

PHY 711 Classical Mechanics and Mathematical Methods

10-10:50 AM MWF Olin 103

Plan for Lecture 2:

- 1. Comments on Maple software**
- 2. Chapter 1 – scattering theory**
 - a) Rutherford scattering**
 - b) Scattering for arbitrary potential**

PHY 711 Classical Mechanics and Mathematical Methods

MWF 10 AM-10:50 PM OPL 103 <http://www.wfu.edu/~natalie/f13phy711/>

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Course schedule

(Preliminary schedule -- subject to frequent adjustment.)

	Date	F&W Reading	Topic	Assignment
1	Wed, 8/28/2013	Chap. 1	Review of basic principles;Scattering theory	#1
2	Fri, 8/30/2013	Chap. 1	Scattering theory continued	#2
3	Mon, 9/02/2013	Chap. 1	Scattering theory continued	#3

Some additional Maple examples

The screenshot shows the Maple 15 software interface. The window title is "D:\Userdata\Userdata\Coursework\12phy711\Lecturenotes\Lecture2\anothermaple.mw - [Server 1] - Maple 15". The menu bar includes File, Edit, View, Insert, Format, Table, Drawing, Plot, Spreadsheet, Tools, Window, and Help. The toolbar contains various icons for file operations, editing, and viewing. On the left, there is a sidebar with categories like Favorites, MapleCloud (Disabled), Variables, Handwriting, Expression, Units (SI), Units (FPS), Common Symbols, Matrix, Components, Greek, Arrows, Relational, Relational Round, Negated, and Large Operators. The main workspace is divided into tabs for Text, Math, Drawing, Plot, and Animation. The Math tab is active, showing a series of commands and their results:

```

> assume(a > 0 and a < 1);
> assume(t > 0);
> Y := (t, a) -> int(sqrt(1 - a^2 * (sin(theta))^2), theta = 0 .. t);

```

$$Y := (t, a) \rightarrow \int_0^t \sqrt{1 - a^2 \sin^2(\theta)} d\theta \quad (1)$$

```

> Y(t, a);

```

$$\frac{\sqrt{1 - \sin^2(t)} \operatorname{EllipticE}(\sin(t), a)}{\cos(t)} + \left(\begin{array}{l} 2 \operatorname{floor}\left(\frac{1}{4} \frac{2t + \pi}{\pi}\right) \operatorname{EllipticE}(a) \\ 0 \end{array} \quad \begin{array}{l} 1 < \frac{1}{4} \frac{2t + \pi}{\pi} \\ \text{otherwise} \end{array} \right) + 2 \left(\right) \quad (2)$$

$$\left(\begin{array}{l} 2 \left(\operatorname{floor}\left(-\frac{1}{4} \frac{-2t + \pi}{\pi}\right) + 1 \right) \operatorname{EllipticE}(a) \\ 0 \end{array} \quad \begin{array}{l} 0 < 2t - \pi \\ \text{otherwise} \end{array} \right) + \left(\right)$$

$$\left(\begin{array}{l} 2 \left(\operatorname{floor}\left(-\frac{1}{4} \frac{-2t + 3\pi}{\pi}\right) + 1 \right) \operatorname{EllipticE}(a) \\ 0 \end{array} \quad \begin{array}{l} 0 < 2t - 3\pi \\ \text{otherwise} \end{array} \right)$$

```

> plot({Y(t, 0.6), Y(t, 0.8), Y(t, 0.99)}, t = 0 .. 5);

```

The plot shows three curves: a blue curve (Y(t, 0.6)), a green curve (Y(t, 0.8)), and a red curve (Y(t, 0.99)). The x-axis ranges from 0 to 5, and the y-axis ranges from 0 to 10. The curves are piecewise linear with sharp jumps at regular intervals, corresponding to the floor functions in the equations above.

At the bottom right of the window, the status bar shows "Memory: 0.74M Time: 0.09s Text Mode".

Scattering theory:

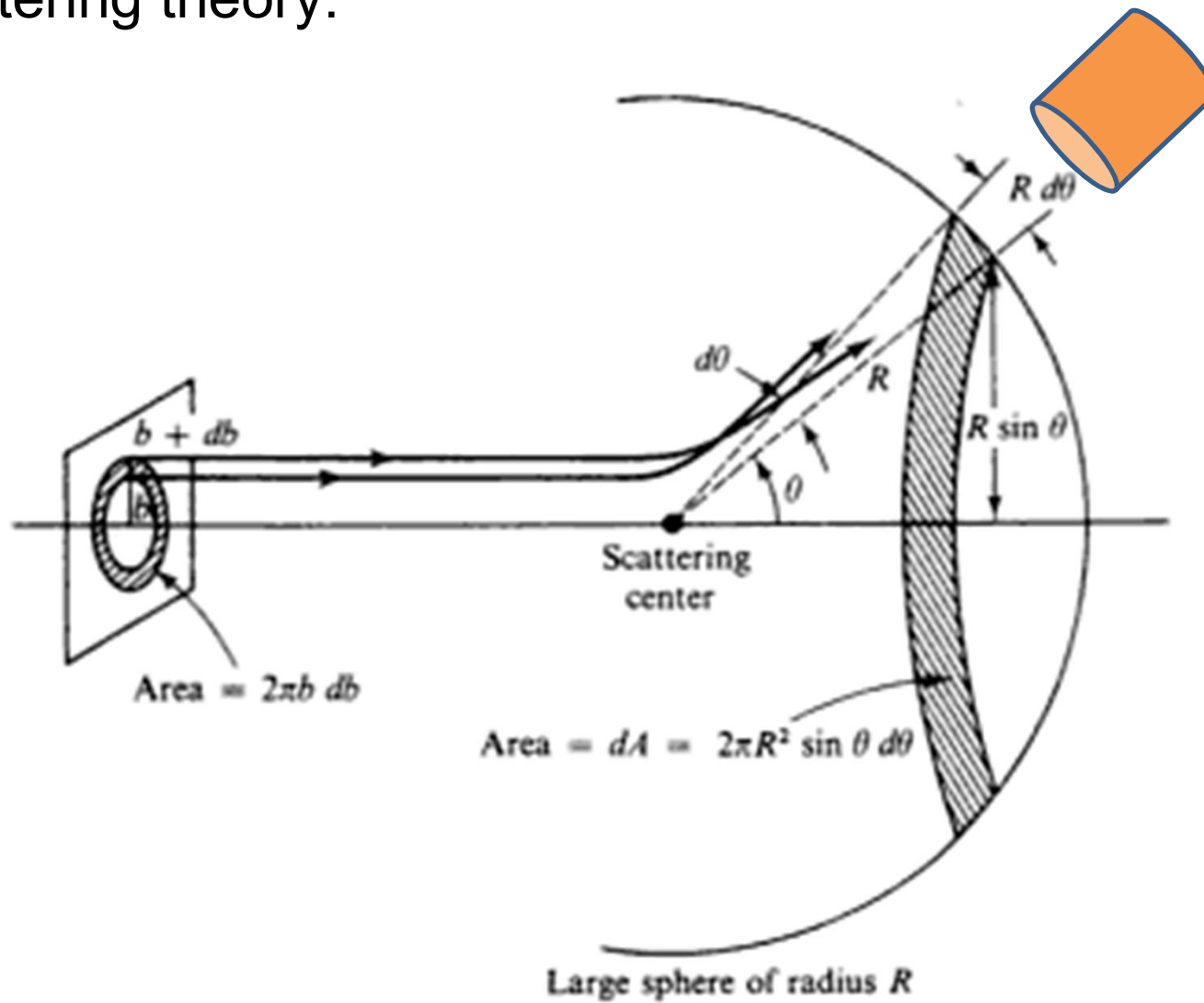
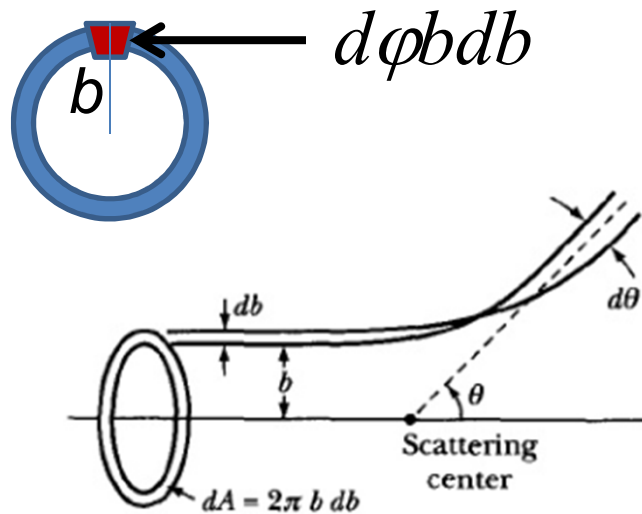


Figure 5.5 The scattering problem and relation of cross section to impact parameter.

Differential cross section

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\text{Number of detected particles at } \theta \text{ per target particle}}{\text{Number of incident particles per unit area}}$$

= Area of incident beam that is scattered into detector
at angle θ

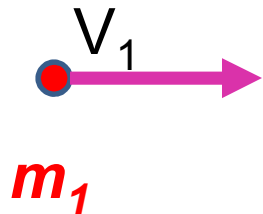


$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{d\phi b db}{d\phi \sin\theta d\theta} = \frac{b}{\sin\theta} \left|\frac{db}{d\theta}\right|$$

Figure from Marion & Thorton, Classical Dynamics

Note: The following analysis will be carried out in the center of mass frame of reference.

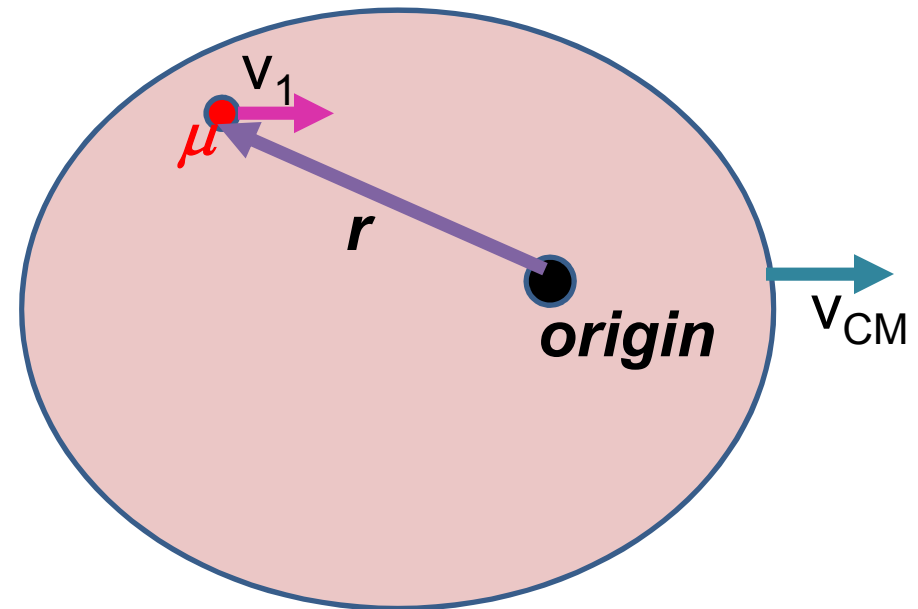
In laboratory frame:



$$\mu = \frac{m_1 m_{\text{target}}}{m_1 + m_{\text{target}}}$$

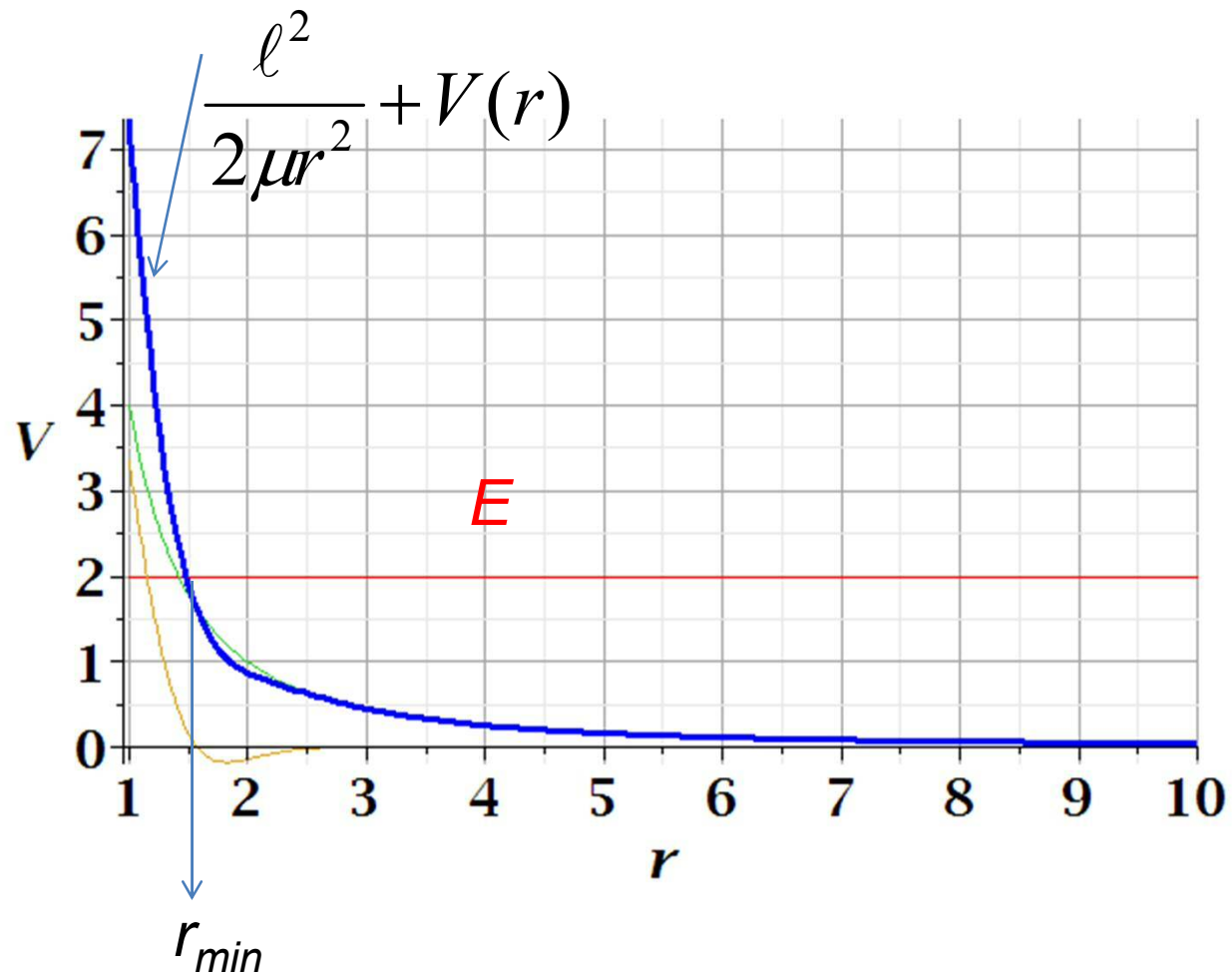
$$\ell = |\mathbf{r} \times \mu \mathbf{v}_1|$$

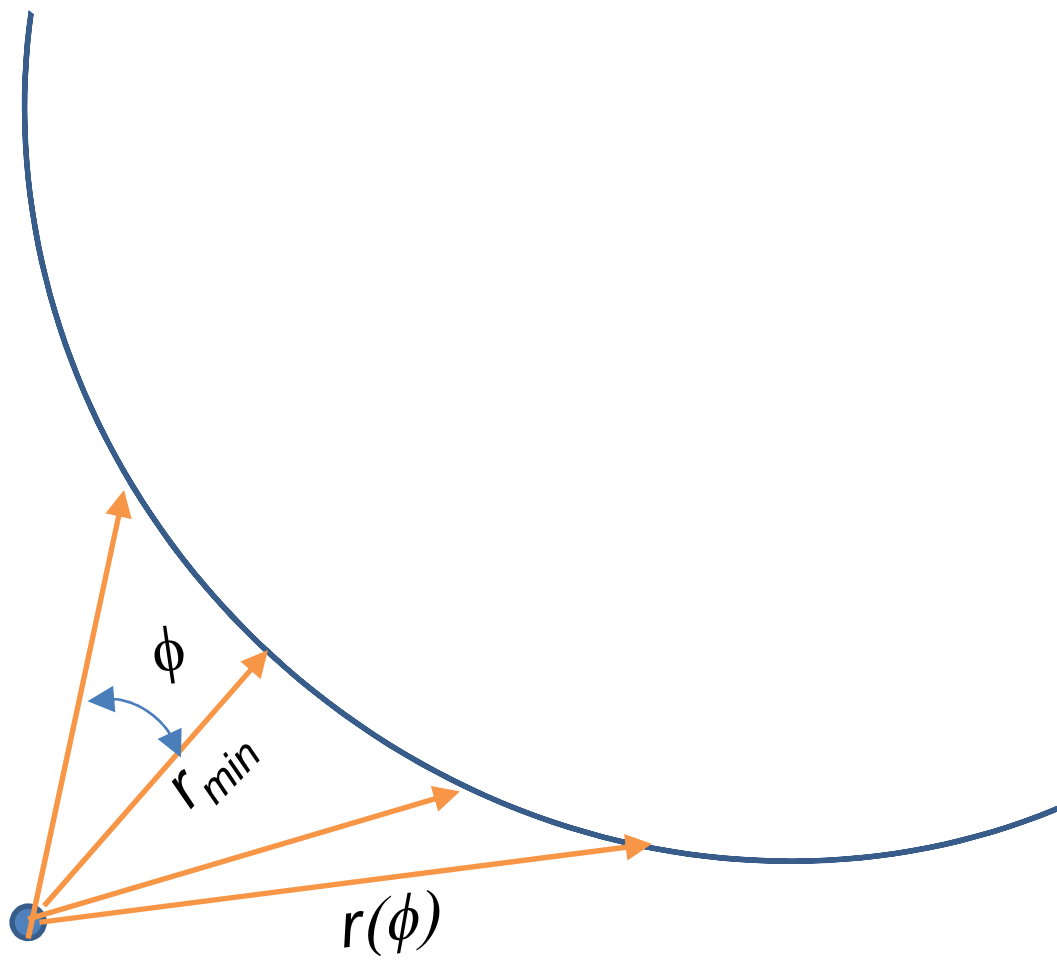
In center-of-mass frame:



Also note: We are assuming that the interaction between particle and target $V(r)$ conserves energy and angular momentum.

In center of mass reference frame:





Conservation of angular momentum:

$$\ell = \mu r^2 \left(\frac{d\phi}{dt} \right)$$

Transformation of trajectory variables :

$$r(t) \Leftrightarrow r(\phi)$$

$$\frac{dr}{dt} = \frac{dr}{d\phi} \frac{d\phi}{dt} = \frac{dr}{d\phi} \frac{\ell}{\mu r^2}$$

Conservation of energy in the center of mass frame :

$$\begin{aligned} E &= \frac{1}{2} \mu \left(\frac{dr}{dt} \right)^2 + \frac{\ell^2}{2\mu r^2} + V(r) \\ &= \frac{1}{2} \mu \left(\frac{dr}{d\phi} \frac{\ell}{\mu r^2} \right)^2 + \frac{\ell^2}{2\mu r^2} + V(r) \end{aligned}$$

$$\begin{aligned} \Rightarrow E &= \frac{1}{2} \mu \left(\frac{dr}{dt} \right)^2 + \frac{\ell^2}{2\mu r^2} + V(r) \\ &= \frac{1}{2} \mu \left(\frac{dr}{d\phi} \frac{\ell}{\mu r^2} \right)^2 + \frac{\ell^2}{2\mu r^2} + V(r) \end{aligned}$$

Solving for $r(\phi) \Leftrightarrow \phi(r)$

$$\left(\frac{dr}{d\phi} \right)^2 = \left(\frac{2\mu r^4}{\ell^2} \right) \left(E - \frac{\ell^2}{2\mu r^2} - V(r) \right)$$

$$d\phi = dr \left(\frac{\ell / r^2}{\sqrt{2\mu \left(E - \frac{\ell^2}{2\mu r^2} - V(r) \right)}} \right)$$

$$d\phi = dr \left(\frac{\ell / r^2}{\sqrt{2\mu \left(E - \frac{\ell^2}{2\mu r^2} - V(r) \right)}} \right)$$

Further simplification at large separation:

$$\ell = \mu v_{\infty} b$$

$$E = \frac{1}{2} \mu v_{\infty}^2$$

$$\Rightarrow \ell = \sqrt{2\mu E} b$$

When the dust clears :

$$d\phi = dr \left(\frac{\ell / r^2}{\sqrt{2\mu \left(E - \frac{\ell^2}{2\mu r^2} - V(r) \right)}} \right)$$

$$d\phi = dr \left(\frac{b / r^2}{\sqrt{1 - \frac{b^2}{r^2} - \frac{V(r)}{E}}} \right)$$

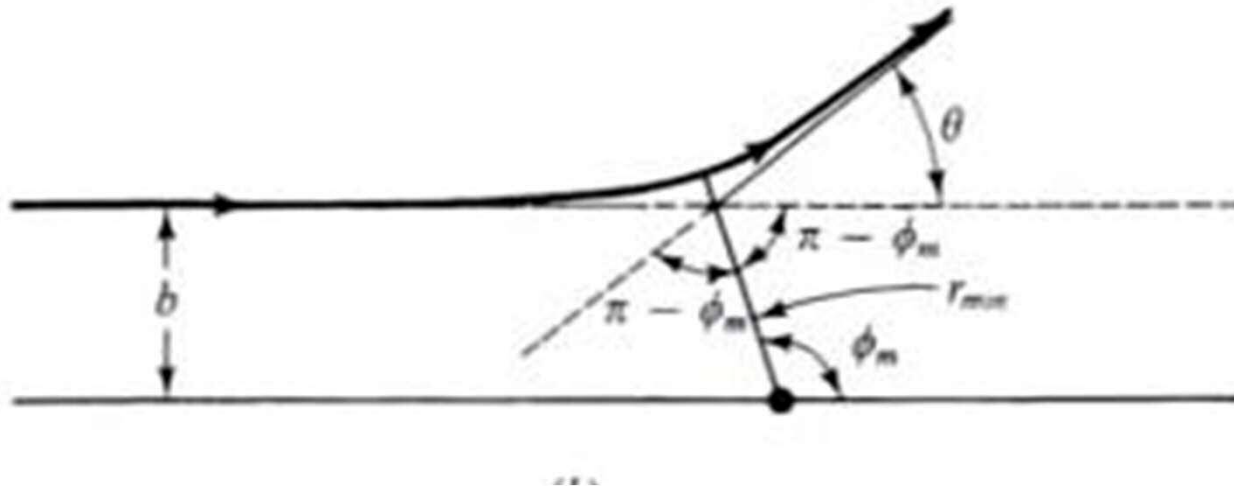
$$\Rightarrow \phi(b, E)$$

$$\int_0^{\phi_{\max}} d\phi = \int_{r_{\min}}^{\infty} dr \left(\frac{b/r^2}{\sqrt{1 - \frac{b^2}{r^2} - \frac{V(r)}{E}}} \right)$$

where :

$$1 - \frac{b^2}{r_{\min}^2} - \frac{V(r_{\min})}{E} = 0$$

Relationship between ϕ_{\max} and θ :



$$2(\pi - \phi_{\max}) + \theta = \pi$$

$$\Rightarrow \phi_{\max} = \frac{\pi}{2} - \frac{\theta}{2}$$

$$\phi_{\max} = \frac{\pi}{2} - \frac{\theta}{2} = \int_{r_{\min}}^{\infty} dr \left(\frac{b/r^2}{\sqrt{1 - \frac{b^2}{r^2} - \frac{V(r)}{E}}} \right)$$

$$\theta = \pi - 2b \int_{r_{\min}}^{\infty} dr \left(\frac{1/r^2}{\sqrt{1 - \frac{b^2}{r^2} - \frac{V(r)}{E}}} \right)$$

$$\theta = \pi - 2b \int_0^{1/r_{\min}} du \left(\frac{1}{\sqrt{1 - b^2 u^2 - \frac{V(1/u)}{E}}} \right)$$

Scattering angle equation :

$$\theta = \pi - 2b \int_0^{1/r_{\min}} du \left(\frac{1}{\sqrt{1 - b^2 u^2 - \frac{V(1/u)}{E}}} \right)$$

Rutherford scattering example :

$$\frac{V(r)}{E} \equiv \frac{\kappa}{r} \quad \frac{1}{r_{\min}} = \frac{1}{b} \left(-\frac{\kappa}{2b} + \sqrt{\left(\frac{\kappa}{2b}\right)^2 + 1} \right)$$

$$\theta = \pi - 2b \int_0^{1/r_{\min}} du \left(\frac{1}{\sqrt{1 - b^2 u^2 - \kappa u}} \right) = 2 \sin^{-1} \left(\frac{1}{\sqrt{(2b/\kappa)^2 + 1}} \right)$$

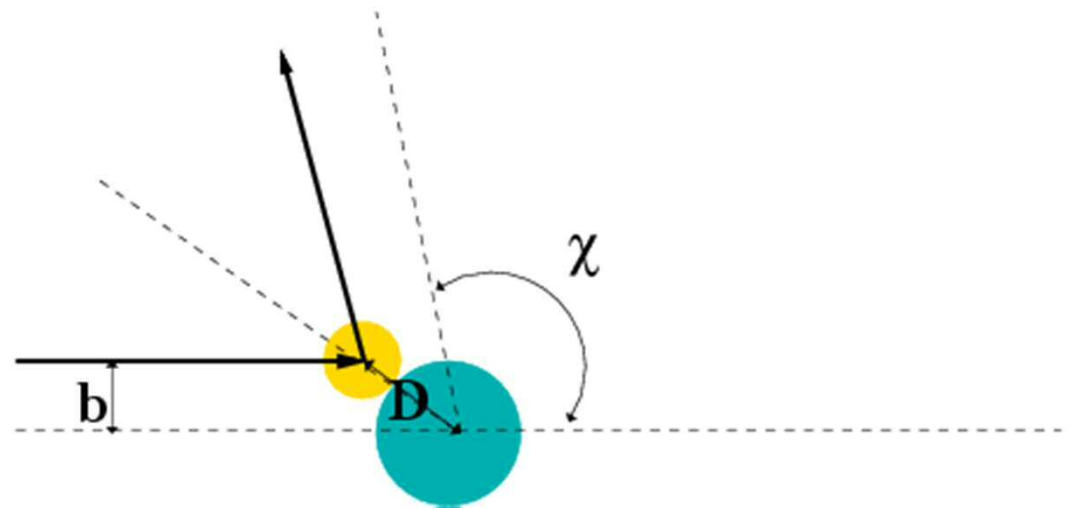
Rutherford scattering continued :

$$\theta = 2 \sin^{-1} \left(\frac{1}{\sqrt{(2b/\kappa)^2 + 1}} \right)$$

$$\frac{2b}{\kappa} = \left| \frac{\cos(\theta/2)}{\sin(\theta/2)} \right|$$

$$\left(\frac{d\sigma}{d\Omega} \right) = \frac{b}{\sin\theta} \left| \frac{db}{d\theta} \right| = \frac{\kappa^2}{16} \frac{1}{\sin^4(\theta/2)}$$

Hard sphere scattering



For your homework you will show that

$$b = D \cos\left(\frac{\chi}{2}\right)$$

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{b}{\sin\chi} \left|\frac{db}{d\chi}\right| = \frac{D^2}{4}$$