PHY 711 Classical Mechanics and Mathematical Methods
10-10:50 AM MWF Olin 103

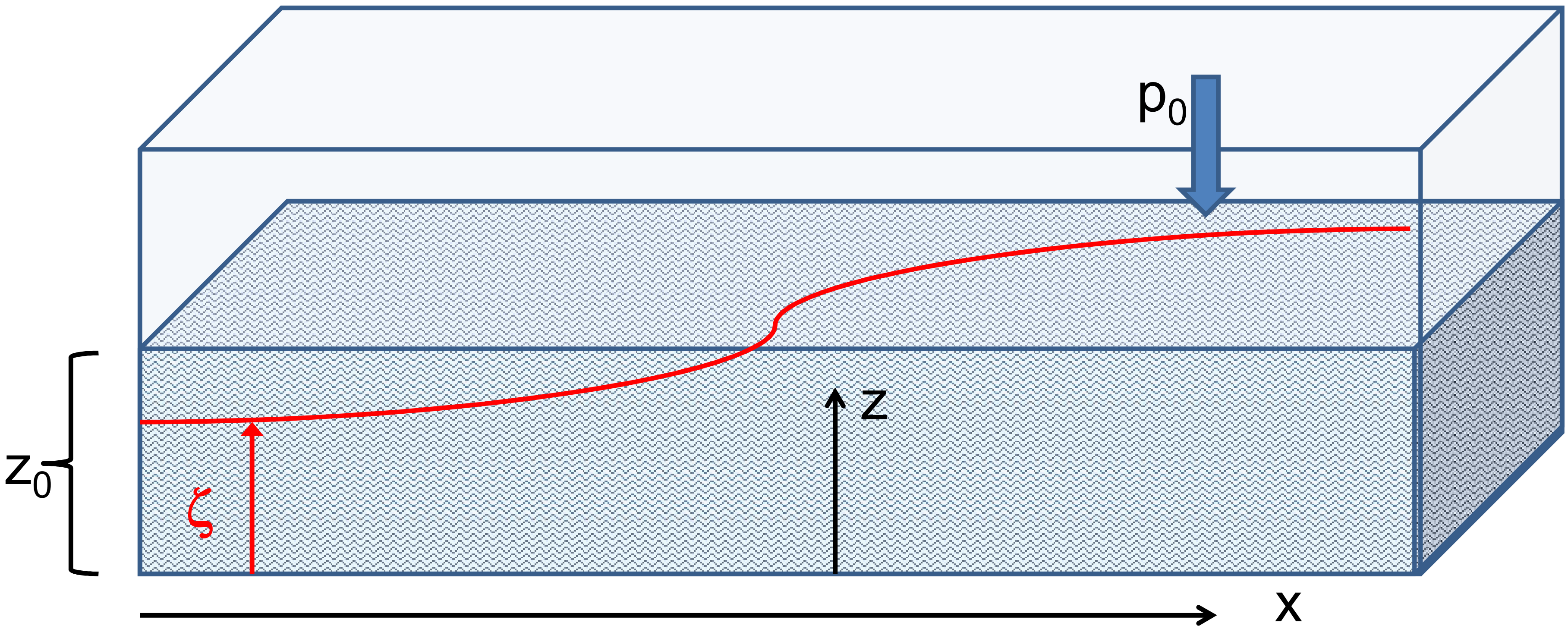
Plan for Lecture 35:

Chapter 10 in F & W: Surface waves

1. Water waves in a channel
2. Wave-like solutions; wave speed

22	Wed, 10/17/2012	Chap. 7, 5	Moment of inertia	
	Fri, 10/19/2012		Fall break	
23	Mon, 10/22/2012	Chap. 5	Rigid body rotation	#16
24	Wed, 10/24/2012	Chap. 5	Rigid body rotation	#17
25	Fri, 10/26/2012	Chap. 5	Rigid body rotation	#18
26	Mon, 10/29/2012	Chap. 8	Waves in elastic membranes	#19
27	Wed, 10/31/2012	Chap. 9	Introduction to hydrodynamics	
28	Fri, 11/01/2012	Chap. 9	Introduction to hydrodynamics	
29	Mon, 11/05/2012	Chap. 9	Introduction to hydrodynamics	#20
30	Wed, 11/07/2012	Chap. 9	Sound waves	
31	Fri, 11/09/2012	Chap. 9	Linear sound waves	#21
32	Mon, 11/12/2012	Chap. 9	Green's function for linear sound	
33	Wed, 11/14/2012	Chap. 9	Non-linear sound	
34	Fri, 11/16/2012	Chap. 9	Non-linear sound	Take Home Exam
35	Mon, 11/19/2012	Chap. 10	Surface waves	Take Home Exam
	Wed, 11/21/2012		*Thanksgiving Holiday*	
	Fri, 11/23/2012		*Thanksgiving Holiday*	
36	Mon, 11/26/2012	Chap. 10	Surface waves	Exam due
37	Wed, 11/28/2012	Chap. 10	Surface waves	
38	Fri, 11/30/2012	Chap. 10	Surface waves	
39	Mon, 12/03/2012		Student presentations I	
40	Wed, 12/05/2012		Student presentations II	

Consider a container of water with average height z_0 and surface ζ:



Euler's equation for incompressible fluid:

$$\frac{d\mathbf{v}}{dt} = f_{applied} - \frac{\nabla p}{\rho} = -g\hat{\mathbf{z}} - \frac{\nabla p}{\rho}$$

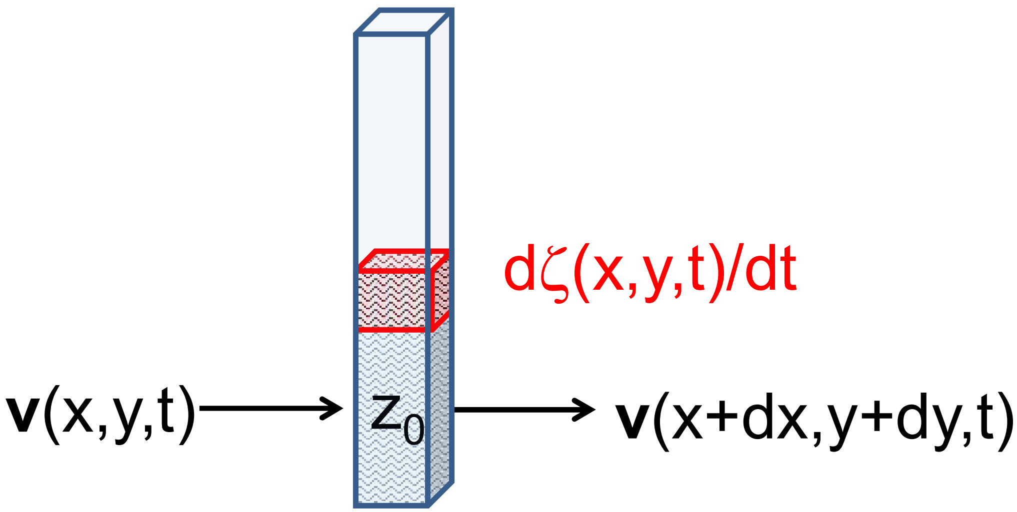
Assume that $v_z << v_x, v_y$ $\qquad \Rightarrow -g - \frac{1}{\rho}\frac{\partial p}{\partial z} = 0$

$$\Rightarrow p(x,y,z,t) = p_0 + \rho g\left(\zeta(x,y,t) - z\right)$$

Horizontal fluid motions:

$$\frac{dv_x}{dt} \approx \frac{\partial v_x}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} = -g\frac{\partial \zeta}{\partial x}$$

$$\frac{dv_y}{dt} \approx \frac{\partial v_y}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial y} = -g\frac{\partial \zeta}{\partial y}$$



Continuity condition for flow of incompressible fluid :

$$\frac{\partial \zeta}{\partial t} + z_0 \nabla \cdot \mathbf{v} = 0$$

From horizontal flow relations : $\frac{\partial \mathbf{v}}{\partial t} = -g\nabla \zeta$

Equation for surface function : $\frac{\partial^2 \zeta}{\partial t^2} - g z_0 \nabla^2 \zeta = 0$

Surface wave equation :

$$\frac{\partial^2 \zeta}{\partial t^2} - c^2 \nabla^2 \zeta = 0 \qquad c^2 = g z_0$$

More complete analysis finds :

$$c^2 = \frac{g}{k} \tanh(k z_0) \qquad \text{where } k = \frac{2\pi}{\lambda}$$