

**PHY 711 Classical Mechanics and
Mathematical Methods
10-10:50 AM MWF Olin 103**

**Plan for Lecture 7:
Continue reading Chapter 3
1. Lagrange's equations
2. D'Alembert's principle**

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Course schedule

(Preliminary schedule -- subject to frequent adjustment.)

Date	F&W Reading	Topic	Assignment
1 Wed, 8/28/2013	Chap. 1	Review of basic principles; Scattering theory	#1
2 Fri, 8/30/2013	Chap. 1	Scattering theory continued	#2
3 Mon, 9/02/2013	Chap. 1	Scattering theory continued	#3
4 Wed, 9/04/2013	Chap. 2	Accelerated Coordinate Systems	#4
5 Fri, 9/06/2013	Chap. 3	Calculus of variations	#5
6 Mon, 9/09/2013	Chap. 3	Calculus of variations -- continued	
7 Wed, 9/11/2013	Chap. 3	Calculus of variations applied to Lagrangians	#6
8 Fri, 9/13/2013	Chap. 3	Lagrangian mechanics	#7

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News

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Thonhauser group receives funding to investigate MOFs for carbon capture and catalysis


Brian Shoemaker and Prof. Thonhauser featured on WFU homepage

Events

Wed. Sept. 11, 2013
WFU Physics Research @ 4:00 PM in Olin 101
Refreshments at 3:30 in Lobby

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Department of Physics

WFU Physics Colloquium

TITLE: "WFU Physics Research -- Part II"
TIME: Wednesday Sept. 11, 2013 at 4:00 PM
PLACE: George P. Williams, Jr. Lecture Hall, (Olin 101)

Refreshments will be served at 3:30 PM in the lounge. All interested persons are cordially invited to attend.

PROGRAM

This colloquium is the second of two which will highlight physics research at Wake Forest University. During the colloquium, Physics Department faculty members will present short overviews of their research programs in the Physics Department. This forum for sharing ideas will hopefully inspire collaborations between students and faculty and between research groups.

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Summary of results from the calculus of variation

For the class of problems where we need to perform an extremization on an integral form :

$$I = \int_{x_1}^{x_2} f\left(y(x), \frac{dy}{dx}, x\right) dx$$

A necessary condition is the Euler - Lagrange equations :

$$\left(\frac{\partial f}{\partial y}\right) - \frac{d}{dx} \left[\left(\frac{\partial f}{\partial (dy/dx)}\right) \right] = 0$$

$$\frac{d}{dx} \left(f - \frac{\partial f}{\partial (dy/dx)} \frac{dy}{dx} \right) = \left(\frac{\partial f}{\partial x}\right)$$

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Application to particle dynamics

Simple example: vertical trajectory of particle of mass m subject to constant downward acceleration $a=-g$.

$$m \frac{d^2 y}{dt^2} = -mg$$

$$y(t) = y_i + v_i t - \frac{1}{2} g t^2$$

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Now consider the Lagrangian defined to be :

$$L\left(\left\{y(t), \frac{dy}{dt}\right\}, t\right) \equiv T - U$$

Kinetic energy
Potential energy

In our example :

$$L\left(\left\{y(t), \frac{dy}{dt}\right\}, t\right) \equiv T - U = \frac{1}{2} m \left(\frac{dy}{dt}\right)^2 - mgy$$

Hamilton's principle states :

$$S \equiv \int_{t_i}^{t_f} \left(\frac{1}{2} m \left(\frac{dy}{dt}\right)^2 - mgy\right) dt \text{ is minimized for physical } y(t) :$$

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<http://www.hamilton2005.ie/>

Sir William Rowan Hamilton

Wednesday, September 11th, 2013

Tribute to Sir William Hamilton

Hello and welcome! This page is dedicated to the life and work of Sir William Rowan Hamilton.

William Rowan Hamilton was Ireland's greatest scientist. He was an mathematician, physicist, and astronomer and made important works in optics, dynamics, and algebra.

His contribution in dynamics plays a important role in the later developed quantum mechanics. His name was perpetuated in one of the fundamental concepts in quantum mechanics, called "Hamiltonian".

The Discovery of Quaternions is probably his most familar invention today.

2005 was the Hamilton Year, celebrating his 200th birthday. The year was dedicated to celebrate Irish Science. 2005 was called the Einstein year also, reminding of three great papers of the year 1905. So UNESCO designated 2005 to the World Year of Physics

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Éire

$i^2 = j^2 = k^2 = -1$

William Rowan Hamilton
1805-1865

48c

<http://rjlipton.wordpress.com>

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Condition for minimizing the action :

$$S \equiv \int_{t_i}^{t_f} \left(\frac{1}{2} m \left(\frac{dy}{dt} \right)^2 - mgy \right) dt$$

Euler - Lagrange relations :

$$\frac{\partial L}{\partial y} - \frac{d}{dt} \frac{\partial L}{\partial \dot{y}} = 0$$

$$\Rightarrow -mg - \frac{d}{dt} m\dot{y} = 0$$

$$\Rightarrow \frac{d}{dt} \dot{y} = -g$$

$$y(t) = y_i + v_i t - \frac{1}{2} g t^2$$

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Check :

$$S \equiv \int_{t_i}^{t_f} \left(\frac{1}{2} m \left(\frac{dy}{dt} \right)^2 - mgy \right) dt$$

Assume $t_i = 0$, $y_i = H = \frac{1}{2} g T^2$; $t_f = T$, $y_f = 0$

Trial trajectories : $y_1(t) = \frac{1}{2} g T^2 (1 - t/T) = H - \frac{1}{2} g T t$
 $y_2(t) = \frac{1}{2} g T^2 (1 - t^2/T^2) = H - \frac{1}{2} g t^2$
 $y_3(t) = \frac{1}{2} g T^2 (1 - t^3/T^3) = H - \frac{1}{2} g t^3 / T$

Maple says :


$$S_1 = -0.125 m g^2 T^3$$

$$S_2 = -0.167 m g^2 T^3$$

$$S_3 = -0.150 m g^2 T^3$$

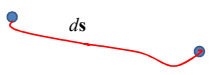
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Jean d'Alembert 1717-1783



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D'Alembert's principle:



Generalized coordinates :
 $q_\sigma(\{x_i\})$

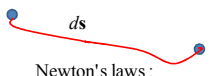
Newton's laws :
 $\mathbf{F} - m\mathbf{a} = 0 \Rightarrow (\mathbf{F} - m\mathbf{a}) \cdot d\mathbf{s} = 0$

$$\mathbf{F} \cdot d\mathbf{s} = \sum_\sigma \sum_i F_i \frac{\partial x_i}{\partial q_\sigma} \delta q_\sigma$$

For a conservative force : $F_i = -\frac{\partial U}{\partial x_i}$

$$\mathbf{F} \cdot d\mathbf{s} = -\sum_\sigma \sum_i \frac{\partial U}{\partial x_i} \frac{\partial x_i}{\partial q_\sigma} \delta q_\sigma = -\sum_\sigma \frac{\partial U}{\partial q_\sigma} \delta q_\sigma$$

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Generalized coordinates :
 $q_\sigma(\{x_i\})$

Newton's laws :
 $\mathbf{F} - m\mathbf{a} = 0 \Rightarrow (\mathbf{F} - m\mathbf{a}) \cdot d\mathbf{s} = 0$

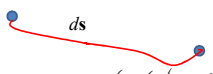
$$m\mathbf{a} \cdot d\mathbf{s} = \sum_\sigma \sum_i m\ddot{x}_i \frac{\partial x_i}{\partial q_\sigma} \delta q_\sigma$$

$$= \sum_\sigma \sum_i \left(\frac{d}{dt} \left(m\dot{x}_i \frac{\partial x_i}{\partial q_\sigma} \right) - m\dot{x}_i \frac{d}{dt} \frac{\partial x_i}{\partial q_\sigma} \right) \delta q_\sigma$$

Claim : $\frac{\partial x_i}{\partial q_\sigma} = \frac{\partial \dot{x}_i}{\partial \dot{q}_\sigma}$ and $\frac{d}{dt} \frac{\partial x_i}{\partial q_\sigma} = \frac{\partial}{\partial q_\sigma} \frac{dx_i}{dt} \equiv \frac{\partial \dot{x}_i}{\partial q_\sigma}$

$$m\mathbf{a} \cdot d\mathbf{s} = \sum_\sigma \sum_i \left(\frac{d}{dt} \left(\frac{\partial (\frac{1}{2} m \dot{x}_i^2)}{\partial \dot{q}_\sigma} \right) - \frac{\partial (\frac{1}{2} m \dot{x}_i^2)}{\partial q_\sigma} \right) \delta q_\sigma$$

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Generalized coordinates :
 $q_\sigma(\{x_i\})$

$$m\mathbf{a} \cdot d\mathbf{s} = \sum_\sigma \sum_i \left(\frac{d}{dt} \left(\frac{\partial (\frac{1}{2} m \dot{x}_i^2)}{\partial \dot{q}_\sigma} \right) - \frac{\partial (\frac{1}{2} m \dot{x}_i^2)}{\partial q_\sigma} \right) \delta q_\sigma$$

Define -- kinetic energy : $T \equiv \sum_i \frac{1}{2} m \dot{x}_i^2$

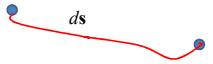
$$m\mathbf{a} \cdot d\mathbf{s} = \sum_\sigma \left(\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_\sigma} - \frac{\partial T}{\partial q_\sigma} \right) \delta q_\sigma$$

Recall :

$$\mathbf{F} \cdot d\mathbf{s} = \sum_\sigma \sum_i \frac{\partial U}{\partial x_i} \frac{\partial x_i}{\partial q_\sigma} \delta q_\sigma = \sum_\sigma \frac{\partial U}{\partial q_\sigma} \delta q_\sigma$$

$$(\mathbf{F} - m\mathbf{a}) \cdot d\mathbf{s} = \sum_\sigma \frac{\partial U}{\partial q_\sigma} \delta q_\sigma - \sum_\sigma \left(\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_\sigma} - \frac{\partial T}{\partial q_\sigma} \right) \delta q_\sigma = 0$$

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Generalized coordinates :
 $q_\sigma(\{x_i\})$

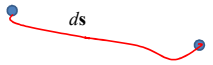
$$(\mathbf{F}-m\mathbf{a}) \cdot d\mathbf{s} = -\sum_\sigma \frac{\partial U}{\partial q_\sigma} \delta q_\sigma - \sum_\sigma \left(\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_\sigma} - \frac{\partial T}{\partial q_\sigma} \right) \delta q_\sigma = 0$$

$$= -\sum_\sigma \left(\frac{d}{dt} \frac{\partial (T-U)}{\partial \dot{q}_\sigma} - \frac{\partial (T-U)}{\partial q_\sigma} \right) \delta q_\sigma = 0$$

$L(q_\sigma, \dot{q}_\sigma; t) = T - U$

Note : This is only true if
 $\frac{\partial U}{\partial \dot{q}_\sigma} = 0$

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Generalized coordinates :
 $q_\sigma(\{x_i\})$

Define -- Lagrangian : $L \equiv T - U$
 $L = L(\{q_\sigma\}, \{\dot{q}_\sigma\}, t)$

$$(\mathbf{F}-m\mathbf{a}) \cdot d\mathbf{s} = -\sum_\sigma \left(\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_\sigma} - \frac{\partial L}{\partial q_\sigma} \right) \delta q_\sigma = 0$$

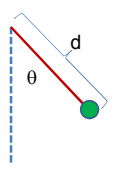
\Rightarrow Minimization integral : $S = \int_{t_i}^{t_f} L(\{q_\sigma\}, \{\dot{q}_\sigma\}, t) dt$

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Euler - Lagrange equations : $L = L(\{q_\sigma\}, \{\dot{q}_\sigma\}, t) = T - U$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_\sigma} - \frac{\partial L}{\partial q_\sigma} = 0$$

Example:



$$L = L(\theta, \dot{\theta}) = \frac{1}{2} m d^2 \dot{\theta}^2 - mg(d - d \cos \theta)$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = 0 \Rightarrow \frac{d}{dt} m d^2 \dot{\theta} - mg d \sin \theta = 0$$

$$\frac{d^2 \theta}{dt^2} = \frac{g}{d} \sin \theta$$

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Another example: $L = L(\{q_\sigma\}, \{\dot{q}_\sigma\}, t) = T - U$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_\sigma} - \frac{\partial L}{\partial q_\sigma} = 0$$

$$L = L(\alpha, \beta, \gamma, \dot{\alpha}, \dot{\beta}, \dot{\gamma}) = \frac{1}{2} I_1 (\dot{\alpha}^2 \sin^2 \beta + \dot{\beta}^2) + \frac{1}{2} I_3 (\dot{\alpha} \cos \beta + \dot{\gamma})^2 - Mgd \cos \beta$$

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