```
> with(LinearAlgebra);
```

(1)

```
[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix,
  BidiagonalForm, BilinearForm, CARE, CharacteristicMatrix,
  CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation,
  ColumnSpace, CompanionMatrix, ConditionNumber, ConstantMatrix,
  ConstantVector, Copy, CreatePermutation, CrossProduct, DARE,
  DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix,
  Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues,
  Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm,
  GaussianElimination, GenerateEquations, GenerateMatrix, Generic,
  GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt,
  HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm,
  HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis,
  IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix,
  JordanForm, KroneckerProduct, LA_Main, LUDecomposition, LeastSquares,
  LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential,
  MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm,
  MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply,
  MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm,
  Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm,
  QRDecomposition, RandomMatrix, RandomVector, Rank,
  RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension,
  RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector,
  SchurForm, SingularValues, SmithForm, StronglyConnectedBlocks,
  SubMatrix, SubVector, SumBasis, SylvesterMatrix, SylvesterSolve,
  ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector,
  VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply,
  VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip]
```

```
> fac := 1/(10*4^2) * (3 - Pi^2/(4*4^2));
```

$$fac := \frac{3}{160} - \frac{1}{10240}\,\pi^2 \qquad (2)$$

```
> rrrhs := evalf(Vector(6, [fac*sin(Pi/4)*sin(Pi*3/4), fac*sin(2*Pi/4)*sin(Pi*3/4),
    fac*sin(Pi/4)*sin(Pi*2/4), fac*sin(2*Pi/4)*sin(Pi*2/4), fac*sin(Pi/4)*sin(Pi/4),
    fac*sin(2*Pi/4)*sin(Pi/4)])):
```

$$rrrhs := \begin{bmatrix} 0.008893085722 \\ 0.01257672244 \\ 0.01257672244 \\ 0.01778617144 \\ 0.008893085722 \\ 0.01257672244 \end{bmatrix} \quad (3)$$

> $$MM := \begin{bmatrix} 1 & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{20} & 0 & 0 \\ -\frac{2}{5} & 1 & -\frac{1}{10} & -\frac{1}{5} & 0 & 0 \\ -\frac{1}{5} & -\frac{1}{20} & 1 & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{20} \\ -\frac{1}{10} & -\frac{1}{5} & -\frac{2}{5} & 1 & -\frac{1}{10} & -\frac{1}{5} \\ 0 & 0 & -\frac{1}{5} & -\frac{1}{20} & 1 & -\frac{1}{5} \\ 0 & 0 & -\frac{1}{10} & -\frac{1}{5} & -\frac{2}{5} & 1 \end{bmatrix};$$

$$MM := \begin{bmatrix} 1 & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{20} & 0 & 0 \\ -\frac{2}{5} & 1 & -\frac{1}{10} & -\frac{1}{5} & 0 & 0 \\ -\frac{1}{5} & -\frac{1}{20} & 1 & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{20} \\ -\frac{1}{10} & -\frac{1}{5} & -\frac{2}{5} & 1 & -\frac{1}{10} & -\frac{1}{5} \\ 0 & 0 & -\frac{1}{5} & -\frac{1}{20} & 1 & -\frac{1}{5} \\ 0 & 0 & -\frac{1}{10} & -\frac{1}{5} & -\frac{2}{5} & 1 \end{bmatrix} \quad (4)$$

> $$res := evalf(MatrixInverse(MM).rrrhs);$$

(5)

$$res := \begin{bmatrix} 0.0266009512800000000 \\ 0.0376194260700000033 \\ 0.0376194260700000033 \\ 0.0532019025499999992 \\ 0.0266009512800000000 \\ 0.0376194260700000033 \end{bmatrix} \quad (5)$$

> $fac1 := \frac{1}{2 \cdot \pi^2};$

$$fac1 := \frac{1}{2\,\pi^2} \quad (6)$$

> $exactres := evalf\left(Vector\left(6, \left[fac1 \cdot \sin\left(\frac{Pi}{4} \right) \cdot \sin\left(\frac{Pi \cdot 3}{4} \right), fac1 \cdot \sin\left(\frac{2 \cdot Pi}{4} \right) \cdot \sin\left(\frac{Pi \cdot 3}{4} \right), fac1 \cdot \sin\left(\frac{Pi}{4} \right) \cdot \sin\left(\frac{Pi \cdot 2}{4} \right), fac1 \cdot \sin\left(\frac{2 \cdot Pi}{4} \right) \cdot \sin\left(\frac{Pi \cdot 2}{4} \right), fac1 \cdot \sin\left(\frac{Pi}{4} \right) \cdot \sin\left(\frac{Pi}{4} \right), fac1 \cdot \sin\left(\frac{2 \cdot Pi}{4} \right) \cdot \sin\left(\frac{Pi}{4} \right) \right] \right) \right);$

$$exactres := \begin{bmatrix} 0.02533029590 \\ 0.03582244800 \\ 0.03582244800 \\ 0.05066059180 \\ 0.02533029590 \\ 0.03582244800 \end{bmatrix} \quad (7)$$

> $assume(a > 0);$

> $g1 := evalf\left(int\left(\sin\left(\frac{Pi \cdot x}{a} \right) \cdot \left(1 - \frac{abs\left(x - \frac{a}{4} \right)}{\frac{a}{4}} \right), x = 0 .. \frac{a}{2} \right) \right);$

$$g1 := 0.1678744335\ a\sim \quad (8)$$

> $g2 := evalf\left(int\left(\sin\left(\frac{Pi \cdot x}{a} \right) \cdot \left(1 - \frac{abs\left(x - \frac{a}{2} \right)}{\frac{a}{4}} \right), x = \frac{a}{4} .. \frac{3 \cdot a}{4} \right) \right);$

$$g2 := 0.2374103010\ a\sim \quad (9)$$

> $g3 := evalf\left(int\left(\sin\left(\frac{Pi \cdot x}{a} \right) \cdot \left(1 - \frac{abs\left(x - \frac{3 \cdot a}{4} \right)}{\frac{a}{4}} \right), x = \frac{a}{2} .. a \right) \right);$

$$g3 := 0.1678744335\, a\sim \tag{10}$$

> $ttrhs := \dfrac{Vector(6, [g1\cdot g3,\ g2\cdot g3,\ g1\cdot g2,\ g2\cdot g2,\ g1\cdot g1,\ g2\cdot g1])}{a^2}$;

$$ttrhs := \begin{bmatrix} 0.02818182542 \\ 0.03985511979 \\ 0.03985511979 \\ 0.05636365102 \\ 0.02818182542 \\ 0.03985511979 \end{bmatrix} \tag{11}$$

> $FEMM := \begin{bmatrix} \frac{8}{3} & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & 0 & 0 \\ -\frac{2}{3} & \frac{8}{3} & -\frac{2}{3} & -\frac{1}{3} & 0 & 0 \\ -\frac{1}{3} & -\frac{1}{3} & \frac{8}{3} & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{2}{3} & -\frac{1}{3} & -\frac{2}{3} & \frac{8}{3} & -\frac{2}{3} & -\frac{1}{3} \\ 0 & 0 & -\frac{1}{3} & -\frac{1}{3} & \frac{8}{3} & -\frac{1}{3} \\ 0 & 0 & -\frac{2}{3} & -\frac{1}{3} & -\frac{2}{3} & \frac{8}{3} \end{bmatrix}$;

$$FEMM := \begin{bmatrix} \frac{8}{3} & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & 0 & 0 \\ -\frac{2}{3} & \frac{8}{3} & -\frac{2}{3} & -\frac{1}{3} & 0 & 0 \\ -\frac{1}{3} & -\frac{1}{3} & \frac{8}{3} & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{2}{3} & -\frac{1}{3} & -\frac{2}{3} & \frac{8}{3} & -\frac{2}{3} & -\frac{1}{3} \\ 0 & 0 & -\frac{1}{3} & -\frac{1}{3} & \frac{8}{3} & -\frac{1}{3} \\ 0 & 0 & -\frac{2}{3} & -\frac{1}{3} & -\frac{2}{3} & \frac{8}{3} \end{bmatrix} \tag{12}$$

> $REres := evalf(MatrixInverse(FEMM).ttrhs)$;

$$RERes := \begin{bmatrix} 0.0266572706000000013 \\ 0.0376990736399999971 \\ 0.0376990736399999971 \\ 0.0533145412499999999 \\ 0.0266572706000000013 \\ 0.0376990736399999971 \end{bmatrix} \quad \textbf{(13)}$$

>