PHY 711 Classical Mechanics and Mathematical Methods
10-10:50 AM MWF Olin 103

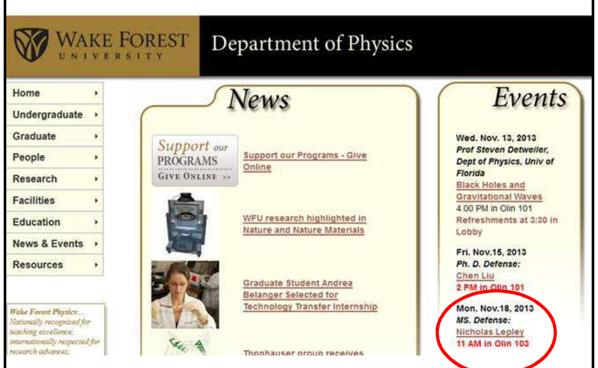
Plan for Lecture 32:

Chapter 10 in F & W: Surface waves

1. **Water waves in a channel**
2. **Wave-like solutions; wave speed**

25	Fri, 10/25/2013	Chap. 5	Rigid rotations	
	Mon, 10/28/2013	No class	Take-home exam	
	Wed, 10/30/2013	No class	Take-home exam	
	Fri, 11/01/2013	No class	Take-home exam	
26	Mon, 11/04/2013	Chap. 8	Oscillations in two-dimensional membranes	Take-home exam due
27	Wed, 11/06/2013	Chap. 9	Physics of fluids	#22
28	Fri, 11/08/2013	Chap. 9	Physics of fluids	#23
29	Mon, 11/11/2013	Chap. 9	Sound Waves	#24
30	Wed, 11/13/2013	Chap. 9	Sound Waves	#25
31	Fri, 11/15/2013	Chap. 9	Non linear effects in Sound	#26
32	Mon, 11/18/2013	Chap. 10	Surface waves	
33	Wed, 11/20/2013	Chap. 10	Surface waves	
34	Fri, 11/22/2013			
35	Mon, 11/25/2013			
	Wed, 11/27/2013		*Thanksgiving Holiday*	
	Fri, 11/29/2013		*Thanksgiving Holiday*	
36	Mon, 12/02/2013		Student presentations I	
37	Wed, 12/04/2013		Student presentations II	
38	Fri, 12/06/2013		Student presentations III	
	Mon, 12/09/2013		Begin Take-home final	

preparation for presentations



WFU Physics Master's Thesis Defense

TITLE: An Investigation of Lithium Solid Electrolyte Materials with First Principles Calculations

SPEAKER: Nicholas Lepley,

Department of Physics
Wake Forest University

TIME: Monday November 18, 2013 at 11 AM

PLACE: Room 103 Olin Physical Laboratory

All interested persons are cordially invited to attend.

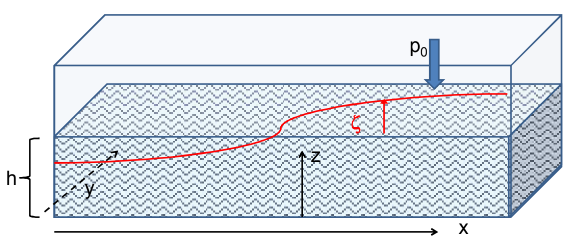
ABSTRACT

Inorganic solid electrolyte materials have recently become the focus of considerable interest due to the discovery of novel compounds with high ionic conductivities (10^{-4}-10^{-3} S/cm). Sulfur-based solid electrolytes are particularly notable in this regard, as well as for their compatibility for Li-S electrode systems. This work focuses on applying computational methods based on density functional theory to identifying and characterizing novel

Physics of incompressible fluids and their surfaces

Reference: Chapter 10 of Fetter and Walecka

Consider a container of water with average height h and surface $h+\zeta(x,y,t)$; ($h \leftrightarrow z_0$ on some of the slides)



Euler's equation for incompressible fluid:

$$\frac{d\mathbf{v}}{dt} = f_{applied} - \frac{\nabla p}{\rho} = -g\hat{\mathbf{z}} - \frac{\nabla p}{\rho}$$

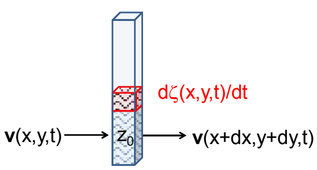
Assume that $v_z << v_x, v_y$ $\quad \Rightarrow -g - \frac{1}{\rho}\frac{\partial p}{\partial z} \approx 0$

$$\Rightarrow p(x,y,z,t) = p_0 + \rho g\left(\zeta(x,y,t) + h - z\right)$$

Horizontal fluid motions:

$$\frac{dv_x}{dt} \approx \frac{\partial v_x}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} = -g\frac{\partial \zeta}{\partial x}$$

$$\frac{dv_y}{dt} \approx \frac{\partial v_y}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial y} = -g\frac{\partial \zeta}{\partial y}$$



Continuity condition for flow of incompressible fluid:

$$\frac{\partial \zeta}{\partial t} + z_0 \nabla \cdot \mathbf{v} = 0$$

From horizontal flow relations: $\quad \frac{\partial \mathbf{v}}{\partial t} = -g\nabla\zeta$

Equation for surface function: $\quad \frac{\partial^2 \zeta}{\partial t^2} - g z_0 \nabla^2 \zeta = 0$

Surface wave equation:

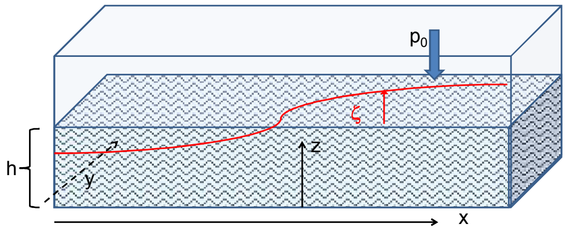
$$\frac{\partial^2 \zeta}{\partial t^2} - c^2 \nabla^2 \zeta = 0 \qquad c^2 = g z_0$$

More complete analysis finds:

$$c^2 = \frac{g}{k}\tanh(k z_0) \qquad \text{where } k = \frac{2\pi}{\lambda}$$

More details: -- recall setup --

Consider a container of water with average height h and surface h+ζ(x,y,t)



Euler's equation for incompressible fluid :

$$\frac{d\mathbf{v}}{dt} = \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v} = \frac{\partial \mathbf{v}}{\partial t} + \nabla\left(\tfrac{1}{2}v^2\right) + \mathbf{v}\times(\nabla\times\mathbf{v}) = -\nabla U - \frac{\nabla p}{\rho}$$

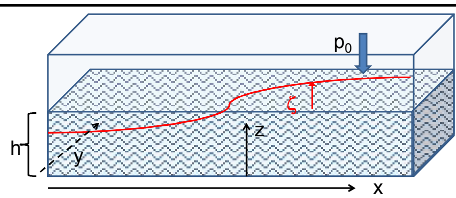
Assume that $\nabla\times\mathbf{v} = 0$ (irrotational flow) $\Rightarrow \mathbf{v} = -\nabla\Phi$

$$\Rightarrow \nabla\left(-\frac{\partial\Phi}{\partial t} + \tfrac{1}{2}v^2 + U + \frac{p}{\rho}\right) = 0$$

$$\Rightarrow -\frac{\partial\Phi}{\partial t} + \tfrac{1}{2}v^2 + U + \frac{p}{\rho} = \text{constant (within the fluid)}$$

For the same system, the continuity condition becomes

$$\nabla\cdot\mathbf{v} = -\nabla^2\Phi = 0$$



Within fluid : $0 \le z \le h + \zeta$

$-\frac{\partial\Phi}{\partial t} + \tfrac{1}{2}v^2 + g(z-h) = \text{constant}$ (We have absorbed

$-\nabla^2\Phi = 0$ p_0 in our constant.)

At surface : $z = h + \zeta$ with $\zeta = \zeta(x,y,t)$

$$\frac{d\zeta}{dt} = \frac{\partial\zeta}{\partial t} + v_x\frac{\partial\zeta}{\partial x} + v_y\frac{\partial\zeta}{\partial y} \quad \text{where } v_{x,y} = v_{x,y}(x,y,h+\zeta,t)$$

Full equations:

Within fluid : $0 \le z \le h+\zeta$

$$-\frac{\partial \Phi}{\partial t}+\tfrac{1}{2}v^2+g(z-h)=\text{constant} \quad \text{(We have absorbed}$$

$$-\nabla^2\Phi=0 \qquad p_0 \text{ in our constant.)}$$

At surface : $z=h+\zeta$ with $\zeta=\zeta(x,y,t)$

$$\frac{d\zeta}{dt}=\frac{\partial \zeta}{\partial t}+v_x\frac{\partial \zeta}{\partial x}+v_y\frac{\partial \zeta}{\partial y} \quad \text{where } v_{x,y}=v_{x,y}(x,y,h+\zeta,t)$$

Linearized equations:

For $0 \le z \le h+\zeta$: $-\frac{\partial \Phi}{\partial t}+g(z-h)=0 \qquad -\nabla^2\Phi=0$

At surface : $z=h+\zeta \quad \frac{d\zeta}{dt}=\frac{\partial \zeta}{\partial t}=v_z(x,y,h+\zeta,t)$

$$-\frac{\partial \Phi(x,y,h+\zeta,t)}{\partial t}+g\zeta=0$$

For simplicity, keep only linear terms and assume that horizontal variation is only along *x*:

For $0 \le z \le h+\zeta$: $\nabla^2\Phi=\left(\frac{\partial^2}{\partial z^2}+\frac{\partial^2}{\partial x^2}\right)\Phi(x,z,t)=0$

Consider and periodic waveform : $\Phi(x,z,t)=Z(z)\cos(k(x-ct))$

$$\Rightarrow \left(\frac{d^2}{dz^2}-k^2\right)Z(z)=0$$

Boundary condition at bottom of tank : $v_z(x,0)=0$

$$\Rightarrow \frac{dZ}{dz}(0)=0 \qquad Z(z)=A\cosh(kz)$$

For simplicity, keep only linear terms and assume that horizontal variation is only along *x* – continued:

At surface : $z=h+\zeta \quad \frac{\partial \zeta}{\partial t}=v_z(x,h+\zeta,t)=-\frac{\partial \Phi(x,h+\zeta,t)}{\partial z}$

$$-\frac{\partial \Phi(x,h+\zeta,t)}{\partial t}+g\zeta=0$$

$$-\frac{\partial^2 \Phi(x,h+\zeta,t)}{\partial t^2}+g\frac{\partial \zeta}{\partial t}=-\frac{\partial^2 \Phi(x,h+\zeta,t)}{\partial t^2}-g\frac{\partial \Phi(x,h+\zeta,t)}{\partial z}=0$$

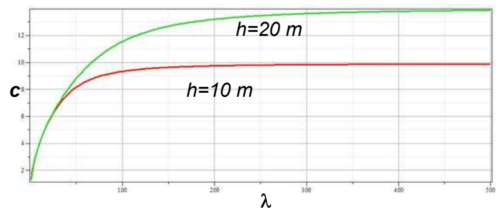
For $\Phi(x,(h+\zeta),t)=A\cosh(k(h+\zeta))\cos(k(x-ct))$

$$A\cosh(k(h+\zeta))\cos(k(x-ct))\left(k^2c^2-gk\frac{\sinh(k(h+\zeta))}{\cosh(k(h+\zeta))}\right)=0$$

For simplicity, keep only linear terms and assume that horizontal variation is only along *x* – continued:

$$c^2 = \frac{g}{k}\frac{\sinh(k(h+\zeta))}{\cosh(k(h+\zeta))} = \frac{g}{k}\tanh(k(h+\zeta))$$

$$\text{Assuming } \zeta << h: \qquad c^2 = \frac{g}{k}\tanh(kh)$$



For simplicity, keep only linear terms and assume that horizontal variation is only along *x* – continued:

$$c^2 \approx \frac{g}{k}\tanh(kh) \qquad \text{For } \lambda >> h,\; c^2 \approx gh$$

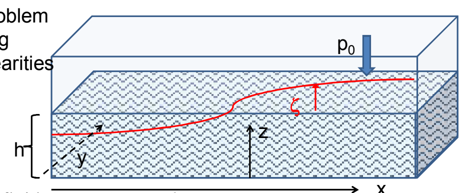
$$\Phi(x,z,t) = A\cosh(kz)\cos(k(x-ct))$$

$$\zeta(x,t) = \frac{1}{g}\frac{\partial\Phi(x,h+\zeta,t)}{\partial t} \approx \frac{kc}{g}A\cosh(kh)\sin(k(x-ct))$$

Note that for $\lambda >> h, \quad c^2 \approx gh$

(solutions are consistent with previous analysis)

General problem including non-linearities



$$\text{Within fluid}: \qquad 0 \le z \le h+\zeta$$

$$-\frac{\partial\Phi}{\partial t} + \tfrac{1}{2}v^2 + g(z-h) = \text{constant} \quad \text{(We have absorbed}$$

$$-\nabla^2\Phi = 0 \qquad p_0 \text{ in our constant.)}$$

$$\text{At surface}: \quad z = h+\zeta \qquad \text{with } \zeta = \zeta(x,y,t)$$

$$\frac{d\zeta}{dt} = \frac{\partial\zeta}{\partial t} + v_x\frac{\partial\zeta}{\partial x} + v_y\frac{\partial\zeta}{\partial y} \qquad \text{where } v_{x,y} = v_{x,y}(x,y,h+\zeta,t)$$