March 24, 2008

PHY 712 – Problem Set # 14

Start reading Chapter 6 of **Jackson**.

1. In the Lorentz gauge and in the absence of sources, we found that the vector $\mathbf{A}(\mathbf{r},t)$ and scalar $\Phi(\mathbf{r},t)$ potentials must satisfy the following equations:

$$\nabla^2\Phi(\mathbf{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Phi(\mathbf{r},t) = 0 \tag{1}$$

and

$$\nabla^2\mathbf{A}(\mathbf{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{A}(\mathbf{r},t) = 0. \tag{2}$$

Assuming solutions of the form:

$$\Phi(\mathbf{r},t) \equiv \Phi_0 \mathrm{e}^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \qquad \mathbf{A}(\mathbf{r},t) \equiv \mathbf{A}_0 \mathrm{e}^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}, \tag{3}$$

where Φ_0 and ω are scalar constants and $\mathbf{A}_0$ and $\mathbf{k}$ are vector constants, find relationships between these constants that must be satisfied in order to satisfy the Maxwell's equations and the Lorentz gauge conditions. Also, determine the corresponding forms of the $\mathbf{E}$ and $\mathbf{B}$ fields.