

**PHY 711 Classical Mechanics and
Mathematical Methods
10-10:50 AM MWF Olin 103**

Plan for Lecture 38:

- 1. Comment on Exam questions**
- 2. Remaining concepts in classical mechanics**
 - a. Stokes equation**
- 3. Class evaluation**

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22	Wed, 10/17/2012	Chap. 7, 5	Moment of inertia	
	Fri, 10/19/2012		Fall break	
23	Mon, 10/22/2012	Chap. 5	Rigid body rotation	#16
24	Wed, 10/24/2012	Chap. 5	Rigid body rotation	#17
25	Fri, 10/26/2012	Chap. 5	Rigid body rotation	#18
26	Mon, 10/29/2012	Chap. 8	Waves in elastic membranes	#19
27	Wed, 10/31/2012	Chap. 9	Introduction to hydrodynamics	
28	Fri, 11/01/2012	Chap. 9	Introduction to hydrodynamics	
29	Mon, 11/05/2012	Chap. 9	Introduction to hydrodynamics	#20
30	Wed, 11/07/2012	Chap. 9	Sound waves	
31	Fri, 11/09/2012	Chap. 9	Linear sound waves	#21
32	Mon, 11/12/2012	Chap. 9	Green's function for linear sound	
33	Wed, 11/14/2012	Chap. 9	Non-linear sound	
34	Fri, 11/16/2012	Chap. 9	Non-linear sound	Take Home Exam
35	Mon, 11/19/2012	Chap. 10	Surface waves	Take Home Exam
	Wed, 11/21/2012		Thanksgiving Holiday	
	Fri, 11/23/2012		Thanksgiving Holiday	
36	Mon, 11/26/2012	Chap. 10	Surface waves	Exam due
37	Wed, 11/28/2012	Chap. 10	Surface waves	
38	Fri, 11/30/2012	Chap. 10	Surface waves	
39	Mon, 12/03/2012		Student presentations I	
40	Wed, 12/05/2012		Student presentations II	

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Schedule for PHY 711 presentations

Monday, Dec. 3, 2012

- 10:00 Pete Diemer -- examples of hydrodynamic phenomena
- 10:15 Katelyn Goetz - Physics and Golf
- 10:30
- 10:45
- 11:00 Xiaohua Liu---

Wednesday, Dec. 5, 2012

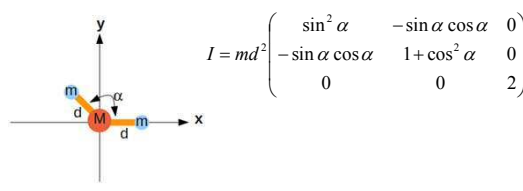
- 10:00 Zach Lampert -- The Physics of Hockey
- 10:15 Jijie Xiao -- Physics of Chinese Flute
- 10:30 David Montgomery--
- 10:45 Chaochao Dun-Membranes
- 11:00

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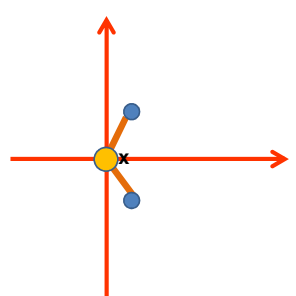
Comment on exam problem 2



$$I = md^2 \begin{pmatrix} \sin^2 \alpha & -\sin \alpha \cos \alpha & 0 \\ -\sin \alpha \cos \alpha & 1 + \cos^2 \alpha & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

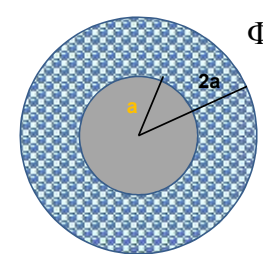
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Principal moments of inertia



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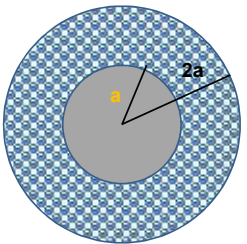
Comment on exam problem 4



$$\Phi(\mathbf{r}, t) = \phi_l(r) Y_{lm}(\hat{\mathbf{r}}) e^{-i\omega t}$$

$$\phi_l(r) = C_l (j_l(kr) + x_l y_l(kr))$$

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$\phi_l(r) = C_l(j_l(kr) + x_l y_l(kr))$
 Boundary values:
 $\frac{d\phi_l(a)}{dr} = 0 = \frac{d\phi_l(2a)}{dr}$

$j'_l(ka) + x_l y'_l(ka) = 0 = j'_l(2ka) + x_l y'_l(2ka)$
 $x_l = -\frac{j'_l(ka)}{y'_l(ka)} = -\frac{j'_l(2ka)}{y'_l(2ka)}$

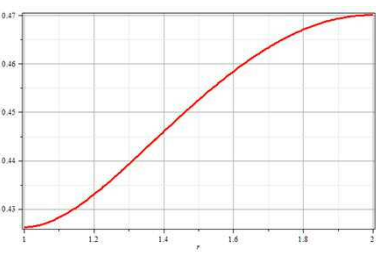
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For $l=0$: $k=3.286/a$ $x=-2.125$



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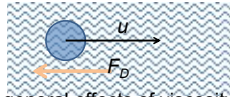
For $l=1$: $k=0.920/a$ $x=-0.092/a$



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Brief introduction to viscous effects in incompressible fluids

Stokes' analysis of viscous drag on a sphere of radius R moving at speed u in medium with viscosity η :

$$F_D = -\eta(6\pi R u)$$


Plan:

1. Consider the general effects of viscosity on fluid equations
2. Consider the solution to the linearized equations for the case of steady-state flow of a sphere of radius R
3. Infer the drag force needed to maintain the steady-state flow

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Newton - Euler equation for incompressible fluid, modified by viscous contribution (Navier - Stokes equation) :

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = \mathbf{f}_{\text{applied}} - \frac{\nabla p}{\rho} + \frac{\eta}{\rho} \nabla^2 \mathbf{v}$$

Continuity equation : $\nabla \cdot \mathbf{v} = 0$ Irrotational flow : $\nabla \times \mathbf{v} = 0$

Assume steady state : $\Rightarrow \frac{\partial \mathbf{v}}{\partial t} = 0$

Assume non - linear effects small

Initially set $\mathbf{f}_{\text{applied}} = 0$; $\nabla p = \eta \nabla^2 \mathbf{v}$

Assume $\mathbf{v} = \nabla \times (\nabla \times f(r) \mathbf{u}) + \mathbf{u}$

where $f(r) \xrightarrow{r \rightarrow \infty} 0$

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$$\mathbf{v} = \nabla \times (\nabla \times f(r) \mathbf{u}) + \mathbf{u}$$

$$\mathbf{u} = u \hat{\mathbf{z}}$$

$$\nabla \times (\nabla \times f(r) \hat{\mathbf{z}}) = \nabla (\nabla \cdot f(r) \hat{\mathbf{z}}) - \nabla^2 f(r) \hat{\mathbf{z}}$$

$$\nabla \times \mathbf{v} = 0 \quad \Rightarrow \nabla^2 (\nabla \times \mathbf{v}) = 0$$

$$\nabla^4 (\nabla \times f(r) \hat{\mathbf{z}}) = 0 \quad \Rightarrow \nabla^4 (\nabla f(r) \times \hat{\mathbf{z}}) = 0 \quad \Rightarrow \nabla^4 f(r) = 0$$

$$f(r) = C_1 r^2 + C_2 r + C_3 + \frac{C_4}{r}$$

$$v_r = u \cos \theta \left(1 - \frac{2}{r} \frac{df}{dr} \right) = u \cos \theta \left(1 - 4C_1 - \frac{2C_2}{r} - \frac{2C_4}{r^3} \right)$$

$$v_\theta = -u \sin \theta \left(1 - \frac{d^2 f}{dr^2} - \frac{1}{r} \frac{df}{dr} \right) = -u \sin \theta \left(1 - 4C_1 - \frac{C_2}{r} + \frac{C_4}{r^3} \right)$$

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$$v_r = u \cos \theta \left(1 - \frac{2}{r} \frac{df}{dr} \right) = u \cos \theta \left(1 - 4C_1 - \frac{2C_2}{r} - \frac{2C_4}{r^3} \right)$$

$$v_r = u \cos \theta \left(1 - \frac{2}{r} \frac{df}{dr} \right) = -u \sin \theta \left(1 - 4C_1 - \frac{C_2}{r} + \frac{C_4}{r^3} \right)$$

To satisfy $\mathbf{v}(r \rightarrow \infty) = \mathbf{u} : \Rightarrow C_1 = 0$

To satisfy $\mathbf{v}(R) = 0$ solve for C_2, C_4

$$v_r = u \cos \theta \left(1 - \frac{3R}{2r} + \frac{R^3}{2r^3} \right)$$

$$v_\theta = -u \sin \theta \left(1 - \frac{3R}{4r} - \frac{R^3}{4r^3} \right)$$

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$$v_r = u \cos \theta \left(1 - \frac{3R}{2r} + \frac{R^3}{2r^3} \right)$$

$$v_\theta = -u \sin \theta \left(1 - \frac{3R}{4r} - \frac{R^3}{4r^3} \right)$$

Determining pressure:

$$\nabla p = \eta \nabla^2 \mathbf{v} = -\eta \nabla \left(u \cos \theta \left(\frac{3R}{2r^2} \right) \right)$$

$$\Rightarrow p = p_0 - \eta u \cos \theta \left(\frac{3R}{2r^2} \right)$$

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$$p = p_0 - \eta u \cos \theta \left(\frac{3R}{2r^2} \right)$$

Corresponds to:

$$F_D = (p(R) - p_0) \frac{4\pi R^2}{\cos \theta} = -\eta u (6\pi R)$$

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