PHY 712 Electrodynamics
11-11:50 AM MWF Olin 107

Plan for Lecture 26:

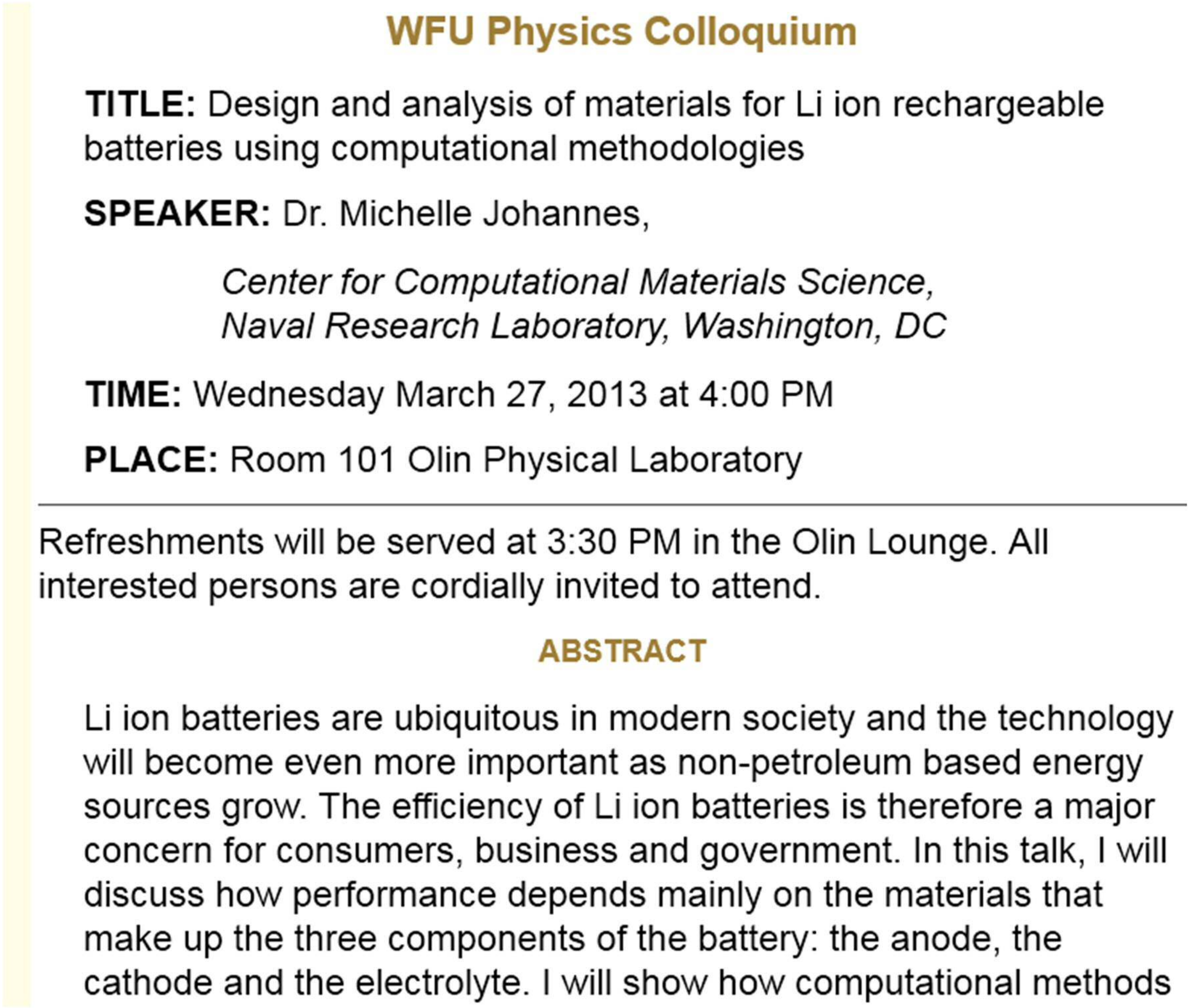
Continue reading Chap. 11 – Theory of Special Relativity

A. Lorentz transformation relations

B. Energy and momentum

C. Electromagnetic field transformations

	03-13(Wed)	*Spring Break*		
	03-15(Fri)	*Spring Break*		
	03-18(Mon)	*APS Meeting*	(no class)	Exam
	03-20(Wed)	*APS Meeting*	(no class)	Exam
	03-22(Fri)	*APS Meeting*	(no class)	Exam
25	03-25(Mon)	Chap. 11	Lorentz transformations	#17
26	03-27(Wed)	Chap. 11	Transformations between electromagnetic fields	#18
27	03-28(Thur)	Chap. 11	Liénard-Wiechert potentials revisited	
	03-29(Fri)	*Good Friday*	(no class)	
28	04-01(Mon)	Chap. 14	Radiation by accelerated charges	
29	04-03(Wed)	Chap. 14	Radiation by accelerated charges	
30	04-05(Fri)	Chap. 14	Synchrotron radiation spectrum	
31	04-08(Mon)			
32	04-10(Wed)			
33	04-12(Fri)			
	04-15(Mon)		(no class -- presentation preparation)	
	04-17(Wed)		(no class -- presentation preparation)	
	04-19(Fri)		(no class -- presentation preparation)	
34	04-22(Mon)			
35	04-24(Wed)			
36	04-26(Fri)			
	04-29(Mon)		Student presentations	
	05-01(Wed)		Student presentations	



WFU Physics Colloquium

TITLE: Design and analysis of materials for Li ion rechargeable batteries using computational methodologies

SPEAKER: Dr. Michelle Johannes,

Center for Computational Materials Science,
Naval Research Laboratory, Washington, DC

TIME: Wednesday March 27, 2013 at 4:00 PM

PLACE: Room 101 Olin Physical Laboratory

Refreshments will be served at 3:30 PM in the Olin Lounge. All interested persons are cordially invited to attend.

ABSTRACT

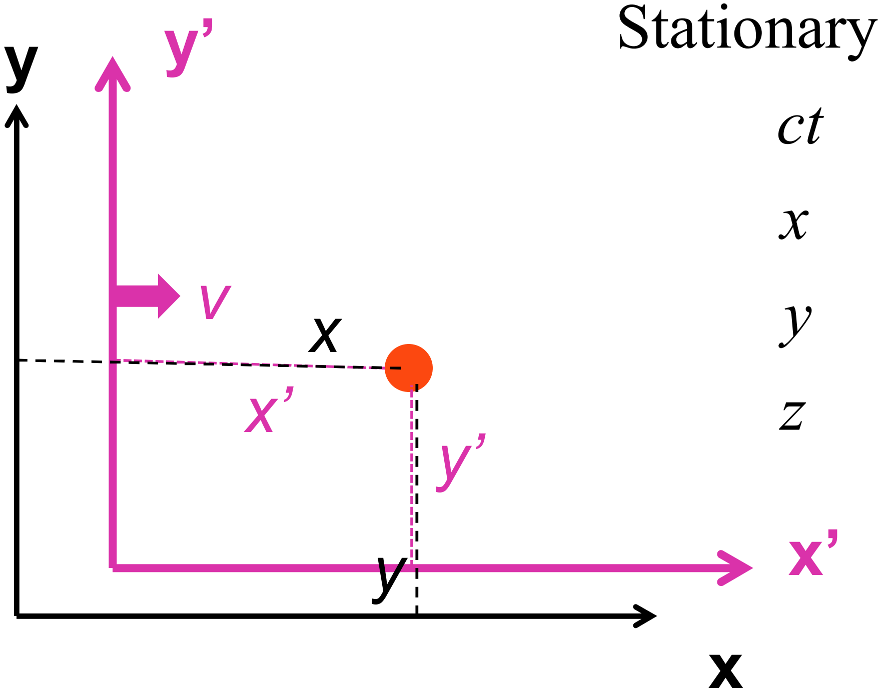
Li ion batteries are ubiquitous in modern society and the technology will become even more important as non-petroleum based energy sources grow. The efficiency of Li ion batteries is therefore a major concern for consumers, business and government. In this talk, I will discuss how performance depends mainly on the materials that make up the three components of the battery: the anode, the cathode and the electrolyte. I will show how computational methods

Lorentz transformations

Convenient notation :

$$\beta_v \equiv \frac{v}{c}$$

$$\gamma_v \equiv \frac{1}{\sqrt{1-\beta_v^{\ 2}}}$$



Stationary frame		Moving frame
ct	$=$	$\gamma(ct' + \beta x')$
x	$=$	$\gamma(x' + \beta ct')$
y	$=$	y'
z	$=$	z'

Lorentz transformations -- continued

For the moving frame with $\mathbf{v} = v\hat{\mathbf{x}}$:

$$\mathcal{L}_v = \begin{pmatrix} \gamma_v & \gamma_v\beta_v & 0 & 0 \\ \gamma_v\beta_v & \gamma_v & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \mathcal{L}_v^{\text{-1}} = \begin{pmatrix} \gamma_v & -\gamma_v\beta_v & 0 & 0 \\ -\gamma_v\beta_v & \gamma_v & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \mathcal{L}_v \begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} \qquad \begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \mathcal{L}_v^{\text{-1}} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

Notice:

$$c^2t^2 - x^2 - y^2 - z^2 = c^2t'^2 - x'^2 - y'^2 - z'^2$$

Lorentz transformation of the velocity

$$\text{Define:} \qquad u_x \equiv \frac{dx}{dt} \qquad u_y \equiv \frac{dy}{dt} \qquad u_z \equiv \frac{dz}{dt}$$

$$u'_x \equiv \frac{dx'}{dt'} \qquad u'_y \equiv \frac{dy'}{dt'} \qquad u'_z \equiv \frac{dz'}{dt'}$$

$$u_x = \frac{u'_x + v}{1 + vu'_x / c^2}$$

$$u_y = \frac{u'_y}{\gamma_v \left(1 + vu'_x / c^2\right)} \qquad u_z = \frac{u'_z}{\gamma_v \left(1 + vu'_x / c^2\right)}$$

$$\text{Where:} \; \gamma_v \equiv \frac{1}{\sqrt{1 - (v/c)^2}} \equiv \frac{1}{\sqrt{1 - \beta_v{}^2}}$$

Note that the velocity components themselves do not obviously transform according to a Lorentz transformation.

Velocity transformations continued:

$$\text{Consider: } u_x = \frac{u'_x + v}{1 + vu'_x/c^2} \qquad u_y = \frac{u'_y}{\gamma_v\left(1 + vu'_x/c^2\right)} \qquad u_z = \frac{u'_z}{\gamma_v\left(1 + vu'_x/c^2\right)}.$$

$$\text{Note that} \qquad \gamma_u \equiv \frac{1}{\sqrt{1-(u/c)^2}} = \frac{1 + vu'_x/c^2}{\sqrt{1-(u'/c)^2}\sqrt{1-(v/c)^2}}$$

$$\Rightarrow \gamma_u c = \gamma_v\left(\gamma_{u'} c + \beta_v \gamma_{u'} u'_x\right)$$

$$\Rightarrow \gamma_u u_x = \gamma_v\left(\gamma_{u'} u'_x + \gamma_{u'} v\right) = \gamma_v\left(\gamma_{u'} u'_x + \beta_v \gamma_{u'} c\right)$$

$$\Rightarrow \gamma_u u_y = \gamma_{u'} u'_y \qquad \gamma_u u_z = \gamma_{u'} u'_z$$

Velocity 4-vector:

$$\begin{pmatrix} \gamma_u c \\ \gamma_u u_x \\ \gamma_u u_y \\ \gamma_u u_z \end{pmatrix} = \mathcal{L}_v \begin{pmatrix} \gamma_{u'} c \\ \gamma_{u'} u'_x \\ \gamma_{u'} u'_y \\ \gamma_{u'} u'_z \end{pmatrix}$$

Significance of 4-velocity vector: $\begin{pmatrix} \gamma_u c \\ \gamma_u u_x \\ \gamma_u u_y \\ \gamma_u u_z \end{pmatrix}$

Introduce the "rest" mass *m* of particle characterized by velocity **u**:

$$mc\begin{pmatrix} \gamma_u c \\ \gamma_u u_x \\ \gamma_u u_y \\ \gamma_u u_z \end{pmatrix} = \begin{pmatrix} \gamma_u mc^2 \\ \gamma_u m u_x c \\ \gamma_u m u_y c \\ \gamma_u m u_z c \end{pmatrix} = \begin{pmatrix} E \\ p_x c \\ p_y c \\ p_z c \end{pmatrix}$$

Properties of energy-moment 4-vector:

$$\begin{pmatrix} E \\ p_x c \\ p_y c \\ p_z c \end{pmatrix} = \mathcal{L}\begin{pmatrix} E' \\ p'_x c \\ p'_y c \\ p'_z c \end{pmatrix} \qquad \begin{pmatrix} E' \\ p'_x c \\ p'_y c \\ p'_z c \end{pmatrix} = \mathcal{L}^{-1}\begin{pmatrix} E \\ p_x c \\ p_y c \\ p_z c \end{pmatrix} \qquad \text{Note}: \; E^2 - p^2c^2 = E'^2 - p'^2 c^2$$

Properties of Energy-momentum 4-vector -- continued

$$\begin{pmatrix} E \\ p_x c \\ p_y c \\ p_z c \end{pmatrix} = \begin{pmatrix} \gamma_u mc^2 \\ \gamma_u m u_x c \\ \gamma_u m u_y c \\ \gamma_u m u_z c \end{pmatrix}$$

$$\text{Note: } E^2 - p^2c^2 = \frac{\left(mc^2\right)^2}{1-\beta_u^{\,2}}\left(1-\left(\frac{u_x}{c}\right)^2-\left(\frac{u_y}{c}\right)^2-\left(\frac{u_z}{c}\right)^2\right) = \left(mc^2\right)^2 = E'^2 - p'^2\,c^2$$

$$\text{Notion of "rest energy": For } \mathbf{p} \equiv 0, \quad E = mc^2$$

$$\text{Define kinetic energy: } E_K \equiv E - mc^2 = \sqrt{p^2c^2 + m^2c^4} - mc^2$$

$$\text{Non-relativistic limit: If } \frac{p}{mc} << 1, \quad E_K = mc^2\left(\sqrt{1+\left(\frac{p}{mc}\right)^2} - 1\right)$$

$$\approx \frac{p^2}{2m}$$

Summary of relativistic energy relationships

$$\begin{pmatrix} E \\ p_x c \\ p_y c \\ p_z c \end{pmatrix} = \begin{pmatrix} \gamma_u mc^2 \\ \gamma_u m u_x c \\ \gamma_u m u_y c \\ \gamma_u m u_z c \end{pmatrix}$$

$$E = \sqrt{p^2c^2 + m^2c^4} = \gamma_u mc^2$$

$$\text{Check}: \quad \sqrt{p^2c^2 + m^2c^4} = mc^2\sqrt{\gamma_u{}^2\beta_u{}^2 + 1} = \gamma_u mc^2$$

$$\text{Example}: \quad \text{for an electron} \quad mc^2 = 0.5\,\text{MeV}$$

$$\text{for} \quad E = 200\,\text{GeV}$$

$$\gamma_u = \frac{E}{mc^2} = 4 \times 10^5$$

$$\beta_u = \sqrt{1 - \frac{1}{\gamma_u{}^2}} \approx 1 - \frac{1}{2\gamma_u{}^2} \approx 1 - 3 \times 10^{-12}$$

Special theory of relativity and Maxwell's equations

Continuity equation: $$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0$$

Lorentz gauge condition: $$\frac{1}{c}\frac{\partial \Phi}{\partial t} + \nabla \cdot \mathbf{A} = 0$$

Potential equations: $$\frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2} + \nabla^2 \Phi = 4\pi\rho$$

$$\frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} + \nabla^2 \mathbf{A} = \frac{4\pi}{c}\mathbf{J}$$

Field relations: $$\mathbf{E} = -\nabla \Phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

More 4-vectors:

$$\text{Time and position}: \quad \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \Rightarrow x^{\alpha}$$

$$\text{Charge and current}: \quad \begin{pmatrix} c\rho \\ J_x \\ J_y \\ J_z \end{pmatrix} \Rightarrow J^{\alpha}$$

$$\text{Vector and scalar potentials}: \quad \begin{pmatrix} \Phi \\ A_x \\ A_y \\ A_z \end{pmatrix} \Rightarrow A^{\alpha}$$

Lorentz transformations

$$\mathcal{L}_v = \begin{pmatrix} \gamma_v & \gamma_v\beta_v & 0 & 0 \\ \gamma_v\beta_v & \gamma_v & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Time and space: $x^\alpha = \mathcal{L}_v x'^\alpha \equiv \mathcal{L}_v{}^{\alpha\beta} x'^\beta$

Charge and current: $J^\alpha = \mathcal{L}_v J'^\alpha \equiv \mathcal{L}_v{}^{\alpha\beta} J'^\beta$

Vector and scalar potential: $A^\alpha = \mathcal{L}_v A'^\alpha \equiv \mathcal{L}_v{}^{\alpha\beta} A'^\beta$

4-vector relationships

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \Leftrightarrow \begin{pmatrix} A^0 \\ A^1 \\ A^2 \\ A^3 \end{pmatrix} \Leftrightarrow \left(A^0, \mathbf{A}\right): \text{ upper index 4-vector } \quad A^\alpha \text{ for } \left(\alpha = 0,1,2,3\right)$$

Keeping track of signs -- lower index 4-vector $A_\alpha = \left(A^0, -\mathbf{A}\right)$

Derivative operators:

$$\partial^\alpha = \left(\frac{\partial}{c\partial t}, -\nabla\right) \qquad \partial_\alpha = \left(\frac{\partial}{c\partial t}, \nabla\right)$$

Special theory of relativity and Maxwell's equations

Continuity equation: $$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0 \qquad \partial_\alpha J^\alpha = 0$$

Lorentz gauge condition: $$\frac{1}{c}\frac{\partial \Phi}{\partial t} + \nabla \cdot \mathbf{A} = 0 \qquad \partial_\alpha A^\alpha = 0$$

Potential equations: $$\frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2} - \nabla^2 \Phi = 4\pi\rho$$

$$\frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} - \nabla^2 \mathbf{A} = \frac{4\pi}{c}\mathbf{J} \qquad \partial_\alpha \partial^\alpha A^\alpha = \frac{4\pi}{c} J^\alpha$$

Field relations: $$\mathbf{E} = -\nabla \Phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

Electric and Magnetic field relationships

$$\mathbf{E} = -\nabla\Phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}$$

$$E_x = -\frac{\partial \Phi}{\partial x} - \frac{\partial A_x}{c\partial t} = -\left(\partial^0 A^1 - \partial^1 A^0\right)$$

$$E_y = -\frac{\partial \Phi}{\partial y} - \frac{\partial A_y}{c\partial t} = -\left(\partial^0 A^2 - \partial^2 A^0\right)$$

$$E_z = -\frac{\partial \Phi}{\partial z} - \frac{\partial A_z}{c\partial t} = -\left(\partial^0 A^3 - \partial^3 A^0\right)$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$B_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = -\left(\partial^2 A^3 - \partial^3 A^2\right)$$

$$B_y = \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} = -\left(\partial^3 A^1 - \partial^1 A^3\right)$$

$$B_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = -\left(\partial^1 A^2 - \partial^2 A^1\right)$$

Field strength tensor $F^{\alpha\beta} \equiv \left(\partial^\alpha A^\beta - \partial^\beta A^\alpha\right)$

$$F^{\alpha\beta} \equiv \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

Transformation of field strength tensor

$$F^{\alpha\beta} = \mathcal{L}_v{}^{\alpha\gamma} F'^{\gamma\delta} \mathcal{L}_v{}^{\delta\beta} \qquad \mathcal{L}_v = \begin{pmatrix} \gamma_v & \gamma_v\beta_v & 0 & 0 \\ \gamma_v\beta_v & \gamma_v & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$F^{\alpha\beta} = \begin{pmatrix} 0 & -E'_x & -\gamma_v\left(E'_y + \beta_v B'_z\right) & -\gamma_v\left(E'_z - \beta_v B'_y\right) \\ E'_x & 0 & -\gamma_v\left(B'_z + \beta_v E'_y\right) & \gamma_v\left(B'_y - \beta_v E'_z\right) \\ \gamma_v\left(E'_y + \beta_v B'_z\right) & \gamma_v\left(B'_z + \beta_v E'_y\right) & 0 & -B'_x \\ \gamma_v\left(E'_z - \beta_v B'_y\right) & -\gamma_v\left(B'_y - \beta_v E'_z\right) & B'_x & 0 \end{pmatrix}$$