PHY 711 Classical Mechanics and Mathematical Methods
10-10:50 AM MWF Olin 103

Plan for Lecture 3:

Chapter 1 – scattering theory continued; center of mass versus laboratory reference frame.

PHY 711 Classical Mechanics and Mathematical Methods

MWF 10 AM-10:50 PM	OPL 103	http://www.wfu.edu/~natalie/f12phy711/

Instructor: Natalie Holzwarth	**Phone:** 758-5510	**Office:** 300 OPL	**e-mail:** natalie@wfu.edu

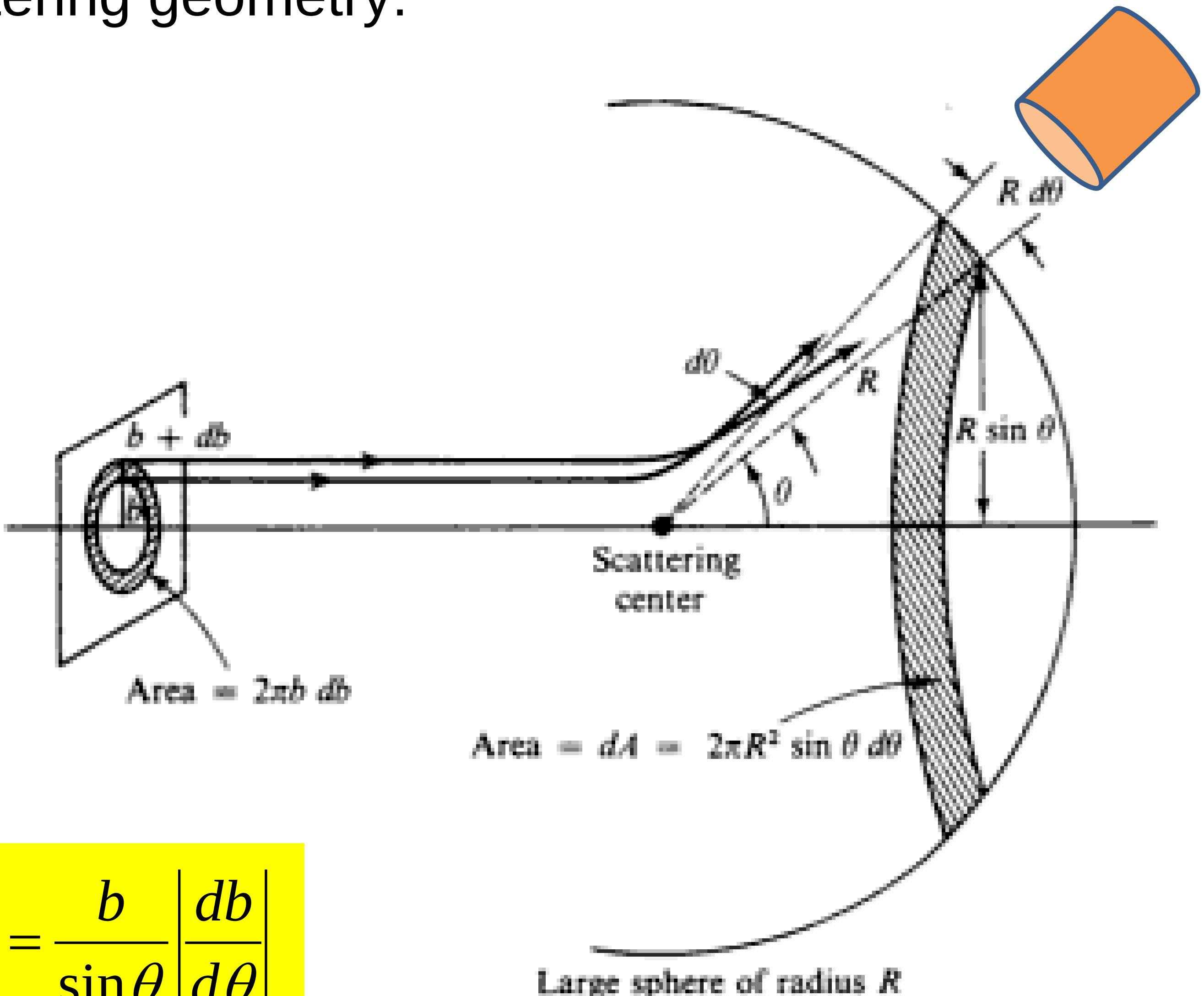
---

Course schedule

(Preliminary schedule -- subject to frequent adjustment.)

Date	F&W Reading	Topic	Assi gnment
Wed, 8/29/2012	Chap. 1	Review of basic principles;Scattering theory	#1
Fri, 8/31/2012	Chap. 1	Scattering theory continued	#2
Mon, 9/03/2012	Chap. 1	Scattering theory continued	#3
Wed, 9/05/2012	Chap. 1	Scattering theory continued	#4

Scattering geometry:



$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

Relationship between scattering angle θ and impact parameter *b* for interaction potential *V(r)*:

$$\theta = \pi - 2b\int_{r_{\min}}^{\infty} dr \left(\frac{1/r^2}{\sqrt{1 - \frac{b^2}{r^2} - \frac{V(r)}{E}}} \right)$$

$$\text{where: } 1 - \frac{b^2}{r_{\min}^2} - \frac{V(r_{\min})}{E} = 0$$

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

Example of cross section analysis

Rutherford scattering:

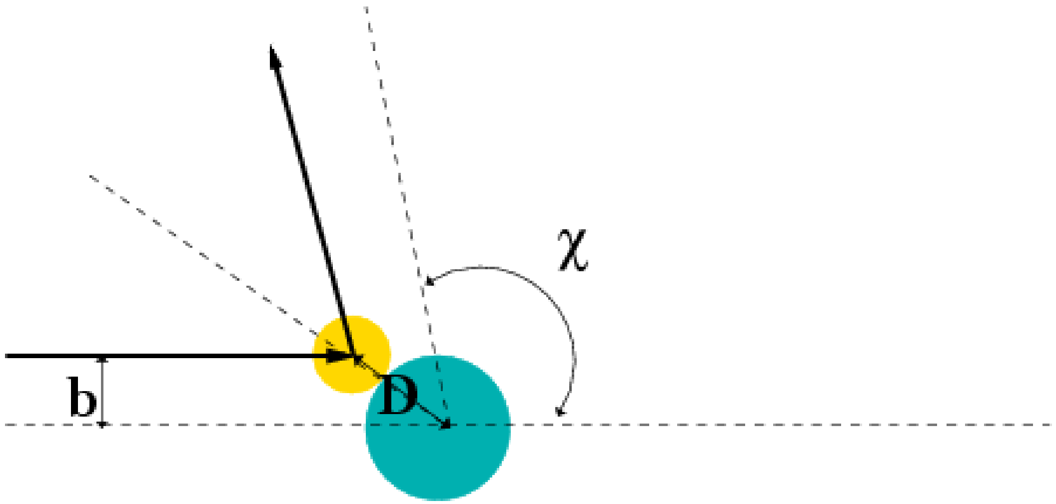
$$\theta = 2\sin^{-1}\left(\frac{1}{\sqrt{(b/\kappa)^2+1}}\right)$$

$$\frac{b}{\kappa} = \left|\frac{\cos(\theta/2)}{\sin(\theta/2)}\right|$$

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right| = \frac{\kappa^2}{4}\frac{1}{\sin^4(\theta/2)}$$

Example of cross section analysis

Hard sphere scattering:



For your homework you showed that

$$b = D\cos\left(\frac{\chi}{2}\right)$$

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{b}{\sin\chi}\left|\frac{db}{d\chi}\right| = \frac{D^2}{4}$$

The results above were derived in the center of mass reference frame; relationship between normal laboratory reference and center of mass:

Laboratory reference frame:

Center of mass reference frame:

Relationship between center of mass and laboratory frames of reference

Definition of center of mass $\mathbf{R}_{CM}$

$$m_1\mathbf{r}_1 + m_2\mathbf{r}_2 = (m_1 + m_2)\mathbf{R}_{CM}$$

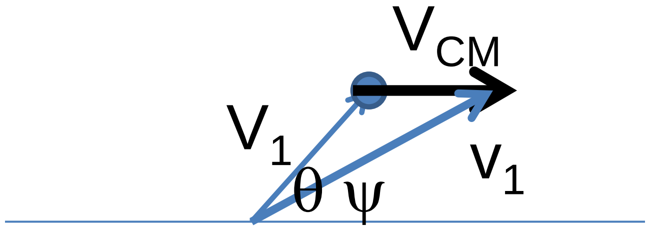
$$m_1\dot{\mathbf{r}}_1 + m_2\dot{\mathbf{r}}_2 = (m_1 + m_2)\dot{\mathbf{R}}_{CM}$$

$$m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = (m_1 + m_2)\mathbf{V}_{CM} = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$$

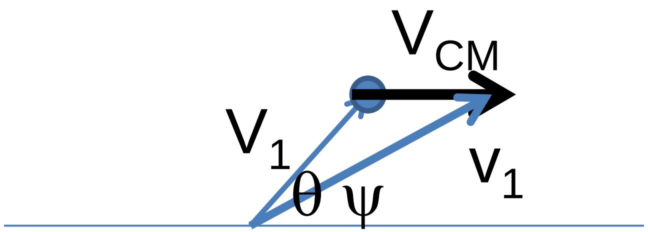
In our case:

$$\mathbf{V}_{CM} = \frac{m_1}{m_1 + m_2}\mathbf{u}_1 = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2}$$

$$\mathbf{v}_1 = \mathbf{V}_1 + \mathbf{V}_{CM}$$



Relationship between center of mass and laboratory frames of reference



Note that for an elastic collision

$$U_1 = V_1 \quad \text{and} \quad U_2 = V_2 = V_{CM}$$

Also note that: $\quad m_1 U_1 = m_2 U_2$

So that: $\quad V_{CM}/V_1 = m_1/\mathrm{m}_2$

$$\mathbf{v}_1 = \mathbf{V}_1 + \mathbf{V}_{CM}$$

$$v_1 \sin\psi = V_1 \sin\theta$$

$$v_1 \cos\psi = V_1 \cos\theta + V_{CM}$$

$$\tan\psi = \frac{\sin\theta}{\cos\theta + V_{CM}/V_1} = \frac{\sin\theta}{\cos\theta + m_1/m_2}$$

Differential cross sections in different reference frames

$$\left(\frac{d\sigma_{LAB}(\psi)}{d\Omega_{LAB}} \right) = \left(\frac{d\sigma_{CM}(\theta)}{d\Omega_{CM}} \right) \frac{d\Omega_{CM}}{d\Omega_{LAB}}$$

$$\frac{d\Omega_{CM}}{d\Omega_{LAB}} = \left| \frac{\sin\theta}{\sin\psi} \frac{d\theta}{d\psi} \right| = \left| \frac{d\cos\theta}{d\cos\psi} \right|$$

From the previous result and/or conservation of momentum and energy, it is possible to show that:

$$\cos\psi = \frac{\cos\theta + m_1/m_2}{\sqrt{1 + 2m_1/m_2 \cos\theta + (m_1/m_2)^2}}$$

Differential cross sections in different reference frames – continued:

$$\left(\frac{d\sigma_{LAB}(\psi)}{d\Omega_{LAB}}\right)=\left(\frac{d\sigma_{CM}(\theta)}{d\Omega_{CM}}\right)\left|\frac{d\cos\theta}{d\cos\psi}\right|$$

$$\left(\frac{d\sigma_{LAB}(\psi)}{d\Omega_{LAB}}\right)=\left(\frac{d\sigma_{CM}(\theta)}{d\Omega_{CM}}\right)\frac{\left(1+2m_1/m_2\cos\theta+(m_1/m_2)^2\right)^{3/2}}{(m_1/m_2)\cos\theta+1}$$

$$\text{where:}\quad \tan\psi=\frac{\sin\theta}{\cos\theta+m_1/m_2}$$

$$\left(\frac{d\sigma_{LAB}(\psi)}{d\Omega_{LAB}}\right)=\left(\frac{d\sigma_{CM}(\theta)}{d\Omega_{CM}}\right)\frac{\left(1+2m_1/m_2\cos\theta+(m_1/m_2)^2\right)^{3/2}}{(m_1/m_2)\cos\theta+1}$$

$$\text{where:}\quad \tan\psi=\frac{\sin\theta}{\cos\theta+m_1/m_2}$$

Example: suppose $m_1 = m_2$

$$\text{In this case:}\quad \tan\psi=\frac{\sin\theta}{\cos\theta+1}\quad\Rightarrow\psi=\frac{\theta}{2}$$

$$\text{note that}\quad 0\le\psi\le\frac{\pi}{2}$$

$$\left(\frac{d\sigma_{LAB}(\psi)}{d\Omega_{LAB}}\right)=\left(\frac{d\sigma_{CM}(2\psi)}{d\Omega_{CM}}\right)\cdot 4\cos\psi$$