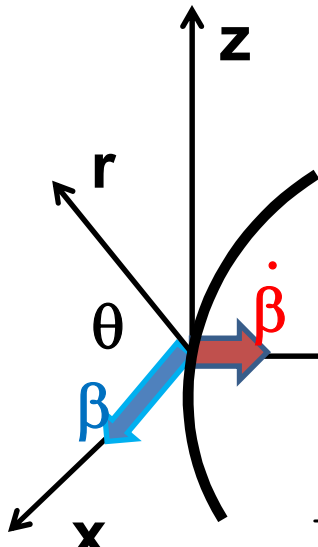
Power distribution for circular acceleration

$$\frac{dP_r(t_r)}{d\Omega} = \frac{q^2}{4\pi c} \frac{\left| \hat{\mathbf{R}} \times \left[(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}} \right] \right|^2}{\left(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}} \right)^5} \Bigg|_{t_r = t - R/c}$$

$$= \frac{q^2}{4\pi c} \frac{\left| \dot{\boldsymbol{\beta}} \right|^2 \left(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}} \right)^2 - \left(\hat{\mathbf{R}} \cdot \dot{\boldsymbol{\beta}} \right)^2 \left(1 - \beta^2 \right)}{\left(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}} \right)^5} \Bigg|_{t_r = t - R/c}$$

$$P_r(t_r) = \int d\Omega \frac{dP_r(t_r)}{d\Omega} = \frac{2}{3} \frac{q^2}{c^3} \left| \dot{\mathbf{v}} \right|^2 \gamma^4$$

Spectral composition of electromagnetic radiation

Previously we determined the power distribution from a charged particle:

$$\frac{dP(t)}{d\Omega} = \mathbf{S} \cdot \hat{\mathbf{R}} R^2 = \frac{q^2}{4\pi c} \left.\frac{\left|\hat{\mathbf{R}} \times \left[\left(\hat{\mathbf{R}} - \boldsymbol{\beta}\right) \times \dot{\boldsymbol{\beta}}\right]\right|^2}{\left(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}}\right)^6}\right|_{t_r = t - R/c}$$

$$\equiv \left|\boldsymbol{a}(t)\right|^2$$

where

$$\boldsymbol{a}(t) \equiv \left.\sqrt{\frac{q^2}{4\pi c}} \frac{\left|\hat{\mathbf{R}} \times \left[\left(\hat{\mathbf{R}} - \boldsymbol{\beta}\right) \times \dot{\boldsymbol{\beta}}\right]\right|}{\left(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}}\right)^3}\right|_{t_r = t - R/c}$$

Time integrated power per solid angle :

$$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty} dt \frac{dP(t)}{d\Omega} = \int_{-\infty}^{\infty} dt \left|\boldsymbol{a}(t)\right|^2 = \int_{-\infty}^{\infty} d\omega \left|\tilde{\boldsymbol{a}}(\omega)\right|^2$$

Spectral composition of electromagnetic radiation -- continued

Time integrated power per solid angle :

$$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty} dt \frac{dP(t)}{d\Omega} = \int_{-\infty}^{\infty} dt \left| \boldsymbol{a}(t) \right|^2 = \int_{-\infty}^{\infty} d\omega \left| \tilde{\boldsymbol{a}}(\omega) \right|^2$$

Fourier amplitude :

$$\tilde{\boldsymbol{a}}(\omega) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt \, \boldsymbol{a}(t) e^{i\omega t} \qquad \boldsymbol{a}(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega \, \tilde{\boldsymbol{a}}(\omega) e^{-i\omega t}$$

Parseval's theorem

Marc-Antoine Parseval des Chênes 1755-1836

http://www-history.mcs.st-andrews.ac.uk/Biographies/Parseval.html

Spectral composition of electromagnetic radiation -- continued

Consequences of Parseval's analysis:

$$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty} dt \frac{dP(t)}{d\Omega} = \int_{-\infty}^{\infty} dt \left| \boldsymbol{a}(t) \right|^2 = \int_{-\infty}^{\infty} d\omega \left| \tilde{\boldsymbol{a}}(\omega) \right|^2$$

Note that : $\tilde{\boldsymbol{a}}(\omega) = \tilde{\boldsymbol{a}}^*(-\omega)$

$$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty} d\omega \left| \tilde{\boldsymbol{a}}(\omega) \right|^2 = \int_{0}^{\infty} d\omega \left(\left| \tilde{\boldsymbol{a}}(\omega) \right|^2 + \left| \tilde{\boldsymbol{a}}(-\omega) \right|^2 \right) \equiv \int_{0}^{\infty} d\omega \frac{\partial^2 I}{\partial \Omega \partial \omega}$$

$$\frac{\partial^2 I}{\partial \Omega \partial \omega} \equiv 2 \left| \tilde{\boldsymbol{a}}(\omega) \right|^2$$

Spectral composition of electromagnetic radiation -- continued

For our case:
$$a(t) \equiv \sqrt{\frac{q^2}{4\pi c}} \left. \frac{\left| \hat{\mathbf{R}} \times \left[(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}} \right] \right|}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^3} \right|_{t_r = t - R/c}$$

Fourier amplitude:

$$\tilde{a}(\omega) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt \, a(t) \, e^{i\omega t}$$

$$= \sqrt{\frac{q^2}{8\pi^2 c}} \int_{-\infty}^{\infty} dt \left. \frac{\left| \hat{\mathbf{R}} \times \left[(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}} \right] \right|}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}})^3} \right|_{t_r = t - R/c} e^{i\omega t}$$

Spectral composition of electromagnetic radiation -- continued

Fourier amplitude :

$$\widetilde{\boldsymbol{a}}(\omega) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt\, \boldsymbol{a}(t)\, e^{i\omega t}$$

$$= \sqrt{\frac{q^2}{8\pi^2 c}} \int_{-\infty}^{\infty} dt \frac{\left|\hat{\mathbf{R}} \times \left[(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}\right]\right|}{\left(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}}\right)^3}\Bigg|_{t_r = t - R/c} e^{i\omega t}$$

$$= \sqrt{\frac{q^2}{8\pi^2 c}} \int_{-\infty}^{\infty} dt_r \frac{dt}{dt_r} \frac{\left|\hat{\mathbf{R}} \times \left[(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}\right]\right|}{\left(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}}\right)^3}\Bigg|_{t_r = t - R/c} e^{i\omega(t_r + R(t_r)/c)}$$

$$= \sqrt{\frac{q^2}{8\pi^2 c}} \int_{-\infty}^{\infty} dt_r \frac{\left|\hat{\mathbf{R}} \times \left[(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}\right]\right|}{\left(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}}\right)^2}\Bigg|_{t_r = t - R/c} e^{i\omega(t_r + R(t_r)/c)}$$

Spectral composition of electromagnetic radiation -- continued

Exact expression :

$$\tilde{a}(\omega) = \sqrt{\frac{q^2}{8\pi^2 c}} \int_{-\infty}^{\infty} dt_r \left.\frac{\left|\hat{\mathbf{R}} \times \left[\left(\hat{\mathbf{R}} - \boldsymbol{\beta}\right) \times \dot{\boldsymbol{\beta}}\right]\right|}{\left(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}}\right)^2}\right|_{t_r = t - R/c} e^{i\omega(t_r + R(t_r)/c)}$$

Recall : $\quad \dot{\mathbf{R}}_q(t_r) \equiv \dfrac{d\mathbf{R}_q(t_r)}{dt_r} \equiv \mathbf{v} \qquad \mathbf{R}(t_r) \equiv \mathbf{r} - \mathbf{R}_q(t_r) \equiv \mathbf{R}$

For $\quad r \gg R_q(t_r) \qquad R(t_r) \approx r - \hat{\mathbf{r}} \cdot \mathbf{R}_q(t_r) \quad$ where $\quad \hat{\mathbf{r}} \equiv \dfrac{\mathbf{r}}{r}$

At the same level of approximation : $\quad \hat{\mathbf{R}} \approx \hat{\mathbf{r}}$

Spectral composition of electromagnetic radiation -- continued

Exact expression :

$$\widetilde{\boldsymbol{a}}(\omega) = \sqrt{\frac{q^2}{8\pi^2 c}} \int_{-\infty}^{\infty} dt_r \left. \frac{\left| \hat{\mathbf{R}} \times \left[(\hat{\mathbf{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}} \right] \right|}{\left(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{R}} \right)^2} \right|_{t_r = t - R/c} e^{i\omega\left(t_r + R(t_r)/c \right)}$$

Approximate expression :

$$\widetilde{\boldsymbol{a}}(\omega) = \sqrt{\frac{q^2}{8\pi^2 c}} e^{i\omega(r/c)} \int_{-\infty}^{\infty} dt_r \left. \frac{\left| \hat{\mathbf{r}} \times \left[(\hat{\mathbf{r}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}} \right] \right|}{\left(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{r}} \right)^2} \right|_{t_r = t - R/c} e^{i\omega\left(t_r - \hat{\mathbf{r}} \cdot \mathbf{R}_q(t_r)/c \right)}$$

It can be shown that :

$$\frac{\hat{\mathbf{r}} \times \left[(\hat{\mathbf{r}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}} \right]}{\left(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{r}} \right)^2} = \frac{d}{dt_r} \left(\frac{\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \boldsymbol{\beta})}{\left(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{r}} \right)} \right)$$

When the dust clears, the spectral intensity depends on the following integral :

$$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{q^2\omega^2}{4\pi^2 c}\left|\int\limits_{-\infty}^{\infty} dt_r \left[\hat{\mathbf{r}}\times\left(\hat{\mathbf{r}}\times\boldsymbol{\beta}(t_r)\right)\right]e^{i\omega\left(t_r-\hat{\mathbf{r}}\cdot\mathbf{R}_q(t_r)/c\right)}\right|^2$$