PHY 711 Classical Mechanics and Mathematical Methods
10-10:50 AM MWF Olin 103

Plan for Lecture 4:

Chapter 2 – Physics described in an accelerated coordinate frame

1. Linear acceleration
2. Angular acceleration
3. Foucault pendulum

PHY 711 Classical Mechanics and Mathematical Methods

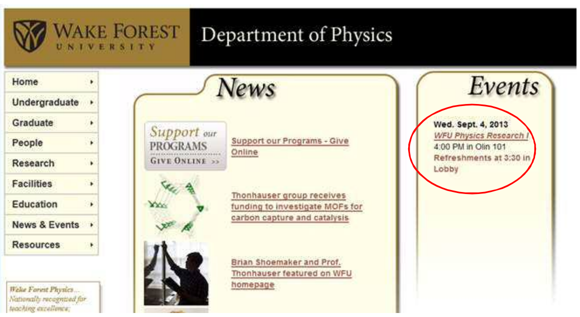
MWF 10 AM-10:50 PM	OPL 103	http://www.wfu.edu/~natalie/f13phy711/

Instructor: Natalie Holzwarth	Phone:758-5510	Office:300 OPL	e-mail:natalie@wfu.edu

Course schedule

(Preliminary schedule -- subject to frequent adjustment.)

	Date	F&W Reading	Topic	Assignment
1	Wed, 8/28/2013	Chap. 1	Review of basic principles;Scattering theory	#1
2	Fri, 8/30/2013	Chap. 1	Scattering theory continued	#2
3	Mon, 9/02/2013	Chap. 1	Scattering theory continued	#3
4	Wed, 9/04/2013	Chap. 2	Accelerated Coordinate Systems	#4



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WFU Physics Colloquium

TITLE: "WFU Physics Research -- Part I"
TIME: Wednesday Sept. 4, 2012 at 4:00 PM
PLACE: George P. Williams, Jr. Lecture Hall, (Olin 101)

Refreshments will be served at 3:30 PM in the lounge. All interested persons are cordially invited to attend.

PROGRAM

This colloquium is the first of two which will highlight physics research at Wake Forest University. During the colloquium, Physics Department faculty members will present short overviews of their research programs in the Physics Department. This forum for sharing ideas will hopefully inspire collaborations between students and faculty and between research groups.

Fall 2013 Schedule
for N. A. W. Holzwarth

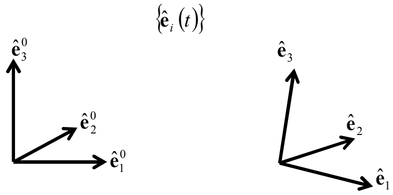
	Monday	Tuesday	Wednesday	Thursday	Friday
8:00-10:00	Lecture Preparation/ Office Hours	Lecture Preparation/ Office Hours	Lecture Preparation/ Office Hours	Lecture Preparation/ Office Hours	Lecture Preparation/ Office Hours
10:00-11:00	Classical Mech PHY711		Classical Mech PHY711		Classical Mech PHY711
11:00-12:30	Office Hours	General Physics I PHY113	Office Hours	General Physics I PHY113	Office Hours
12:30-2:00	Condensed Matter Theory Journal Club	Office Hours	Physics Research	Office Hours	Physics Research
2:00-3:30	Condensed Matter Monthly Meeting	Physics Research		Physics Research	
3:30-5:00	Physics Research		Physics Colloquium		

avel dates:

- Oct. 27 -- Nov. 1, 2013 Electrochemical Society Meeting

Physical laws as described in non-inertial coordinate systems

- Newton's laws are formulated in an inertial frame of reference $\{\hat{\mathbf{e}}_i^0\}$
- For some problems, it is convenient to transform the the equations into a non-inertial coordinate system $\{\hat{\mathbf{e}}_i(t)\}$



Comparison of analysis in "inertial frame" versus "non-inertial frame"

Denote by $\hat{e}_i^0$ a fixed coordinate system

Denote by $\hat{e}_i$ a moving coordinate system

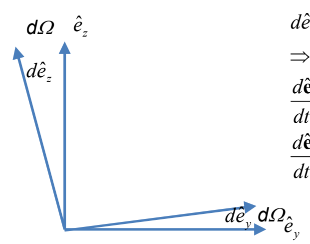
$$\mathbf{V} = \sum_{i=1}^{3} V_i^0 \hat{e}_i^0 = \sum_{i=1}^{3} V_i \hat{e}_i$$

$$\left(\frac{d\mathbf{V}}{dt}\right)_{inertial} = \sum_{i=1}^{3} \frac{dV_i^0}{dt}\hat{e}_i^0 = \sum_{i=1}^{3} \frac{dV_i}{dt}\hat{e}_i + \sum_{i=1}^{3} V_i \frac{d\hat{e}_i}{dt}$$

Define : $\left(\frac{d\mathbf{V}}{dt}\right)_{body} \equiv \sum_{i=1}^{3} \frac{dV_i}{dt}\hat{e}_i$

$$\Rightarrow \left(\frac{d\mathbf{V}}{dt}\right)_{inertial} = \left(\frac{d\mathbf{V}}{dt}\right)_{body} + \sum_{i=1}^{3} V_i \frac{d\hat{e}_i}{dt}$$

Properties of the frame motion (rotation only):



$$d\hat{e}_y = d\Omega \hat{e}_z$$

$$d\hat{e}_z = -d\Omega \hat{e}_y$$

$$\Rightarrow d\hat{\mathbf{e}} = d\mathbf{\Omega} \times \hat{\mathbf{e}}$$

$$\frac{d\hat{\mathbf{e}}}{dt} = \frac{d\mathbf{\Omega}}{dt} \times \hat{\mathbf{e}}$$

$$\frac{d\hat{\mathbf{e}}}{dt} = \boldsymbol{\omega} \times \hat{\mathbf{e}}$$

Properties of the frame motion (rotation only) -- continued

$$\left(\frac{d\mathbf{V}}{dt}\right)_{inertial} = \left(\frac{d\mathbf{V}}{dt}\right)_{body} + \sum_{i=1}^{3} V_i \frac{d\hat{e}_i}{dt}$$

$$\left(\frac{d\mathbf{V}}{dt}\right)_{inertial} = \left(\frac{d\mathbf{V}}{dt}\right)_{body} + \boldsymbol{\omega} \times \mathbf{V}$$

Effects on acceleration (rotation only):

$$\left(\frac{d}{dt}\frac{d\mathbf{V}}{dt}\right)_{inertial} = \left(\left(\frac{d}{dt}\right)_{body} + \boldsymbol{\omega} \times\right)\left\{\left(\frac{d\mathbf{V}}{dt}\right)_{body} + \boldsymbol{\omega} \times \mathbf{V}\right\}$$

$$\left(\frac{d^2\mathbf{V}}{dt^2}\right)_{inertial} = \left(\frac{d^2\mathbf{V}}{dt^2}\right)_{body} + 2\boldsymbol{\omega} \times \left(\frac{d\mathbf{V}}{dt}\right)_{body} + \frac{d\boldsymbol{\omega}}{dt} \times \mathbf{V} + \boldsymbol{\omega} \times \boldsymbol{\omega} \times \mathbf{V}$$

Application of Newton's laws in a coordinate system which has an angular velocity $\boldsymbol{\omega}$ and linear acceleration $\left(\frac{d^2\mathbf{a}}{dt^2}\right)_{inertial}$

Newton's laws; Let **r** denote the position of particle of mass *m*:

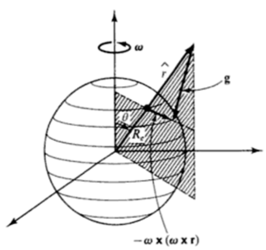
$$m\left(\frac{d^2\mathbf{r}}{dt^2}\right)_{inertial} = \mathbf{F}_{ext}$$

$$m\left(\frac{d^2\mathbf{r}}{dt^2}\right)_{body} = \mathbf{F}_{ext} - m\left(\frac{d^2\mathbf{a}}{dt^2}\right)_{inertial} - 2m\boldsymbol{\omega}\times\left(\frac{d\mathbf{r}}{dt}\right)_{body} - m\frac{d\boldsymbol{\omega}}{dt}\times\mathbf{r} - m\boldsymbol{\omega}\times\boldsymbol{\omega}\times\mathbf{r}$$

Coriolis force

Centrifugal force

Motion on the surface of the Earth:



$$\omega = \frac{2\pi}{\tau} \approx 7.3\times10^{-5}\,\text{rad/s}$$

$$\mathbf{F}_{ext} = -\frac{GM_e m}{r^2}\hat{\mathbf{r}} + \mathbf{F}'$$

Main contributions :

$$m\left(\frac{d^2\mathbf{r}}{dt^2}\right)_{earth} = -\frac{GM_e m}{r^2}\hat{\mathbf{r}} + \mathbf{F}' - 2m\boldsymbol{\omega}\times\left(\frac{d\mathbf{r}}{dt}\right)_{earth} - m\frac{d\boldsymbol{\omega}}{dt}\times\mathbf{r} - m\boldsymbol{\omega}\times\boldsymbol{\omega}\times\mathbf{r}$$

Non-inertial effects on effective gravitational "constant"

$$m\left(\frac{d^2\mathbf{r}}{dt^2}\right)_{earth} = -\frac{GM_e m}{r^2}\hat{\mathbf{r}} + \mathbf{F}' - 2m\boldsymbol{\omega}\times\left(\frac{d\mathbf{r}}{dt}\right)_{earth} - m\frac{d\boldsymbol{\omega}}{dt}\times\mathbf{r} - m\boldsymbol{\omega}\times\boldsymbol{\omega}\times\mathbf{r}$$

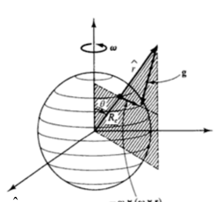
For $\left(\frac{d\mathbf{r}}{dt}\right)_{earth} = 0$ and $\left(\frac{d^2\mathbf{r}}{dt^2}\right)_{earth} = 0$,

$$0 = -\frac{GM_e m}{r^2}\hat{\mathbf{r}} + \mathbf{F}' - m\boldsymbol{\omega}\times\boldsymbol{\omega}\times\mathbf{r}$$

$$\mathbf{F}' = -m\mathbf{g}$$

$$\Rightarrow \mathbf{g} = -\frac{GM_e}{r^2}\hat{\mathbf{r}} - \boldsymbol{\omega}\times\boldsymbol{\omega}\times\mathbf{r}\Big|_{r\approx R_e}$$

$$= \left(-\frac{GM_e}{R_e^2} + \omega^2 R_e \sin^2\theta\right)\hat{\mathbf{r}} + \sin\theta\cos\theta\,\omega^2 R_e\hat{\boldsymbol{\theta}}$$



0.03 m/s²

9.80 m/s²

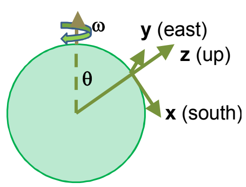
Foucault pendulum

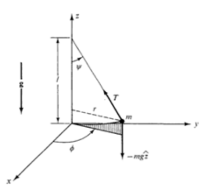
http://www.si.edu/Encyclopedia_SI/nmah/pendulum.htm

The Foucault pendulum was displayed for many years in the Smithsonian's National Museum of American History. It is named for the French physicist Jean Foucault who first used it in 1851 to demonstrate the rotation of the earth.

Equation of motion on Earth's surface

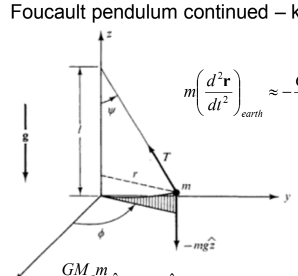
$$m\left(\frac{d^2\mathbf{r}}{dt^2}\right)_{earth} = -\frac{GM_e m}{r^2}\hat{\mathbf{r}} + \mathbf{F}' - 2m\boldsymbol{\omega}\times\left(\frac{d\mathbf{r}}{dt}\right)_{earth} - m\frac{d\boldsymbol{\omega}}{dt}\times\mathbf{r} - m\boldsymbol{\omega}\times\boldsymbol{\omega}\times\mathbf{r}$$





$$\boldsymbol{\omega} \approx -\omega\sin\theta\hat{\mathbf{x}} + \omega\cos\theta\hat{\mathbf{z}}$$

Foucault pendulum continued – keeping leading terms:



$$m\left(\frac{d^2\mathbf{r}}{dt^2}\right)_{earth} \approx -\frac{GM_e m}{R_e^2}\hat{\mathbf{r}} + \mathbf{F}' - 2m\boldsymbol{\omega}\times\left(\frac{d\mathbf{r}}{dt}\right)_{earth}$$

$$-\frac{GM_e m}{r^2}\hat{\mathbf{r}} \approx -mg\hat{\mathbf{z}}$$

$$\mathbf{F}' \approx -T\sin\psi\cos\phi\hat{\mathbf{x}} - T\sin\psi\sin\phi\hat{\mathbf{y}} + T\cos\psi\hat{\mathbf{z}}$$

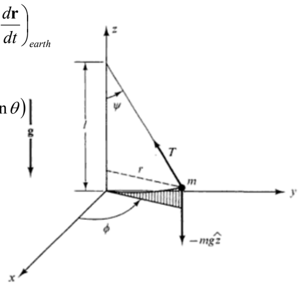
$$\boldsymbol{\omega} \approx -\omega\sin\theta\hat{\mathbf{x}} + \omega\cos\theta\hat{\mathbf{z}}$$

$$\boldsymbol{\omega}\times\left(\frac{d\mathbf{r}}{dt}\right)_{earth} \approx \omega\left(-\dot{y}\cos\theta\hat{\mathbf{x}} + (\dot{x}\cos\theta + \dot{z}\sin\theta)\hat{\mathbf{y}} - \dot{y}\sin\theta\hat{\mathbf{z}}\right)$$

Foucault pendulum continued – keeping leading terms:

$$m\left(\frac{d^2\mathbf{r}}{dt^2}\right)_{earth} \approx -\frac{GM_e m}{R_e^2}\hat{\mathbf{r}} + \mathbf{F}' - 2m\boldsymbol{\omega}\times\left(\frac{d\mathbf{r}}{dt}\right)_{earth}$$

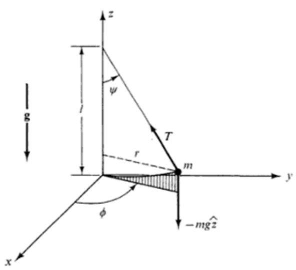
$$m\ddot{x} \approx -T\sin\psi\cos\phi + 2m\omega\dot{y}\cos\theta$$
$$m\ddot{y} \approx -T\sin\psi\sin\phi - 2m\omega(\dot{x}\cos\theta + \dot{z}\sin\theta)$$
$$m\ddot{z} \approx T\cos\psi - mg + 2m\omega\dot{y}\sin s\theta$$



Further approximation :
$\psi << 1;\quad \ddot{z} \approx 0;\quad T \approx mg$
$$m\ddot{x} \approx -mg\sin\psi\cos\phi + 2m\omega\dot{y}\cos\theta$$
$$m\ddot{y} \approx -mg\sin\psi\sin\phi - 2m\omega\dot{x}\cos\theta$$
Also note that :
$$x \approx \ell\sin\psi\cos\phi$$
$$y \approx \ell\sin\psi\sin\phi$$

Foucault pendulum continued – coupled equations:

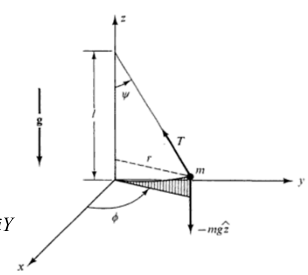
$$\ddot{x} \approx -\frac{g}{\ell}x + 2\omega\cos\theta\dot{y}$$
$$\ddot{y} \approx -\frac{g}{\ell}y - 2\omega\cos\theta\dot{x}$$



Try to find a solution of the form :
$x(t) = Xe^{-iqt} \qquad y(t) = Ye^{-iqt}$
Denote $\quad \omega_\perp \equiv \omega\cos\theta$
$$\begin{pmatrix} -q^2 + \frac{g}{\ell} & i2\omega_\perp q \\ -i2\omega_\perp q & -q^2 + \frac{g}{\ell} \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} = 0$$
Non - trivial solutions :
$$q_\pm = \alpha \pm \beta \equiv \omega_\perp \pm \sqrt{\omega_\perp^2 + \frac{g}{\ell}}$$

Foucault pendulum continued – coupled equations:

Solution continued :
$x(t) = Xe^{-iqt} \qquad y(t) = Ye^{-iqt}$
$$\begin{pmatrix} -q^2 + \frac{g}{\ell} & i2\omega_\perp q \\ -i2\omega_\perp q & -q^2 + \frac{g}{\ell} \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} = 0$$
Non - trivial solutions :
$$q_\pm = \alpha \pm \beta \equiv \omega_\perp \pm \sqrt{\omega_\perp^2 + \frac{g}{\ell}}$$
Amplitude relationship : $\quad X = iY$



General solution with complex amplitudes C and D :
$$x(t) = \mathrm{Re}\left\{iCe^{-i(\alpha+\beta)t} + iDe^{-i(\alpha-\beta)t}\right\}$$
$$y(t) = \mathrm{Re}\left\{Ce^{-i(\alpha+\beta)t} + De^{-i(\alpha-\beta)t}\right\}$$

General solution with complex amplitudes C and D:

$$x(t) = \mathrm{Re}\left\{iCe^{-i(\alpha+\beta)t} + iDe^{-i(\alpha-\beta)t}\right\}$$
$$y(t) = \mathrm{Re}\left\{Ce^{-i(\alpha+\beta)t} + De^{-i(\alpha-\beta)t}\right\}$$

$$q_\pm = \alpha \pm \beta \equiv \omega_\perp \pm \sqrt{\omega_\perp{}^2 + \frac{g}{\ell}} \approx \omega_\perp \pm \sqrt{\frac{g}{\ell}}$$

since $\omega_\perp \approx 7\times 10^{-5}\cos\theta \ \ rad/s << \sqrt{\frac{g}{\ell}}$

Suppose: $x(0) = X_0 \qquad y(0) = 0$

$$x(t) = X_0 \cos\left(\sqrt{\frac{g}{\ell}}t\right)\cos(\omega_\perp t)$$
$$y(t) = -X_0 \cos\left(\sqrt{\frac{g}{\ell}}t\right)\sin(\omega_\perp t)$$

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