

**PHY 711 Classical Mechanics and
Mathematical Methods
10-10:50 AM MWF Olin 103**

Plan for Lecture 26:

Traveling and standing waves in 1 and 2 dimensions

- 1. Brief review of 1-d from Chap. 7**
- 2. Waves in 2-d elastic membranes – Chap. 8**

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6	Mon, 9/09/2013	Chap 3	Calculus of variations – continued	
7	Wed, 9/11/2013	Chap 3	Calculus of variations applied to Lagrangians	#6
8	Fri, 9/13/2013	Chap 3	Lagrangian mechanics	#7
9	Mon, 9/16/2013	Chap 3 & 6	Lagrangian mechanics	#8
10	Wed, 9/18/2013	Chap 3 & 6	Lagrangian mechanics	#9
11	Fri, 9/20/2013	Chap 3 & 6	Lagrangian & Hamiltonian mechanics	#10
12	Mon, 9/23/2013	Chap 3 & 6	Hamiltonian formalism	#11
13	Wed, 9/25/2013	Chap 3 & 6	Hamiltonian formalism	#12
14	Fri, 9/27/2013	Chap 3 & 6	Hamiltonian formalism	#13
15	Mon, 9/30/2013	Chap 4	Small Oscillations	#14
16	Wed, 10/02/2013	Chap 4	Small Oscillations	
17	Fri, 10/04/2013	Chap 4	Small Oscillations	#15
18	Mon, 10/07/2013	Chap 4 & 7	Small Oscillations and waves	#16
19	Wed, 10/09/2013	Chap 7	Wave equation	
	Fri, 10/11/2013		No class (Fall Break)	
20	Mon, 10/14/2013	Chap 7	Wave equation (Presentation topic due)	#17
21	Wed, 10/16/2013	Chap 7	Mathematical methods	#18
22	Fri, 10/18/2013	Chap 7	Mathematical methods	#19
23	Mon, 10/21/2013	Chap 5	Rigid rotations	#20
24	Wed, 10/23/2013	Chap 5	Rigid rotations	#21
25	Fri, 10/25/2013	Chap 5	Rigid rotations	
	Mon, 10/28/2013	No class	Take-home exam	
	Wed, 10/30/2013	No class	Take-home exam	
	Fri, 11/01/2013	No class	Take-home exam	
26	Mon, 11/04/2013	Chap 8	Oscillations in two-dimensional membranes	Take-home exam due

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Review of wave equation in one-dimension – here $\mu(x,t)$ can describe either a longitudinal or transverse wave.

Traveling wave solutions --

$$\frac{\partial^2 \mu}{\partial t^2} - c^2 \frac{\partial^2 \mu}{\partial x^2} = 0$$

Note that for any function $f(q)$ or $g(q)$:

$$\mu(x,t) = f(x-ct) + g(x+ct)$$

satisfies the wave equation.

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Initial value problem : $\mu(x,0) = \phi(x)$ and $\frac{\partial \mu}{\partial t}(x,0) = \psi(x)$

then : $\mu(x,0) = \phi(x) = f(x) + g(x)$

$$\frac{\partial \mu}{\partial t}(x,0) = \psi(x) = -c \left(\frac{df(x)}{dx} - \frac{dg(x)}{dx} \right)$$

$$\Rightarrow f(x) - g(x) = -\frac{1}{c} \int \psi(x') dx'$$

For each x , find $f(x)$ and $g(x)$:

$$f(x) = \frac{1}{2} \left(\phi(x) - \frac{1}{c} \int \psi(x') dx' \right)$$

$$g(x) = \frac{1}{2} \left(\phi(x) + \frac{1}{c} \int \psi(x') dx' \right)$$

$$\Rightarrow \mu(x,t) = \frac{1}{2} (\phi(x-ct) + \phi(x+ct)) + \frac{1}{2c} \int_{x-ct}^{x+ct} \psi(x') dx'$$

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Standing wave solutions of wave equation:

$$\frac{\partial^2 \mu}{\partial t^2} - c^2 \frac{\partial^2 \mu}{\partial x^2} = 0$$

with $\mu(0,t) = \mu(L,t) = 0$.

Assume : $\mu(x,t) = \Re(e^{-i\omega t} \rho(x))$

where $\frac{d^2 \rho(x)}{dx^2} + k^2 \rho(x) = 0$ $k = \frac{\omega}{c}$

$$\rho_v(x) = A \sin\left(\frac{v\pi x}{L}\right)$$

$$k_v = \frac{v\pi}{L} \quad \omega_v = ck_v$$

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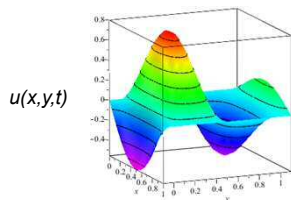
Fundamental, or first harmonic: $n=1$, $L = \frac{1}{2} \lambda$

Second harmonic: $n=2$, $L = \lambda$

Third harmonic: $n=3$, $L = \frac{3}{2} \lambda$

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Extension of ideas to wave motion on a two-dimensional surface – elastic membrane (transverse wave; linear regime).



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Lagrangian density: $\mathcal{L}\left(u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial t}; x, y, t\right)$

$$L = \int \mathcal{L}\left(u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial t}; x, y, t\right) dx dy$$

Hamilton's principle:

$$\delta \int_{t_1}^{t_2} L dt = 0$$

$$\frac{\partial \mathcal{L}}{\partial u} - \frac{\partial}{\partial t} \frac{\partial \mathcal{L}}{\partial (\partial u / \partial t)} - \frac{\partial}{\partial x} \frac{\partial \mathcal{L}}{\partial (\partial u / \partial x)} - \frac{\partial}{\partial y} \frac{\partial \mathcal{L}}{\partial (\partial u / \partial y)} = 0$$

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Lagrangian density for elastic membrane with constant σ and τ :

$$\mathcal{L}\left(u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial t}; x, y, t\right) = \frac{1}{2} \sigma \left(\frac{\partial u}{\partial t}\right)^2 - \frac{1}{2} \tau (\nabla u)^2$$

$$\frac{\partial \mathcal{L}}{\partial u} - \frac{\partial}{\partial t} \frac{\partial \mathcal{L}}{\partial (\partial u / \partial t)} - \frac{\partial}{\partial x} \frac{\partial \mathcal{L}}{\partial (\partial u / \partial x)} - \frac{\partial}{\partial y} \frac{\partial \mathcal{L}}{\partial (\partial u / \partial y)} = 0$$

$$\frac{\partial^2 u}{\partial t^2} - c^2 \nabla^2 u = 0 \quad \text{where } c^2 = \frac{\tau}{\sigma}$$

Two - dimensional wave equation :

$$\frac{\partial^2 u}{\partial t^2} - c^2 \nabla^2 u = 0 \quad \text{where } c^2 = \frac{\tau}{\sigma}$$

Standing wave solutions:

$$u(x, y, t) = \Re\left(e^{-i\omega t} \rho(x, y)\right)$$

$$(\nabla^2 + k^2) \rho(x, y) = 0 \quad \text{where } k = \frac{\omega}{c}$$

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Consider a rectangular boundary:



Clamped boundary conditions:

$$\rho(0, y) = \rho(a, y) = \rho(x, 0) = \rho(x, b) = 0$$

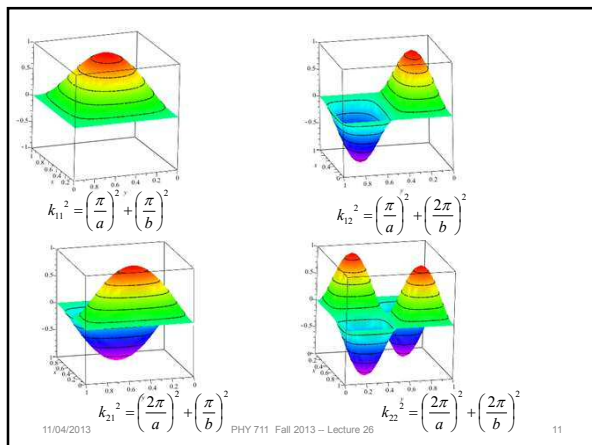
$$\Rightarrow \rho_{mn}(x, y) = A \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

$$k_{mn}^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \quad \omega_{mn} = ck_{mn}$$

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More general boundary conditions:

$\tau \nabla u|_b = \kappa u|_b$ represents boundary side constrained with spring

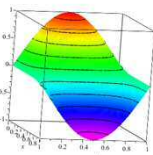
$\tau \nabla u|_b = 0$ represents "free" side

Mixed boundary conditions:

$$\rho(x, 0) = \rho(x, b) = \frac{\partial \rho(0, y)}{\partial x} = \frac{\partial \rho(a, y)}{\partial x} = 0$$

$$\Rightarrow \rho_{mn}(x, y) = A \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

$$k_{mn}^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \quad \omega_{mn} = ck_{mn}$$



$$k_{11}^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2$$

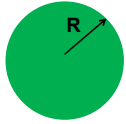
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Consider a circular boundary:

Clamped boundary conditions for $\rho(r, \varphi)$:
 $\rho(R, \varphi) = 0$



$(\nabla^2 + k^2)\rho(r, \varphi) = 0$ where $k = \frac{\omega}{c}$

In cylindrical coordinate system

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \varphi^2}$$

Assume: $\rho(r, \varphi) = f(r)\Phi(\varphi)$

Let: $\Phi(\varphi) = e^{im\varphi}$

Note: $\Phi(\varphi) = \Phi(\varphi + 2\pi)$
 $\Rightarrow m = \text{integer}$

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Consider circular boundary -- continued

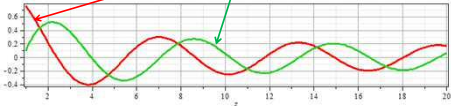
Differential equation for radial function:

$$\left(\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{m^2}{r^2} + k^2 \right) f(r) = 0$$

\Rightarrow Bessel equation of integer order with transcendental solutions

Cylindrical Bessel function $J_m(z)$

Cylindrical Neumann function $N_m(z)$



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Some properties of Bessel functions

Ascending series: $J_m(z) = \left(\frac{z}{2}\right)^m \sum_{j=0}^{\infty} \frac{(-1)^j}{j!(j+m)!} \left(\frac{z}{2}\right)^{2j}$

Recursion relations: $J_{m-1}(z) + J_{m+1}(z) = \frac{2m}{z} J_m(z)$

$$J_{m-1}(z) - J_{m+1}(z) = 2 \frac{dJ_m(z)}{dz}$$

Asymptotic form: $J_m(z) \xrightarrow{z \gg 1} \sqrt{\frac{2}{\pi z}} \cos\left(z - \frac{m\pi}{2} - \frac{\pi}{4}\right)$

Zeros of Bessel functions $J_m(z_{mn}) = 0$

$m = 0$: $z_{0n} = 2.406, 5.520, 8.654, \dots$

$m = 1$: $z_{1n} = 3.832, 7.016, 10.173, \dots$

$m = 2$: $z_{2n} = 5.136, 8.417, 11.620, \dots$

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Some properties of Bessel functions -- continued

Note: It is possible to prove the following

identity for the functions $J_m\left(\frac{z_{mn}}{R}r\right)$:

$$\int_0^R J_m\left(\frac{z_{mn}}{R}r\right) J_m\left(\frac{z_{m'n'}}{R}r\right) r dr = \frac{R^2}{2} (J_{m+1}(z_{mn}))^2 \delta_{nn'}$$

Returning to differential equation for radial function :

$$\left(\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{m^2}{r^2} + k^2\right) f(r) = 0$$

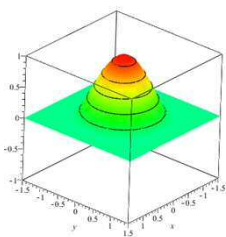
$$\Rightarrow f_{mn}(r) = AJ_m\left(\frac{z_{mn}}{R}r\right); \quad k_{mn} = \frac{z_{mn}}{R}$$

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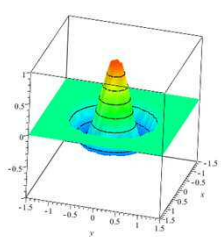
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$$\rho_{01}(r, \varphi) = f_{01}(r) = AJ_0\left(\frac{z_{01}}{R}r\right)$$



$$k_{01} = \frac{2.406}{R}$$

$$\rho_{02}(r, \varphi) = f_{02}(r) = AJ_0\left(\frac{z_{02}}{R}r\right)$$



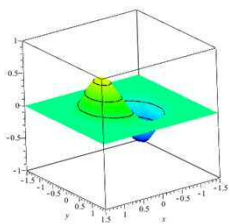
$$k_{02} = \frac{5.520}{R}$$

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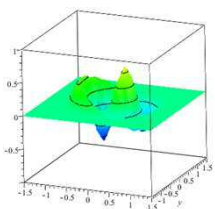
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$$\rho_{11}(r, \varphi) = f_{11}(r) \cos(\varphi) = AJ_1\left(\frac{z_{11}}{R}r\right) \cos(\varphi)$$



$$k_{11} = \frac{3.832}{R}$$

$$\rho_{12}(r, \varphi) = f_{12}(r) \cos(\varphi) = AJ_1\left(\frac{z_{12}}{R}r\right) \cos(\varphi)$$



$$k_{12} = \frac{7.016}{R}$$

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Maple server evaluation project

- Request from Rick Matthews, *Associate Provost for Technology and Information Systems*
- The University is exploring the possibility of making Maple available on a server rather than on individual computers
 - ❖ Need students to exercise the software
 - ❖ Need students to assess the performanc
 - ❖ Need students to identify issues
- Entirely voluntary

Possible pizza party or ??? rewards

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Introduction to algebraic manipulation software



Available from the webpage:
<http://help.wfu.edu/public/vcl>

If you have any trouble with this installation and setup, please contact Ching-Wan Yip at yipcw@wfu.edu

Note: This is a remote server that the university is testing. If you are off campus you will need to use VPN to gain access.

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Installing and Connecting to Virtual Computing Labs

Beginning Fall of 2013 certain academic departments at Wake Forest University will begin utilizing Virtual Computing Labs to access specific software programs needed for coursework. This page will include the instructions as well as the files needed to download and install the VMware View client and establish a connection for the first time.

Instructions

- [Installing and Connecting with Windows](#)
- [Installing and Connecting with Macintosh](#)

Downloads

- [VMware View Client 32-Bit](#) (ThinkPad T430s and lower)
- [VMware View Client 64-Bit](#) (ThinkPad X1 Carbon and newer)

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