PHY 711 Classical Mechanics and Mathematical Methods
10-10:50 AM MWF Olin 103

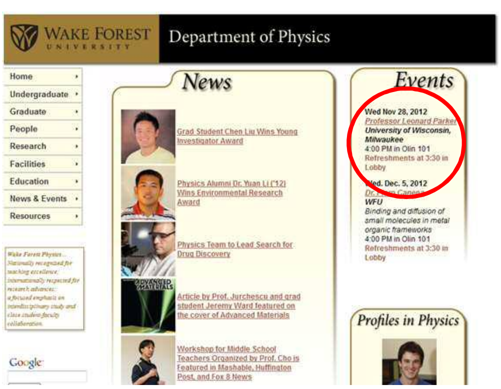
Plan for Lecture 37:

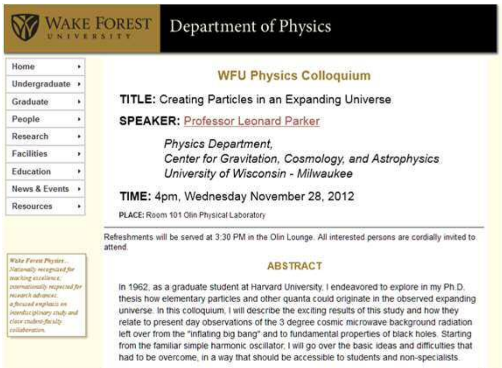
Chapter 10 in F & W:

Soliton surface waves

1. **Nonlinear water surface waves – soliton solutions**

22	Wed, 10/17/2012	Chap. 7, 5	Moment of inertia	
	Fri, 10/19/2012		Fall break	
23	Mon, 10/22/2012	Chap. 5	Rigid body rotation	#16
24	Wed, 10/24/2012	Chap. 5	Rigid body rotation	#17
25	Fri, 10/26/2012	Chap. 5	Rigid body rotation	#18
26	Mon, 10/29/2012	Chap. 8	Waves in elastic membranes	#19
27	Wed, 10/31/2012	Chap. 9	Introduction to hydrodynamics	
28	Fri, 11/01/2012	Chap. 9	Introduction to hydrodynamics	
29	Mon, 11/05/2012	Chap. 9	Introduction to hydrodynamics	#20
30	Wed, 11/07/2012	Chap. 9	Sound waves	
31	Fri, 11/09/2012	Chap. 9	Linear sound waves	#21
32	Mon, 11/12/2012	Chap. 9	Green's function for linear sound	
33	Wed, 11/14/2012	Chap. 9	Non-linear sound	
34	Fri, 11/16/2012	Chap. 9	Non-linear sound	Take Home Exam
35	Mon, 11/19/2012	Chap. 10	Surface waves	Take Home Exam
	Wed, 11/21/2012		*Thanksgiving Holiday*	
	Fri, 11/23/2012		*Thanksgiving Holiday*	
36	Mon, 11/26/2012	Chap. 10	Surface waves	Exam due
7	Wed, 11/28/2012	Chap. 10	Surface waves	
38	Fri, 11/30/2012	Chap. 10	Surface waves	
39	Mon, 12/03/2012		Student presentations I	
40	Wed, 12/05/2012		Student presentations II	





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WFU Physics Colloquium

TITLE: Creating Particles in an Expanding Universe

SPEAKER: Professor Leonard Parker

Physics Department,
Center for Gravitation, Cosmology, and Astrophysics
University of Wisconsin - Milwaukee

TIME: 4pm, Wednesday November 28, 2012

PLACE: Room 101 Olin Physical Laboratory

Refreshments will be served at 3:30 PM in the Olin Lounge. All interested persons are cordially invited to attend.

ABSTRACT

In 1962, as a graduate student at Harvard University, I endeavored to explore in my Ph.D. thesis how elementary particles and other quanta could originate in the observed expanding universe. In this colloquium, I will describe the exciting results of this study and how they relate to present day observations of the 3 degree cosmic microwave background radiation left over from the "inflating big bang" and to fundamental properties of black holes. Starting from the familiar simple harmonic oscillator, I will go over the basic ideas and difficulties that had to be overcome, in a way that should be accessible to students and non-specialists.

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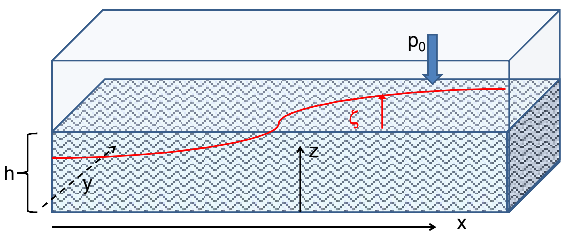
Consider a container of water with average height h and surface h+ζ(x,y,t) (slightly different notation than last time):



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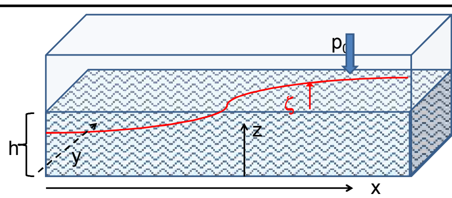

Within fluid: $0 \le z \le h+\zeta$

$$-\frac{\partial \Phi}{\partial t} + \tfrac{1}{2} v^2 + g(z-h) = \text{constant} \quad \text{(We have absorbed}$$

$$-\nabla^2 \Phi = 0 \qquad p_0 \text{ in our constant.)}$$

At surface: $z = h+\zeta$ with $\zeta = \zeta(x,y,t)$

$$\frac{d\zeta}{dt} = \frac{\partial \zeta}{\partial t} + v_x \frac{\partial \zeta}{\partial x} + v_y \frac{\partial \zeta}{\partial y} \quad \text{where } v_{x,y} = v_{x,y}(x,y,h+\zeta,t)$$

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Convenient assumptions: trivial y dependence
express problem in terms of $\Phi(x,z,t)$
and $\zeta(x,t)$

Bernoulli's equation at water surface

$$-\frac{\partial\Phi(x,z,t)}{\partial t}+\frac{1}{2}\left[\left(\frac{\partial\Phi(x,z,t)}{\partial x}\right)^2+\left(\frac{\partial\Phi(x,z,t)}{\partial z}\right)^2\right]_{z=h+\zeta}+g\zeta(x,t)=0$$

Consistent vertical velocity at water surface

$$v_z(x,z,t)\rfloor_{z=h+\zeta}=\frac{d\zeta}{dt}=\mathbf{v}\cdot\nabla\zeta+\frac{\partial\zeta}{\partial t}.$$

$$-\frac{\partial\Phi(x,z,t)}{\partial z}+\frac{\partial\Phi(x,z,t)}{\partial x}\frac{\partial\zeta(x,t)}{\partial x}-\frac{\partial\zeta(x,t)}{\partial t}\Big\rfloor_{z=h+\zeta}=0$$

Boundary condition at z=0

Zero vertical velocity at bottom of tank

$$\frac{\partial\Phi(x,0,t)}{\partial z}=0.$$

Taylor's expansion about $z=0$

$$\Phi(x,z,t)\approx\Phi(x,0,t)+z\frac{\partial\Phi}{\partial z}(x,0,t)+\frac{z^2}{2}\frac{\partial^2\Phi}{\partial z^2}(x,0,t)+\frac{z^3}{3!}\frac{\partial^3\Phi}{\partial z^3}(x,0,t)+\frac{z^4}{4!}\frac{\partial^4\Phi}{\partial z^4}(x,0,t)\cdots$$

$$\Rightarrow\Phi(x,z,t)\approx\Phi(x,0,t)+\frac{z^2}{2}\frac{\partial^2\Phi(x,0,t)}{\partial z^2}+\frac{z^4}{4!}\frac{\partial^4\Phi(x,0,t)}{\partial z^4}\cdots$$

$$\Phi(x,z,t)\approx\Phi(x,0,t)+\frac{z^2}{2}\frac{\partial^2\Phi(x,0,t)}{\partial z^2}+\frac{z^4}{4!}\frac{\partial^4\Phi(x,0,t)}{\partial z^4}\cdots$$

From Laplace equation: $\frac{\partial^2\Phi(x,z,t)}{\partial x^2}+\frac{\partial^2\Phi(x,z,t)}{\partial z^2}=0$

Modified Taylor's expansion

$$\Phi(x,z,t)\approx\Phi(x,0,t)-\frac{z^2}{2}\frac{\partial^2\Phi}{\partial x^2}(x,0,t)+\frac{z^4}{4!}\frac{\partial^4\Phi}{\partial x^4}(x,0,t)\cdots$$

Bernoulli's equation at water surface

$$-\frac{\partial\Phi(x,z,t)}{\partial t}+\frac{1}{2}\left[\left(\frac{\partial\Phi(x,z,t)}{\partial x}\right)^2+\left(\frac{\partial\Phi(x,z,t)}{\partial z}\right)^2\right]_{z=h+\zeta}+g\zeta(x,t)=0$$