PHY 711 Classical Mechanics and Mathematical Methods
10-10:50 AM MWF Olin 103

Plan for Lecture 4:

1. **Chapter 1 – scattering theory summary**
2. **Chapter 2 – Physics described in an accelerated coordinate frame**

PHY 711 Classical Mechanics and Mathematical Methods

MWF 10 AM-10:50 PM	OPL 103	http://www.wfu.edu/~natalie/f12phy711/

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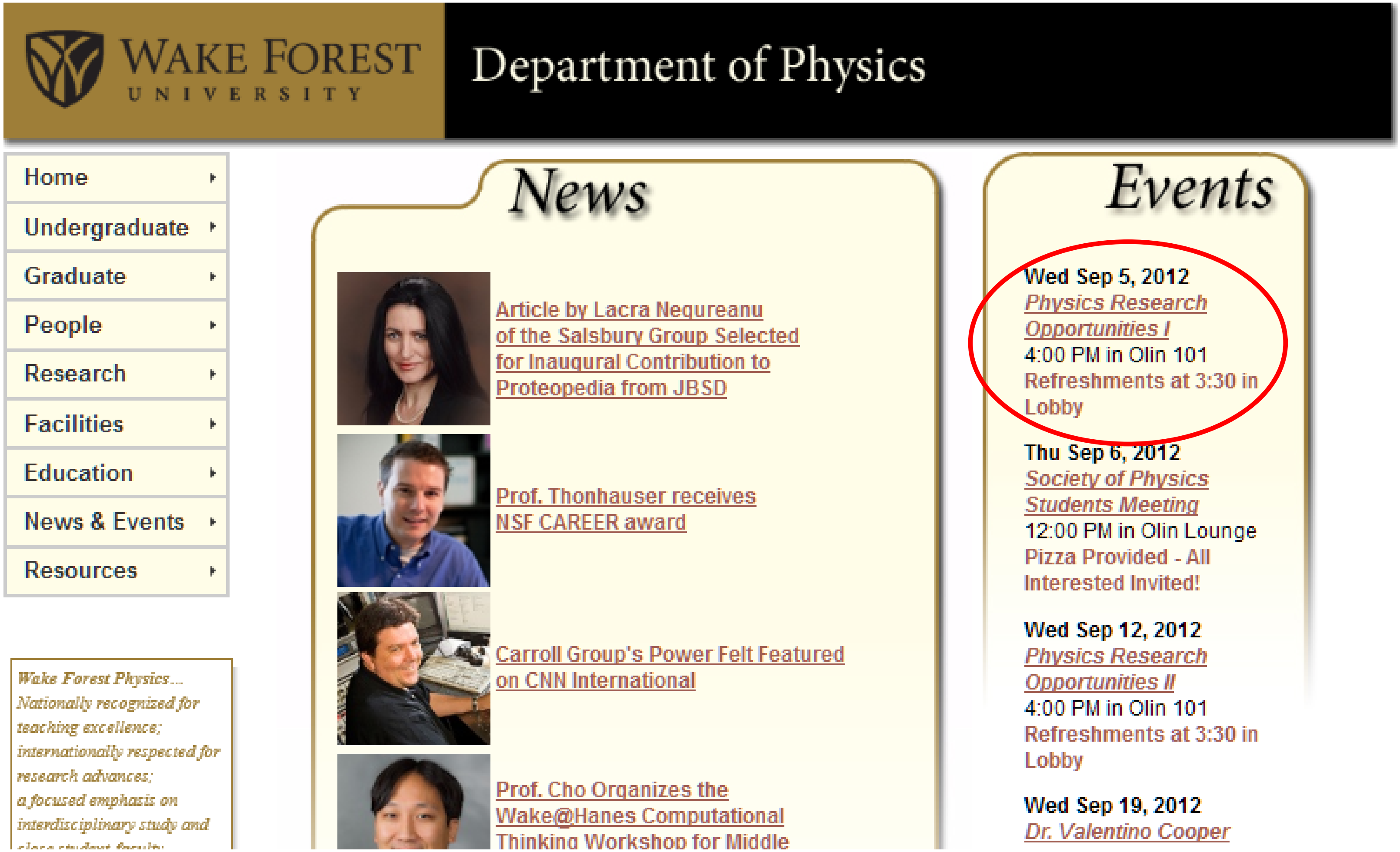
Course schedule

(Preliminary schedule -- subject to frequent adjustment.)

	Date	F&W Reading	Topic	Assignment
1	Wed, 8/29/2012	Chap. 1	Review of basic principles;Scattering theory	#1
2	Fri, 8/31/2012	Chap. 1	Scattering theory continued	#2
3	Mon, 9/03/2012	Chap. 1	Scattering theory continued	#3
4	Wed, 9/05/2012	Chap. 1 & 2	Scattering theory/Accelerated coordinate frame	#4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Wake Forest Physics... Nationally recognized for teaching excellence; internationally respected for research advances; a focused emphasis on interdisciplinary study and
News
Article by Lacra Negureanu of the Salsbury Group Selected for Inaugural Contribution to Proteopedia from JBSD
Prof. Thonhauser receives NSF CAREER award
Carroll Group's Power Felt Featured on CNN International
Prof. Cho Organizes the Wake@Hanes Computational Thinking Workshop for Middle
Events
Wed Sep 5, 2012
Physics Research Opportunities I
4:00 PM in Olin 101
Refreshments at 3:30 in Lobby
Thu Sep 6, 2012
Society of Physics Students Meeting
12:00 PM in Olin Lounge
Pizza Provided - All Interested Invited!
Wed Sep 12, 2012
Physics Research Opportunities II
4:00 PM in Olin 101
Refreshments at 3:30 in Lobby
Wed Sep 19, 2012
Dr. Valentino Cooper

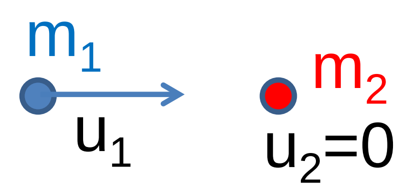
Fall 2012 Schedule
for N. A. W. Holzwarth

	Monday	Tuesday	Wednesday	Thursday	Friday
8:00-9:00	Lecture Preparation/ Office Hours	Lecture Preparation/ Office Hours	Lecture Preparation/ Office Hours	Lecture Preparation/ Office Hours	Lecture Preparation/ Office Hours
9:00-10:00	General Physics I PHY113		General Physics I PHY113		General Physics I PHY113
10:00-11:00	Classical Mech PHY711		Classical Mech PHY711		Classical Mech PHY711
11:00-12:30	Office Hours	Physics Research	Office Hours	Physics Research	Office Hours
12:30-2:00	Condensed Matter Theory Journal Club		Physics Research		Physics Research
2:00-3:30	Physics Research				
3:30-5:00			Physics Colloquium		CEES -- Renewable Energy Research

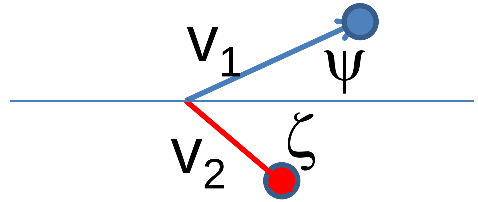
Some more details on the laboratory and center of mass reference frames

Laboratory reference frame:

Before

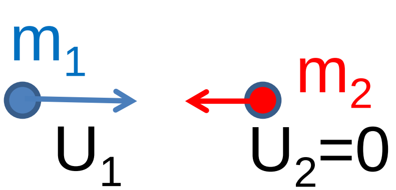


After

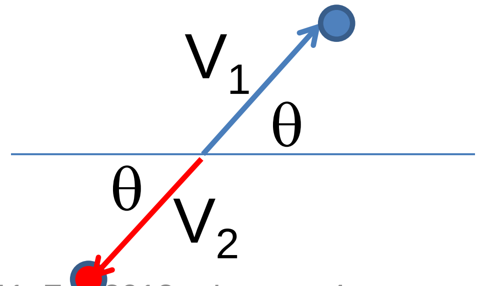


Center of mass reference frame:

Before



After



Laboratory reference frame:

$$\text{Total energy of the system:}$$

$$E_{LAB} = \tfrac{1}{2} m_1 \dot{\mathbf{r}}_1^{\,2} + \tfrac{1}{2} m_2 \dot{\mathbf{r}}_2^{\,2} + V(|\mathbf{r}_2 - \mathbf{r}_1|)$$

$$\text{Relative coordinate:} \quad \mathbf{r} \equiv \mathbf{r}_2 - \mathbf{r}_1$$

$$\text{Center of mass coordinate:} \ \mathbf{R}_{CM} \equiv \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2}{m_1 + m_2}$$

$$E_{LAB} = \tfrac{1}{2}(m_1 + m_2)\left|\dot{\mathbf{R}}_{CM}\right|^2 + \tfrac{1}{2}\mu\left|\dot{\mathbf{r}}\right|^2 + V(r)$$

$$\equiv \tfrac{1}{2}(m_1 + m_2)\left|\dot{\mathbf{R}}_{CM}\right|^2 + E_{CM}$$

$$\text{where} \quad \mu \equiv \frac{m_1 m_2}{m_1 + m_2}$$

Analysis before and after collision in CM frame:

Assume that before and after the collision, $V(r) \approx 0$:

$$E_{CM} = \tfrac{1}{2}\mu|\dot{\mathbf{r}}|^2 = \tfrac{1}{2}m_1U_1^2 + \tfrac{1}{2}m_2U_2^2 = \tfrac{1}{2}m_1V_1^2 + \tfrac{1}{2}m_2V_2^2$$

Conservation of momentum requires:

$$m_1\mathbf{U}_1 + m_2\mathbf{U}_2 = 0 = m_1\mathbf{V}_1 + m_2\mathbf{V}_2$$

$$\Rightarrow m_1\mathbf{U}_1 = -m_2\mathbf{U}_2 \text{ and } m_1\mathbf{V}_1 = -m_2\mathbf{V}_2$$

More algebra:

$$m_1\left(U_1^2 - V_1^2\right) = -m_2\left(U_2^2 - V_2^2\right)$$

$$m_1\left(U_1^2 - V_1^2\right) = -\frac{m_1^2}{m_2}\left(U_1^2 - V_1^2\right)$$

$$\Rightarrow U_1 = V_1 \quad \text{and} \quad U_2 = V_2$$

Physical laws as described in non-inertial coordinate systems

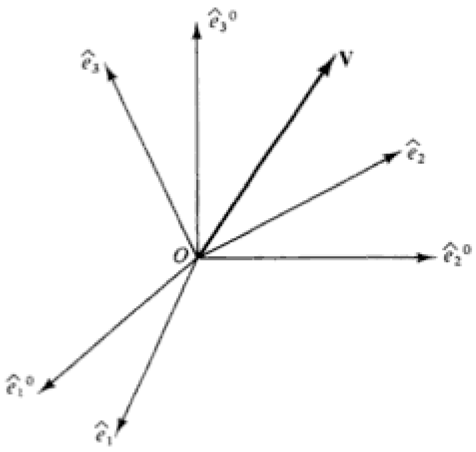


Figure 6.1 Transformation to a rotating coordinate system.

Let **V** be a general vector, e.g., the position of a particle. This vector can be characterized by its components with respect to either orthonormal triad. Thus we can write

$$\mathbf{V} = \sum_{i=1}^{3} V_i^0 \hat{e}_i^0 \tag{6.1a}$$

$$\mathbf{V} = \sum_{i=1}^{3} V_i \hat{e}_i \tag{6.1b}$$

Comparison of analysis in “inertial frame” versus “non-inertial frame”

Denote by $\hat{e}_i^0$ a fixed coordinate system

Denote by $\hat{e}_i$ a moving coordinate system

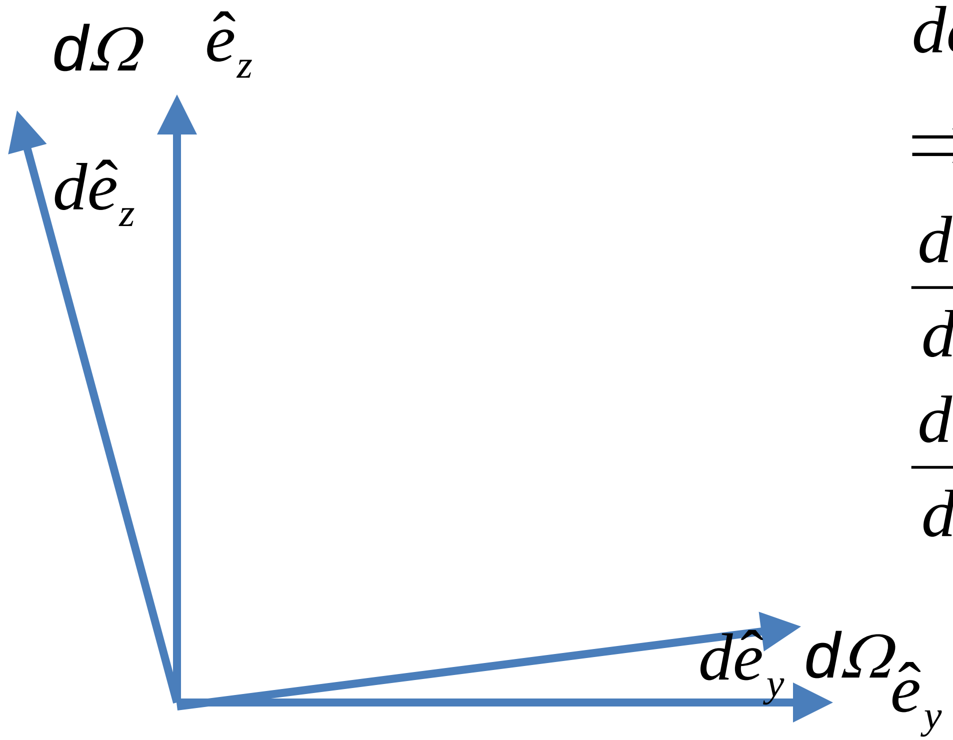
$$\mathbf{V} = \sum_{i=1}^{3} V_i^{\,0} \hat{e}_i^0 = \sum_{i=1}^{3} V_i \hat{e}_i$$

$$\left(\frac{d\mathbf{V}}{dt}\right)_{inertial} = \sum_{i=1}^{3} \frac{dV_i^{\,0}}{dt} \hat{e}_i^0 = \sum_{i=1}^{3} \frac{dV_i}{dt} \hat{e}_i + \sum_{i=1}^{3} V_i \frac{d\hat{e}_i}{dt}$$

$$\text{Define:} \quad \left(\frac{d\mathbf{V}}{dt}\right)_{body} \equiv \sum_{i=1}^{3} \frac{dV_i}{dt} \hat{e}_i$$

$$\Rightarrow \left(\frac{d\mathbf{V}}{dt}\right)_{inertial} = \left(\frac{d\mathbf{V}}{dt}\right)_{body} + \sum_{i=1}^{3} V_i \frac{d\hat{e}_i}{dt}$$

Properties of the frame motion (rotation):



$$d\hat{e}_y = d\Omega \hat{e}_z$$

$$d\hat{e}_z = -d\Omega \hat{e}_y$$

$$\Rightarrow d\hat{\mathbf{e}} = d\mathbf{\Omega} \times \hat{\mathbf{e}}$$

$$\frac{d\hat{\mathbf{e}}}{dt} = \frac{d\mathbf{\Omega}}{dt} \times \hat{\mathbf{e}}$$

$$\frac{d\hat{\mathbf{e}}}{dt} = \boldsymbol{\omega} \times \hat{\mathbf{e}}$$

$$\left(\frac{d\mathbf{V}}{dt}\right)_{inertial} = \left(\frac{d\mathbf{V}}{dt}\right)_{body} + \sum_{i=1}^{3} V_i \frac{d\hat{e}_i}{dt}$$

$$\left(\frac{d\mathbf{V}}{dt}\right)_{inertial} = \left(\frac{d\mathbf{V}}{dt}\right)_{body} + \boldsymbol{\omega} \times \mathbf{V}$$

Effects on acceleration:

$$\left(\frac{d}{dt}\frac{d\mathbf{V}}{dt}\right)_{inertial} = \left(\left(\frac{d}{dt}\right)_{body} + \boldsymbol{\omega} \times\right)\left\{\left(\frac{d\mathbf{V}}{dt}\right)_{body} + \boldsymbol{\omega} \times \mathbf{V}\right\}$$

$$\left(\frac{d^2\mathbf{V}}{dt^2}\right)_{inertial} = \left(\frac{d^2\mathbf{V}}{dt^2}\right)_{body} + 2\boldsymbol{\omega} \times \left(\frac{d\mathbf{V}}{dt}\right)_{body} + \frac{d\boldsymbol{\omega}}{dt} \times \mathbf{V} + \boldsymbol{\omega} \times \boldsymbol{\omega} \times \mathbf{V}$$

Extension to rotation and translation of coordinate system

$$\text{Denote by } \left(\frac{d^2\mathbf{a}}{dt^2}\right)_{inertial} \quad \text{the acceleration of the coordinate system}$$

$$\left(\frac{d^2\mathbf{V}}{dt^2}\right)_{inertial} = \left(\frac{d^2\mathbf{a}}{dt^2}\right)_{inertial} + \left(\frac{d^2\mathbf{V}}{dt^2}\right)_{body} + 2\boldsymbol{\omega}\times\left(\frac{d\mathbf{V}}{dt}\right)_{body} + \frac{d\boldsymbol{\omega}}{dt}\times\mathbf{V} + \boldsymbol{\omega}\times\boldsymbol{\omega}\times\mathbf{V}$$

Newton's laws; Let **V** = **r**, the position of particle of mass *m*:

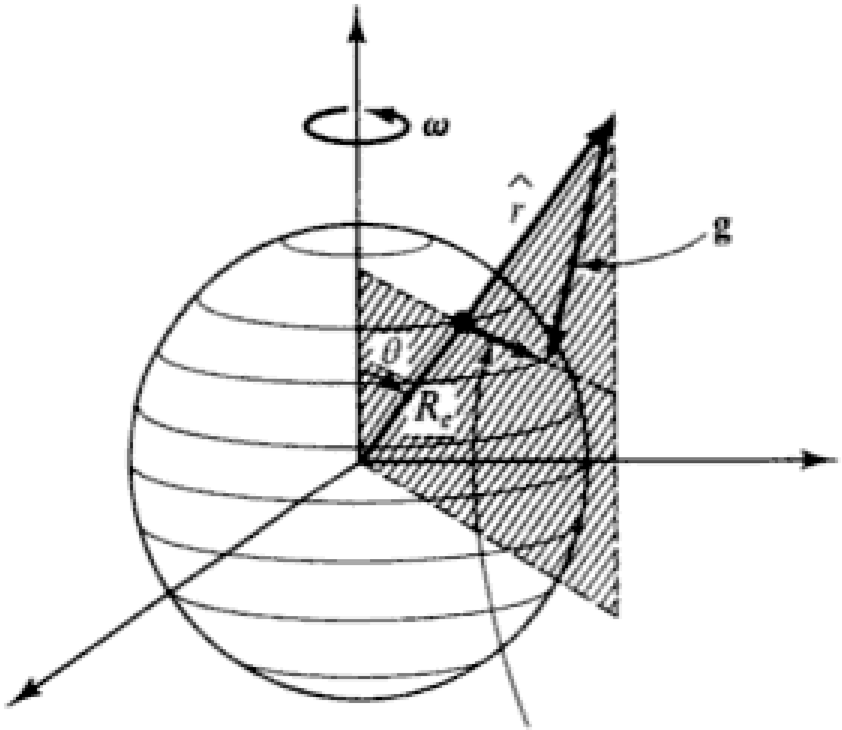
$$m\left(\frac{d^2\mathbf{r}}{dt^2}\right)_{inertial} = \mathbf{F}_{ext}$$

$$m\left(\frac{d^2\mathbf{r}}{dt^2}\right)_{body} = \mathbf{F}_{ext} - m\left(\frac{d^2\mathbf{a}}{dt^2}\right)_{inertial} - 2m\boldsymbol{\omega}\times\left(\frac{d\mathbf{r}}{dt}\right)_{body} - m\frac{d\boldsymbol{\omega}}{dt}\times\mathbf{r} - m\boldsymbol{\omega}\times\boldsymbol{\omega}\times\mathbf{r}$$

Coriolis force

Centrifugal force

Motion on the surface of the Earth:



$$\omega = \frac{2\pi}{\tau} \approx 7.3\times 10^{-5}\,\text{rad/s}$$

$$\mathbf{F}_{ext} = -\frac{GM_e m}{r^2}\hat{\mathbf{r}} + \mathbf{F}'$$

Main contributions:

$$m\left(\frac{d^2\mathbf{r}}{dt^2}\right)_{earth} = -\frac{GM_e m}{r^2}\hat{\mathbf{r}} + \mathbf{F}' - 2m\boldsymbol{\omega}\times\left(\frac{d\mathbf{r}}{dt}\right)_{earth} - m\frac{d\boldsymbol{\omega}}{dt}\times\mathbf{r} - m\boldsymbol{\omega}\times\boldsymbol{\omega}\times\mathbf{r}$$

Non-inertial effects on effective gravitational "constant"

$$m\left(\frac{d^2\mathbf{r}}{dt^2}\right)_{earth} = -\frac{GM_e m}{r^2}\hat{\mathbf{r}} + \mathbf{F}' - 2m\boldsymbol{\omega}\times\left(\frac{d\mathbf{r}}{dt}\right)_{earth} - m\frac{d\boldsymbol{\omega}}{dt}\times\mathbf{r} - m\boldsymbol{\omega}\times\boldsymbol{\omega}\times\mathbf{r}$$

For $\left(\frac{d\mathbf{r}}{dt}\right)_{earth} = 0$ and $\left(\frac{d^2\mathbf{r}}{dt^2}\right)_{earth} = 0,$

$$0 = -\frac{GM_e m}{r^2}\hat{\mathbf{r}} + \mathbf{F}' - m\boldsymbol{\omega}\times\boldsymbol{\omega}\times\mathbf{r}$$

$$\mathbf{F}' = -m\mathbf{g}$$

$$\Rightarrow \mathbf{g} = -\frac{GM_e}{r^2}\hat{\mathbf{r}} - \boldsymbol{\omega}\times\boldsymbol{\omega}\times\mathbf{r}\Big|_{r\approx R_e}$$

$$= \left(-\frac{GM_e}{R_e^2} + \omega^2 R_e \sin^2\theta\right)\hat{\mathbf{r}} + \sin\theta\cos\theta\,\omega^2 R_e\hat{\boldsymbol{\theta}}$$

0.03 m/s^2

9.80 m/s^2