PHY 711 Classical Mechanics and Mathematical Methods
10-10:50 AM MWF Olin 103

Plan for Lecture 22:

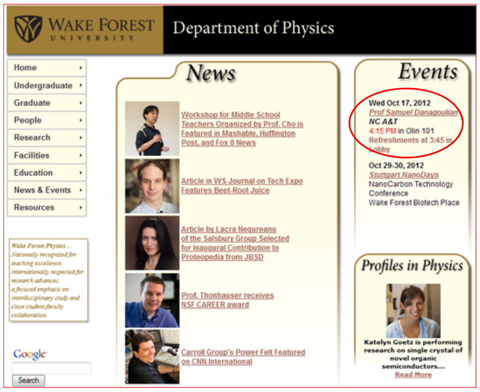
Back to the wave equation; begin rotational motion (Chap. 5)

1. Standing waves ←→ periodic waves
2. Moment of inertia tensor

Course schedule

(Preliminary schedule -- subject to frequent adjustment.)

	Date	F&W Reading	Topic	Assignment
1	Wed, 8/29/2012	Chap. 1	Review of basic principles;Scattering theory	#1
2	Fri, 8/31/2012	Chap. 1	Scattering theory continued	#2
3	Mon, 9/03/2012	Chap. 1	Scattering theory continued	#3
4	Wed, 9/05/2012	Chap. 1 & 2	Scattering theory/Accelerated coordinate frame	#4
5	Fri, 9/07/2012	Chap. 2	Accelerated coordinate frame	#5
6	Mon, 9/10/2012	Chap. 3	Calculus of Variation	#6
7	Wed, 9/12/2012	Chap. 3	Calculus of Variation continued	
8	Fri, 9/14/2012	Chap. 3	Lagrangian	#7
9	Mon, 9/17/2012	Chap. 3 & 6	Lagrangian	#8
10	Wed, 9/19/2012	Chap. 3 & 6	Lagrangian	#9
11	Fri, 9/21/2012	Chap. 3 & 6	Lagrangian	#10
12	Mon, 9/24/2012	Chap. 3 & 6	Lagrangian and Hamiltonian	#11
13	Wed, 9/26/2012	Chap. 6	Lagrangian and Hamiltonian	#12
14	Fri, 9/28/2012	Chap. 6	Lagrangian and Hamiltonian	#13
15	Mon, 10/01/2012	Chap. 4	Small oscillations	#14
16	Wed, 10/03/2012	Chap. 4	Small oscillations	#15
17	Fri, 10/05/2012	Chap. 4	Small oscillations	
18	Mon, 10/08/2012	Chap. 7	Wave equation	Take Home Exam
19	Wed, 10/10/2012	Chap. 7	Wave equation	Take Home Exam
20	Fri, 10/12/2012	Chap. 7	Wave equation	Take Home Exam
21	Mon, 10/15/2012	Chap. 7	Wave equation	Exam due
22	Wed, 10/17/2012	Chap. 7, 5	Moment of inertia	
	Fri, 10/19/2012		Fall break	



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WFU Physics Colloquium

TITLE: A Precise Measurement of the Neutral Pi-meson Lifetime at Jefferson Laboratory

SPEAKER: Professor Samuel Danagoulian ,

Department of Physics
North Carolina A&T State University

TIME: Wednesday October 17, 2012 at 4:15 PM*

PLACE: Room 101 Olin Physical Laboratory

* Note: late starting time.

Refreshments will be served at 3:45 PM in the Olin Lounge. All interested persons are cordially invited to attend.

ABSTRACT

The lifetime of the neutral pi-meson was measured with an accuracy of 2.8% using Primakoff process of photoproduction of pions from nuclear targets. The experiment was conducted on 6-GeV electron beam at Jefferson Laboratory using high precision photon tagger in experimental Hall B and multichannel lead-glass - lead tungstate (PbWO_4) crystal calorimeter HYCAL (experiment PrimEx). The second phase of the experiment was conducted recently, data analysis is in progress. The accuracy will be improved to about 1.5% which is compatible with the level of accuracy of theoretical prediction.

The wave equation

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0$$

Suppose $u(x,t) = e^{-i\omega t}\tilde{F}(x,\omega)$

Then $\tilde{F}(x,\omega)$ must satisfy an eigenvalue equation :

$$\frac{\partial^2 \tilde{F}(x,\omega)}{\partial x^2} = -k^2 \tilde{F}(x,\omega) \qquad \text{where } k^2 \equiv \frac{\omega^2}{c^2}$$

For fixed boundary conditions :

for example $\tilde{F}(0,\omega) = 0$ and $\tilde{F}(L,\omega) = 0$

$$\tilde{F}_n(x,\omega) = \sin\left(\frac{n\pi x}{L}\right) \qquad k \to k_n = \frac{n\pi}{L} \equiv \frac{\omega_n}{c}$$

$$\Rightarrow u(x,t) = \sum_n C_n e^{-i\omega_n t} \sin(k_n x) = \sum_n \frac{C_n}{2i} e^{-i\omega_n t}\left(e^{ik_n x} - e^{-ik_n x}\right)$$

$$= \sum_n \frac{C_n}{2i}\left(e^{ik_n(x-ct)} - e^{-ik_n(x+ct)}\right) \equiv f(x-ct) + g(x+ct)$$

The physics of rigid body motion; body fixed frame vs inertial frame:

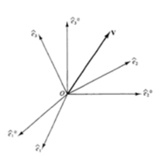


Figure 6.1 Transformation to a rotating coordinate system.

Let **V** be a general vector, e.g., the position of a particle. This vector can be characterized by its components with respect to either orthonormal triad. Thus we can write

$$\mathbf{V} = \sum_{i=1}^{3} V_i^0 \hat{e}_i^0 \qquad (6.1a)$$

$$\mathbf{V} = \sum_{i=1}^{3} V_i \hat{e}_i \qquad (6.1b)$$

Comparison of analysis in "inertial frame" versus "non-inertial frame"

Denote by $\hat{e}_i^0$ a fixed coordinate system

Denote by $\hat{e}_i$ a moving coordinate system

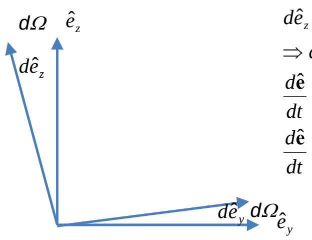
$$\mathbf{V} = \sum_{i=1}^{3} V_i^{\,0} \hat{e}_i^0 = \sum_{i=1}^{3} V_i \hat{e}_i$$

$$\left(\frac{d\mathbf{V}}{dt}\right)_{inertial} = \sum_{i=1}^{3} \frac{dV_i^{\,0}}{dt}\hat{e}_i^0 = \sum_{i=1}^{3} \frac{dV_i}{dt}\hat{e}_i + \sum_{i=1}^{3} V_i \frac{d\hat{e}_i}{dt}$$

$$\text{Define : } \left(\frac{d\mathbf{V}}{dt}\right)_{body} \equiv \sum_{i=1}^{3} \frac{dV_i}{dt}\hat{e}_i$$

$$\Rightarrow \left(\frac{d\mathbf{V}}{dt}\right)_{inertial} = \left(\frac{d\mathbf{V}}{dt}\right)_{body} + \sum_{i=1}^{3} V_i \frac{d\hat{e}_i}{dt}$$

Properties of the frame motion (rotation):



$$d\hat{e}_y = d\Omega \hat{e}_z$$

$$d\hat{e}_z = -d\Omega \hat{e}_y$$

$$\Rightarrow d\hat{\mathbf{e}} = d\boldsymbol{\Omega} \times \hat{\mathbf{e}}$$

$$\frac{d\hat{\mathbf{e}}}{dt} = \frac{d\boldsymbol{\Omega}}{dt} \times \hat{\mathbf{e}}$$

$$\frac{d\hat{\mathbf{e}}}{dt} = \boldsymbol{\omega} \times \hat{\mathbf{e}}$$

$$\left(\frac{d\mathbf{V}}{dt}\right)_{inertial} = \left(\frac{d\mathbf{V}}{dt}\right)_{body} + \sum_{i=1}^{3} V_i \frac{d\hat{e}_i}{dt}$$

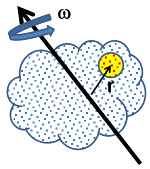
$$\left(\frac{d\mathbf{V}}{dt}\right)_{inertial} = \left(\frac{d\mathbf{V}}{dt}\right)_{body} + \boldsymbol{\omega} \times \mathbf{V}$$

Effects on acceleration:

$$\left(\frac{d}{dt}\frac{d\mathbf{V}}{dt}\right)_{inertial} = \left(\left(\frac{d}{dt}\right)_{body} + \boldsymbol{\omega} \times\right)\left\{\left(\frac{d\mathbf{V}}{dt}\right)_{body} + \boldsymbol{\omega} \times \mathbf{V}\right\}$$

$$\left(\frac{d^2\mathbf{V}}{dt^2}\right)_{inertial} = \left(\frac{d^2\mathbf{V}}{dt^2}\right)_{body} + 2\boldsymbol{\omega} \times \left(\frac{d\mathbf{V}}{dt}\right)_{body} + \frac{d\boldsymbol{\omega}}{dt} \times \mathbf{V} + \boldsymbol{\omega} \times \boldsymbol{\omega} \times \mathbf{V}$$

Kinetic energy of rigid body :



$$\left(\frac{d\mathbf{r}}{dt}\right)_{inertial} = \left(\frac{d\mathbf{r}}{dt}\right)_{body} + \boldsymbol{\omega}\times\mathbf{r}$$

$$\left(\frac{d\mathbf{r}}{dt}\right)_{inertial} = \boldsymbol{\omega}\times\mathbf{r}$$

$$T = \sum_p \frac{1}{2} m_p v_p^2 = \sum_p \frac{1}{2} m_p \left(\left|\boldsymbol{\omega}\times\mathbf{r}_p\right|\right)^2$$

$$= \sum_p \frac{1}{2} m_p \left(\boldsymbol{\omega}\times\mathbf{r}_p\right)\cdot\left(\boldsymbol{\omega}\times\mathbf{r}_p\right)$$

$$= \sum_p \frac{1}{2} m_p \left[(\boldsymbol{\omega}\cdot\boldsymbol{\omega})(\mathbf{r}_p\cdot\mathbf{r}_p) - (\mathbf{r}_p\cdot\boldsymbol{\omega})^2\right]$$

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$$T = \sum_p \frac{1}{2} m_p \left[(\boldsymbol{\omega}\cdot\boldsymbol{\omega})(\mathbf{r}_p\cdot\mathbf{r}_p) - (\mathbf{r}_p\cdot\boldsymbol{\omega})^2\right]$$

$$= \boldsymbol{\omega}\cdot\ddot{\mathbf{I}}\cdot\boldsymbol{\omega}$$

Moment of inertia tensor :

$$\ddot{\mathbf{I}} \equiv \sum_p m_p \left(\mathbf{1} r_p^2 - \mathbf{r}_p\mathbf{r}_p\right) \qquad \text{(dyad notation)}$$

Matrix notation :

$$\ddot{\mathbf{I}} \equiv \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix}$$

$$I_{ij} \equiv \sum_p m_p \left(\delta_{ij} r_p^2 - r_{pi} r_{pj}\right)$$

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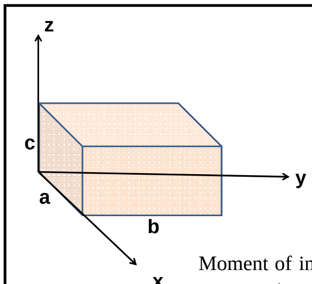

Moment of inertia tensor :

$$\ddot{\mathbf{I}} = M \begin{pmatrix} \frac{1}{3}(b^2+c^2) & -\frac{1}{4}ab & -\frac{1}{4}ac \\ -\frac{1}{4}ab & \frac{1}{3}(a^2+c^2) & -\frac{1}{4}bc \\ -\frac{1}{4}ac & -\frac{1}{4}bc & \frac{1}{3}(a^2+b^2) \end{pmatrix}$$

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Properties of moment of inertia tensor:

- Symmetric matrix → real eigenvalues I_1, I_2, I_3
- → orthogonal eigenvectors

$$\vec{\mathbf{I}} \cdot \hat{\mathbf{e}}_i = I_i \hat{\mathbf{e}}_i \qquad i = 1,2,3$$