PHY 712 Electrodynamics
11-11:50 AM MWF Olin 107

Plan for Lecture 18:

Start reading Chapter 7

A. Sourceless solutions of Maxwell's Equations

B. Plane polarized electromagnetic waves

C. Reflectance and transmittance

FOREST
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Department of Physics

WFU Physics Colloquium

TITLE: Threading the Needle: Solid-State Nanopores for Biomolecule Detection and Characterization

SPEAKER: Professor Adam Hall,

Joint School of Nanoscience and Nanoengineering, University of North Carolina at Greensboro, Greensboro, North Carolina

TIME: Wednesday February 27, 2013 at 4:00 PM

PLACE: Room 101 Olin Physical Laboratory

Refreshments will be served at 3:30 PM in the Olin Lounge. All interested persons are cordially invited to attend.

10	02-08(Fri)	Chap. 4	Microscopic and macroscopic polarizability	
11	02-11(Mon)	Chap. 5	Magnetostatics	#10
12	02-13(Wed)	Chap. 5	Magnetostatic fields	
13	02-15(Fri)	Chap. 5	Magnetic dipole fields	Exam
14	02-18(Mon)	Chap. 5	Permeable media	Exam
15	02-20(Wed)	Chap. 5	Magnetic susceptibility and permeability	Exam
16	02-22(Fri)	Chap. 6	Maxwell's equations	Exam
17	02-25(Mon)	Chap. 6	Poynting Vector	#11
18	02-27(Wed)	Chap. 7	Reflectance and transmittance of electromagnetic plane waves	#12
19	02-28(Thur)	Chap. 7	Anisotropic media	#13
20	03-01(Fri)			
21	03-04(Mon)			
22	03-06(Wed)			
23	03-07(Thur)			
24	03-08(Fri)			
	03-11(Mon)	*Spring Break*		
	03-13(Mon)	*Spring Break*		
	03-15(Fri)	*Spring Break*		
	03-18(Mon)	*APS Meeting*		Exam
	03-20(Wed)	*APS Meeting*		Exam
	03-22(Fri)	*APS Meeting*		Exam

Maxwell's equations

Coulomb's law: $\nabla \cdot \mathbf{D} = \rho_{free}$

Ampere-Maxwell's law: $\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}_{free}$

Faraday's law: $\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$

No magnetic monopoles: $\nabla \cdot \mathbf{B} = 0$

Maxwell's equations

For linear isotropic media -- $\mathbf{D} = \varepsilon\mathbf{E}; \quad \mathbf{B} = \mu\mathbf{H}$

and no sources:

Coulomb's law: $\nabla \cdot \mathbf{E} = 0$

Ampere-Maxwell's law: $\nabla \times \mathbf{B} - \mu\varepsilon \frac{\partial \mathbf{E}}{\partial t} = 0$

Faraday's law: $\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$

No magnetic monopoles: $\nabla \cdot \mathbf{B} = 0$

Analysis of Maxwell's equations without sources -- continued:

Coulomb's law: $\nabla \cdot \mathbf{E} = 0$

Ampere-Maxwell's law: $\nabla \times \mathbf{B} - \mu\varepsilon \dfrac{\partial \mathbf{E}}{\partial t} = 0$

Faraday's law: $\nabla \times \mathbf{E} + \dfrac{\partial \mathbf{B}}{\partial t} = 0$

No magnetic monopoles: $\nabla \cdot \mathbf{B} = 0$

$$\nabla \times \left(\nabla \times \mathbf{B} - \mu\varepsilon \frac{\partial \mathbf{E}}{\partial t} \right) = -\nabla^2 \mathbf{B} - \mu\varepsilon \frac{\partial (\nabla \times \mathbf{E})}{\partial t}$$

$$= -\nabla^2 \mathbf{B} + \mu\varepsilon \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0$$

$$\nabla \times \left(\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} \right) = -\nabla^2 \mathbf{E} + \frac{\partial (\nabla \times \mathbf{B})}{\partial t}$$

$$= -\nabla^2 \mathbf{E} + \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

Analysis of Maxwell's equations without sources -- continued:
Both E and B fields are solutions to a wave equation:

$$\nabla^2 \mathbf{B} - \frac{1}{v^2} \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0$$

$$\nabla^2 \mathbf{E} - \frac{1}{v^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

$$\text{where} \quad v^2 \equiv c^2 \frac{\mu_0 \varepsilon_0}{\mu \varepsilon} \equiv \frac{c^2}{n^2}$$

Plane wave solutions to wave equation:

$$\mathbf{B}(\mathbf{r}, t) = \Re\left(\mathbf{B}_0 e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}\right) \qquad \mathbf{E}(\mathbf{r}, t) = \Re\left(\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}\right)$$

Analysis of Maxwell's equations without sources -- continued:

Plane wave solutions to wave equation:

$$\mathbf{B}(\mathbf{r},t) = \Re\left(\mathbf{B}_0 e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right) \qquad \mathbf{E}(\mathbf{r},t) = \Re\left(\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right)$$

$$|\mathbf{k}|^2 = \left(\frac{\omega}{v}\right)^2 = \left(\frac{n\omega}{c}\right)^2 \qquad \text{where } n \equiv \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$$

Note: ε, μ, *n*, *k* can all be complex; for the moment we will assume that they are all real (no dissipation).

Note that $\mathbf{E}_0$ and $\mathbf{B}_0$ are not independent;

from Faraday's law: $\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$

$$\Rightarrow \mathbf{B}_0 = \frac{\mathbf{k}\times\mathbf{E}_0}{\omega} = \frac{n\hat{\mathbf{k}}\times\mathbf{E}_0}{c}$$

also note: $\hat{\mathbf{k}}\cdot\mathbf{E}_0 = 0$ and $\hat{\mathbf{k}}\cdot\mathbf{B}_0 = 0$

Analysis of Maxwell's equations without sources -- continued:

Summary of plane electromagnetic waves:

$$\mathbf{B}(\mathbf{r},t) = \Re\left(\frac{n\hat{\mathbf{k}}\times\mathbf{E}_0}{c}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right) \qquad \mathbf{E}(\mathbf{r},t) = \Re\left(\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right)$$

$$|\mathbf{k}|^2 = \left(\frac{\omega}{v}\right)^2 = \left(\frac{n\omega}{c}\right)^2 \quad \text{where } n \equiv \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}} \quad \text{and } \hat{\mathbf{k}}\cdot\mathbf{E}_0 = 0$$

Poynting vector for plane electromagnetic waves:

$$\langle\mathbf{S}\rangle_{avg} = \frac{1}{2}\Re\left(\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\times\frac{1}{\mu}\left(\frac{n\hat{\mathbf{k}}\times\mathbf{E}_0}{c}e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right)^*\right)$$

$$= \frac{n|\mathbf{E}_0|^2}{2\mu c}\hat{\mathbf{k}} = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}|\mathbf{E}_0|^2\hat{\mathbf{k}}$$

Analysis of Maxwell's equations without sources -- continued:

Summary of plane electromagnetic waves:

$$\mathbf{B}(\mathbf{r},t) = \Re\left(\frac{n\hat{\mathbf{k}} \times \mathbf{E}_0}{c} e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t} \right) \qquad \mathbf{E}(\mathbf{r},t) = \Re\left(\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right)$$

$$|\mathbf{k}|^2 = \left(\frac{\omega}{v}\right)^2 = \left(\frac{n\omega}{c}\right)^2 \quad \text{where } n \equiv \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}} \quad \text{and } \hat{\mathbf{k}}\cdot\mathbf{E}_0 = 0$$

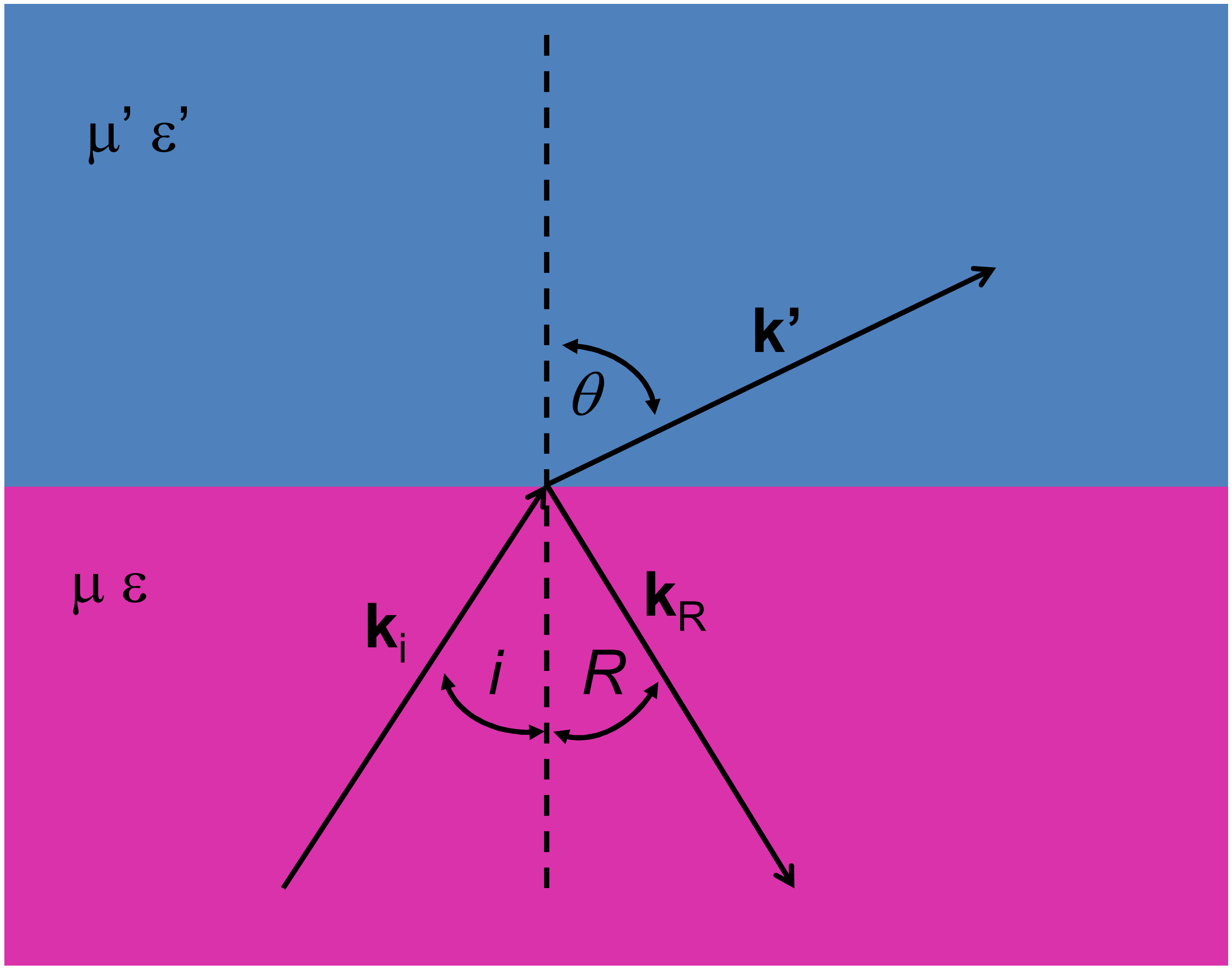
Energy density for plane electromagnetic waves:

$$\langle u \rangle_{avg} = \frac{1}{4}\Re\left(\varepsilon \mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t} \cdot \left(\mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t}\right)^* \right) +$$

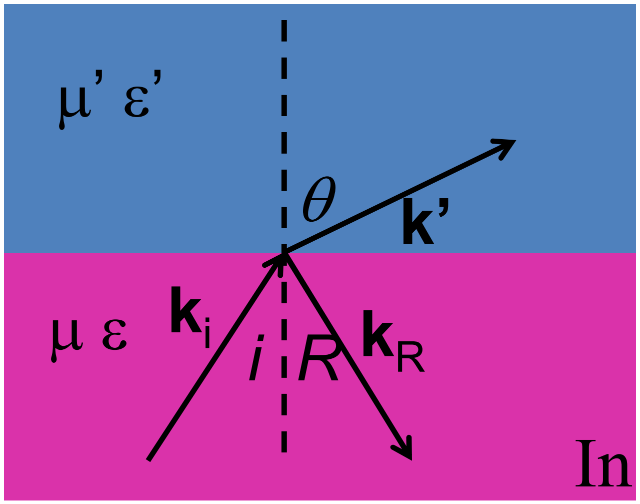
$$\frac{1}{4}\Re\left(\frac{1}{\mu} \frac{n\hat{\mathbf{k}} \times \mathbf{E}_0}{c} e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t} \cdot \left(\frac{n\hat{\mathbf{k}} \times \mathbf{E}_0}{c} e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t} \right)^* \right)$$

$$= \frac{1}{2}\varepsilon|\mathbf{E}_0|^2$$

Reflection and refraction of plane electromagnetic waves at a plane interface between dielectrics (assumed to be lossless)



Reflection and refraction -- continued



In medium $\mu'\varepsilon'$:

$$\mathbf{E}'(\mathbf{r},t)=\Re\left(\mathbf{E}'_0\, e^{i\frac{\omega}{c}\left(n'\hat{\mathbf{k}}'\cdot\mathbf{r}-ct\right)}\right)$$

$$\mathbf{B}'(\mathbf{r},t)=\frac{n'}{c}\hat{\mathbf{k}}'\times\mathbf{E}'(\mathbf{r},t)=\sqrt{\mu'\varepsilon'}\hat{\mathbf{k}}'\times\mathbf{E}(\mathbf{r},t)$$

In medium $\mu\varepsilon$:

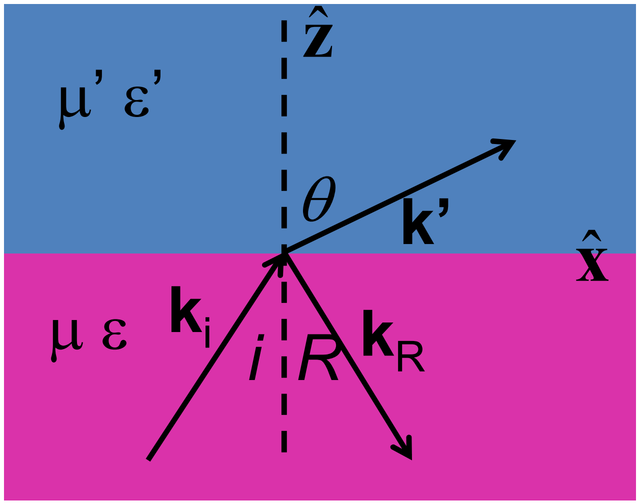
$$\mathbf{E}_i(\mathbf{r},t)=\Re\left(\mathbf{E}_{0i}e^{i\frac{\omega}{c}\left(n\hat{\mathbf{k}}_i\cdot\mathbf{r}-ct\right)}\right)$$

$$\mathbf{B}_i(\mathbf{r},t)=\frac{n}{c}\hat{\mathbf{k}}_i\times\mathbf{E}_i(\mathbf{r},t)=\sqrt{\mu\varepsilon}\hat{\mathbf{k}}_i\times\mathbf{E}_i(\mathbf{r},t)$$

$$\mathbf{E}_R(\mathbf{r},t)=\Re\left(\mathbf{E}_{0R}e^{i\frac{\omega}{c}\left(n\hat{\mathbf{k}}_R\cdot\mathbf{r}-ct\right)}\right)$$

$$\mathbf{B}_R(\mathbf{r},t)=\frac{n}{c}\hat{\mathbf{k}}_R\times\mathbf{E}_R(\mathbf{r},t)=\sqrt{\mu\varepsilon}\hat{\mathbf{k}}_R\times\mathbf{E}_R(\mathbf{r},t)$$

Reflection and refraction -- continued



Snell's law -- matching phase factors at boundary plane:

$$\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + 0\hat{\mathbf{z}}$$

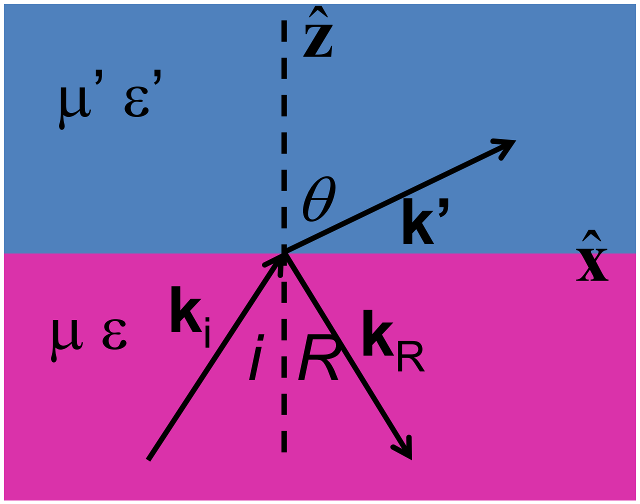
$$\hat{\mathbf{k}}'\cdot\mathbf{r} = x\sin\theta$$

$$\hat{\mathbf{k}}_i\cdot\mathbf{r} = x\sin i = \hat{\mathbf{k}}_R\cdot\mathbf{r} \quad \Rightarrow i = R$$

$$n'\hat{\mathbf{k}}'\cdot\mathbf{r} = n\hat{\mathbf{k}}_i\cdot\mathbf{r} \quad \Rightarrow n'x\sin\theta = nx\sin i$$

Snell's law: $n'\sin\theta = n\sin i$

Reflection and refraction -- continued



Continuity equations at boundary with no sources:

$$\nabla \cdot \mathbf{D} = 0 \qquad\qquad \nabla \cdot \mathbf{B} = 0$$

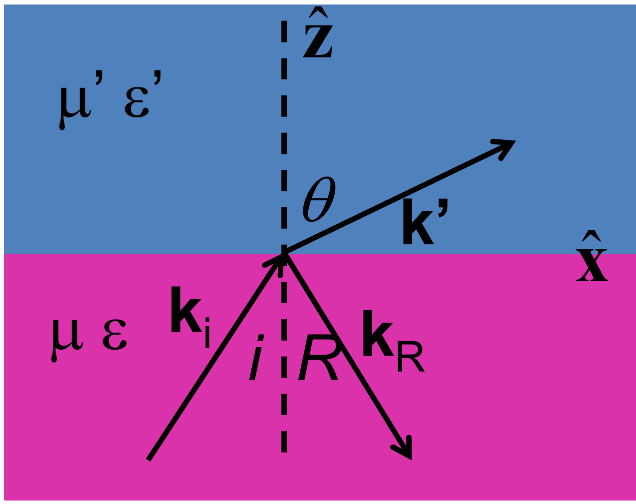
$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = 0 \qquad\qquad \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$

Matching field amplitudes at boundary plane:

$\mathbf{D} \cdot \hat{\mathbf{z}},\ \mathbf{B} \cdot \hat{\mathbf{z}}$ continuous

$\mathbf{H} \times \hat{\mathbf{z}}, \mathbf{E} \times \hat{\mathbf{z}}$ continuous

Reflection and refraction -- continued



Matching field amplitudes at boundary plane:

$\mathbf{D}\cdot\hat{\mathbf{z}}$ continuous:

$$\varepsilon\left(\mathbf{E}_{0i}+\mathbf{E}_{0R}\right)\cdot\hat{\mathbf{z}}=\varepsilon'\mathbf{E}'_0\cdot\hat{\mathbf{z}}$$

$\mathbf{B}\cdot\hat{\mathbf{z}}$ continuous:

$$n\left(\hat{\mathbf{k}}_i\times\mathbf{E}_{0i}+\hat{\mathbf{k}}_R\times\mathbf{E}_{0R}\right)\cdot\hat{\mathbf{z}}=$$
$$n'\hat{\mathbf{k}}'\times\mathbf{E}'_{0i}\cdot\hat{\mathbf{z}}$$

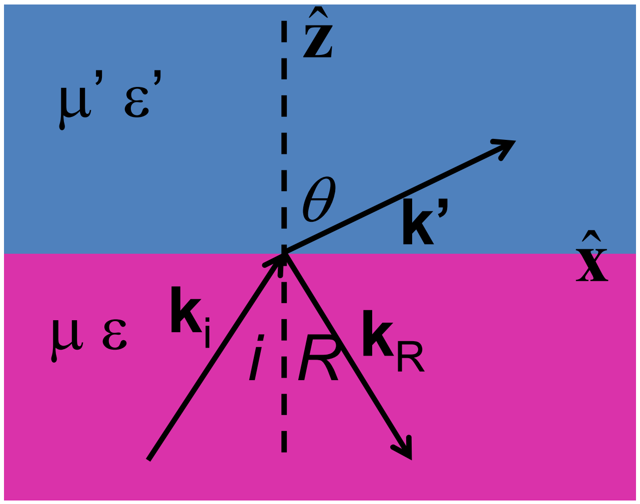
$\mathbf{E}\times\hat{\mathbf{z}}$ continuous:

$$\left(\mathbf{E}_{0i}+\mathbf{E}_{0R}\right)\times\hat{\mathbf{z}}=\mathbf{E}'_0\times\hat{\mathbf{z}}$$

$\mathbf{H}\times\hat{\mathbf{z}}$ continuous:

$$\frac{n}{\mu}\left(\hat{\mathbf{k}}_i\times\mathbf{E}_{0i}+\hat{\mathbf{k}}_R\times\mathbf{E}_{0R}\right)\times\hat{\mathbf{z}}=\frac{n'}{\mu'}\hat{\mathbf{k}}'\times\mathbf{E}'_{0i}\times\hat{\mathbf{z}}$$

Reflection and refraction -- continued



s-polarization – **E** field "polarized" perpendicular to plane of incidence

$$\mathbf{E}\times\hat{\mathbf{z}} \quad \text{continuous:}$$

$$\left(\mathbf{E}_{0i}+\mathbf{E}_{0R}\right)\times\hat{\mathbf{z}}=\mathbf{E}'_0\times\hat{\mathbf{z}}$$

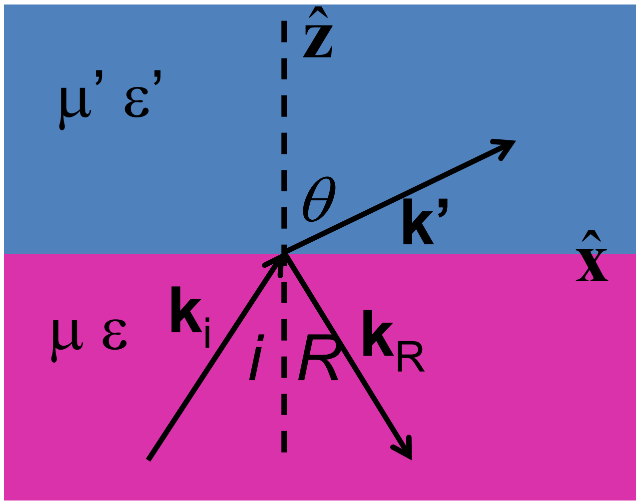
$$\mathbf{H}\times\hat{\mathbf{z}} \quad \text{continuous:}$$

$$\frac{n}{\mu}\left(\hat{\mathbf{k}}_i\times\mathbf{E}_{0i}+\hat{\mathbf{k}}_R\times\mathbf{E}_{0R}\right)\times\hat{\mathbf{z}}=\frac{n'}{\mu'}\hat{\mathbf{k}}'\times\mathbf{E}'_{0i}\times\hat{\mathbf{z}}$$

$$\frac{E_{0R}}{E_{0i}}=\frac{n\cos i-\frac{\mu}{\mu'}n'\cos\theta}{n\cos i+\frac{\mu}{\mu'}n'\cos\theta} \qquad \frac{E'_0}{E_{0i}}=\frac{2n\cos i}{n\cos i+\frac{\mu}{\mu'}n'\cos\theta}$$

$$\text{Note that:}\quad n'\cos\theta=\sqrt{n'^2-n^2\sin^2 i}$$

Reflection and refraction -- continued



p-polarization – **E** field "polarized" parallel to plane of incidence

$$\mathbf{D}\cdot\hat{\mathbf{z}} \quad \text{continuous:}$$

$$\varepsilon\left(\mathbf{E}_{0i}+\mathbf{E}_{0R}\right)\cdot\hat{\mathbf{z}}=\varepsilon'\mathbf{E}'_0\cdot\hat{\mathbf{z}}$$

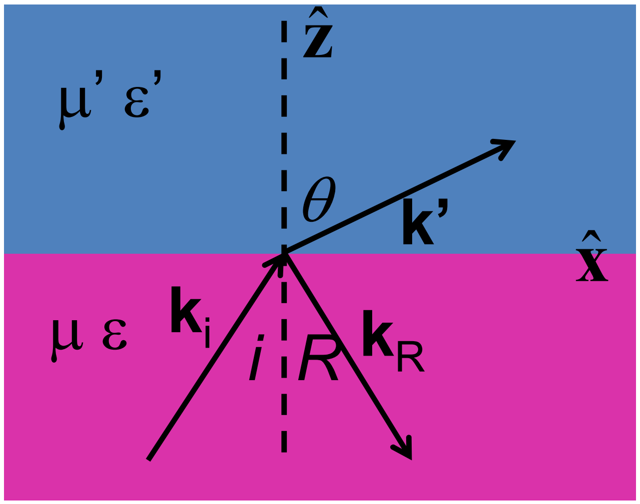
$$\mathbf{H}\times\hat{\mathbf{z}} \quad \text{continuous:}$$

$$\frac{n}{\mu}\left(\hat{\mathbf{k}}_i\times\mathbf{E}_{0i}+\hat{\mathbf{k}}_R\times\mathbf{E}_{0R}\right)\times\hat{\mathbf{z}}=\frac{n'}{\mu'}\hat{\mathbf{k}}'\times\mathbf{E}'_{0i}\times\hat{\mathbf{z}}$$

$$\frac{E_{0R}}{E_{0i}}=\frac{\frac{\mu}{\mu'}n'\cos i-n\cos\theta}{\frac{\mu}{\mu'}n'\cos i+n\cos\theta} \qquad \frac{E'_0}{E_{0i}}=\frac{2n\cos i}{\frac{\mu}{\mu'}n'\cos i+n\cos\theta}$$

$$\text{Note that:} \quad n'\cos\theta=\sqrt{n'^2-n^2\sin^2 i}$$

Reflection and refraction -- continued



Reflectance, transmittance:

$$R = \frac{\mathbf{S}_R \cdot \hat{\mathbf{z}}}{\mathbf{S}_i \cdot \hat{\mathbf{z}}} = \left| \frac{E_{0R}}{E_{0i}} \right|^2 \qquad T = \frac{\mathbf{S}' \cdot \hat{\mathbf{z}}}{\mathbf{S}_i \cdot \hat{\mathbf{z}}} = \left| \frac{E'_0}{E_{0i}} \right|^2 \frac{n'}{n} \frac{\mu}{\mu'} \frac{\cos\theta}{\cos i}$$

Note that $\quad R + T = 1$