PHY 712 – Problem Set # 15

Continue reading Chapter 6 of **Jackson**.

1. Suppose that an electromagnetic wave of pure frequency ω is traveling along the z-axis of a wave guide having a square cross section with side dimension a composed of a medium having permittivity constant ϵ and permeability constant μ. It is known to have the form:

$$\mathbf{E}(\mathbf{r}, t) = \Re \left\{ H_0 e^{ikz - i\omega t} (i\mu\omega) \frac{\pi}{a} \sin \left(\frac{\pi x}{a} \right) \hat{\mathbf{y}} \right\}$$

$$\mathbf{H}(\mathbf{r}, t) = \Re \left\{ H_0 e^{ikz - i\omega t} \left[-ik \frac{\pi}{a} \sin \left(\frac{\pi x}{a} \right) \hat{\mathbf{x}} + \cos \left(\frac{\pi x}{a} \right) \hat{\mathbf{z}} \right] \right\}.$$

Here H_0 denotes a real amplitude, and the parameter k is assumed to be real and equal to

$$k \equiv \sqrt{\omega^2 - \left(\frac{\pi}{a} \right)^2}.$$

Find the form of the time-averaged Poynting vector

$$\langle \mathbf{S} \rangle_{avg} \equiv \frac{1}{2} \Re \left\{ \mathbf{E}(\mathbf{r}, t) \times \mathbf{H}^*(\mathbf{r}, t) \right\}$$

for this electromagnetic wave.