## PHY 711 Classical Mechanics and Mathematical Methods
**10-10:50 AM MWF Olin 103**

**Plan for Lecture 20:**

**Summary of mathematical methods**

1. **Sturm-Liouville equations**
2. **Green's function methods**
3. **Contour integration**
4. **Laplace and Fourier transforms**



### Course schedule

(Preliminary schedule -- subject to frequent adjustment.)

| | Date | F&W Reading | Topic | Assignment |
|---|---|---|---|---|
| 1 | Wed, 8/29/2012 | Chap. 1 | Review of basic principles;Scattering theory | #1 |
| 2 | Fri, 8/31/2012 | Chap. 1 | Scattering theory continued | #2 |
| 3 | Mon, 9/03/2012 | Chap. 1 | Scattering theory continued | #3 |
| 4 | Wed, 9/05/2012 | Chap. 1 & 2 | Scattering theory/Accelerated coordinate frame | #4 |
| 5 | Fri, 9/07/2012 | Chap. 2 | Accelerated coordinate frame | #5 |
| 6 | Mon, 9/10/2012 | Chap. 3 | Calculus of Variation | #6 |
| 7 | Wed, 9/12/2012 | Chap. 3 | Calculus of Variation continued | |
| 8 | Fri, 9/14/2012 | Chap. 3 | Lagrangian | #7 |
| 9 | Mon, 9/17/2012 | Chap. 3 & 6 | Lagrangian | #8 |
| 10 | Wed, 9/19/2012 | Chap. 3 & 6 | Lagrangian | #9 |
| 11 | Fri, 9/21/2012 | Chap. 3 & 6 | Lagrangian | #10 |
| 12 | Mon, 9/24/2012 | Chap. 3 & 6 | Lagrangian and Hamiltonian | #11 |
| 13 | Wed, 9/26/2012 | Chap. 6 | Lagrangian and Hamiltonian | #12 |
| 14 | Fri, 9/28/2012 | Chap. 6 | Lagrangian and Hamiltonian | #13 |
| 15 | Mon, 10/01/2012 | Chap. 4 | Small oscillations | #14 |
| 16 | Wed, 10/03/2012 | Chap. 4 | Small oscillations | #15 |
| 17 | Fri, 10/05/2012 | Chap. 4 | Small oscillations | |
| 18 | Mon, 10/08/2012 | Chap. 7 | Wave equation | Take Home Exam |
| 19 | Wed, 10/10/2012 | Chap. 7 | Wave equation | Take Home Exam |
| 20 | Fri, 10/12/2012 | Chap. 7 | Wave equation | Take Home Exam |
| 21 | Mon, 10/15/2012 | Chap. 7 | Wave equation | Exam due |



Sturm-Liouville equation:

Homogenous problem: $\left(-\frac{d}{dx}\tau(x)\frac{d}{dx}+v(x)-\lambda\sigma(x)\right)\phi_0(x)=0$

Inhomogenous problem: $\left(-\frac{d}{dx}\tau(x)\frac{d}{dx}+v(x)-\lambda\sigma(x)\right)\phi(x)=F(x)$

Eigenfunctions:

$$\left(-\frac{d}{dx}\tau(x)\frac{d}{dx}+v(x)\right)f_n(x)=\lambda_n\sigma(x)f_n(x)$$

Example: $\tau(x)=1;\quad \sigma(x)=1;\quad v(x)=0;\quad a=0$ and $b=L$

$$\lambda=1;\qquad F(x)=F_0\sin\left(\frac{\pi x}{L}\right)$$

Inhomogenous equation:

$$\left(-\frac{d^2}{dx^2}-1\right)\phi(x)=F_0\sin\left(\frac{\pi x}{L}\right)$$

Eigenvalue equation:

$$\left(-\frac{d^2}{dx^2}\right) f_n(x) = \lambda_n f_n(x)$$

Eigenfunctions

$$f_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

Eigenvalues:

$$\lambda_n = \left(\frac{n\pi}{L}\right)^2$$

Green's function for this example:

$$G(x,x') = \sum_n \frac{f_n(x) f_n(x')/N_n}{\lambda_n - \lambda} = \frac{2}{L}\sum_n \frac{\sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{n\pi x'}{L}\right)}{\left(\frac{n\pi}{L}\right)^2 - 1}$$

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Using Green's function to solve inhomogenous equation:

$$\left(-\frac{d^2}{dx^2} - 1\right)\phi(x) = F_0 \sin\left(\frac{\pi x}{L}\right)$$

$$\phi(x) = \phi_0(x) + \int_0^L G(x,x') F_0 \sin\left(\frac{\pi x'}{L}\right) dx'$$

$$= \phi_0(x) + \frac{2}{L}\sum_n \left[\frac{\sin\left(\frac{n\pi x}{L}\right)}{\left(\frac{n\pi}{L}\right)^2 - 1}\int_0^L \sin\left(\frac{n\pi x'}{L}\right) F_0 \sin\left(\frac{\pi x'}{L}\right) dx'\right]$$

$$= \phi_0(x) + \frac{F_0}{\left(\frac{\pi}{L}\right)^2 - 1}\sin\left(\frac{\pi x}{L}\right)$$

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Alternate Green's function method:

$$G(x,x') = \frac{1}{W} g_a(x_<) g_B(x_>)$$

$$\left(-\frac{d^2}{dx^2} - 1\right) g_i(x) = 0 \qquad \Rightarrow g_a(x) = \sin(x); \quad g_b(x) = \sin(L-x);$$

$$W = g_b(x)\frac{dg_a(x)}{dx} - g_a(x)\frac{dg_b(x)}{dx} = \sin(L-x)\cos(x) + \sin(x)\cos(L-x)$$

$$= \sin(L)$$

$$\phi(x) = \phi_0(x) + \frac{\sin(L-x)}{\sin(L)}\int_0^x \sin(x') F_0 \sin\left(\frac{\pi x'}{L}\right) dx'$$

$$+ \frac{\sin(x)}{\sin(L)}\int_x^L \sin(L-x') F_0 \sin\left(\frac{\pi x'}{L}\right) dx'$$

$$\phi(x) = \phi_0(x) + \frac{F_0}{\left(\frac{\pi}{L}\right)^2 - 1}\sin\left(\frac{\pi x}{L}\right)$$

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