

# **PHY 711 Classical Mechanics and Mathematical Methods**

**10-10:50 AM MWF Olin 103**

## **Plan for Lecture 2:**

- 1. Comments on Maple software**
- 2. Chapter 1 – scattering theory**
  - a) Rutherford scattering**
  - b) Scattering for arbitrary potential**


# PHY 711 Classical Mechanics and Mathematical Methods

MWF 10 AM-10:50 PM OPL 103 <http://www.wfu.edu/~natalie/f12phy711/>

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## Course schedule

(Preliminary schedule -- subject to frequent adjustment.)

Date	F&W Reading	Topic	Assignment
Wed, 8/29/2012	Chap. 1	Review of basic principles; Scattering theory	<a href="#">#1</a>
 Fri, 8/31/2012	Chap. 1	Scattering theory continued	<a href="#">#2</a>
Fri, 9/03/2012	Chap. 1	Scattering theory continued	<a href="#">#3</a>

## PHY 711 - Assignment #1

In order to calculate the differential cross section for Rutherford scattering, it is necessary to evaluate the following relation

# Some additional Maple examples

The screenshot shows the Maple 15 software interface. The window title is "D:\Userdata\Userdata\Coursework\phys711\Lecture2\anothermaple.mw - [Server 1] - Maple 15". The menu bar includes File, Edit, View, Insert, Format, Table, Drawing, Plot, Spreadsheet, Tools, Window, and Help. The toolbar contains various icons for file operations, editing, and plotting. On the left, there is a sidebar with categories like Favorites, MapleCloud (Disabled), Variables, Handwriting, Expression, Units (SI), Units (FPS), Common Symbols, Matrix, Components, Greek, Arrows, Relational, Relational Round, and Negated. The main workspace is divided into tabs: Text, Math, Drawing, Plot, and Animation. The Math tab is active, showing a Maple Plot with Times New Roman font and size 12. The code in the workspace is as follows:

```

> assume(a > 0 and a < 1);
> assume(t > 0);
> Y := (t, a) -> int(sqrt(1 - a^2 * (sin(theta))^2), theta = 0 .. t);

```

$$Y := (t, a) \rightarrow \int_0^t \sqrt{1 - a^2 \sin(\theta)^2} d\theta \quad (1)$$

```

> Y(t, a);

```

$$\frac{\sqrt{1 - \sin(t)^2} \operatorname{EllipticE}(\sin(t), a)}{\cos(t)} + \begin{cases} 2 \operatorname{floor}\left(\frac{1}{4} \frac{2t + \pi}{\pi}\right) \operatorname{EllipticE}(a) & 1 < \frac{1}{4} \frac{2t + \pi}{\pi} \\ 0 & \text{otherwise} \end{cases} + 2 \begin{cases} 2 \left( \operatorname{floor}\left(-\frac{1}{4} \frac{-2t + \pi}{\pi}\right) + 1 \right) \operatorname{EllipticE}(a) & 0 < 2t - \pi \\ 0 & \text{otherwise} \end{cases} + \begin{cases} 2 \left( \operatorname{floor}\left(-\frac{1}{4} \frac{-2t + 3\pi}{\pi}\right) + 1 \right) \operatorname{EllipticE}(a) & 0 < 2t - 3\pi \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

```

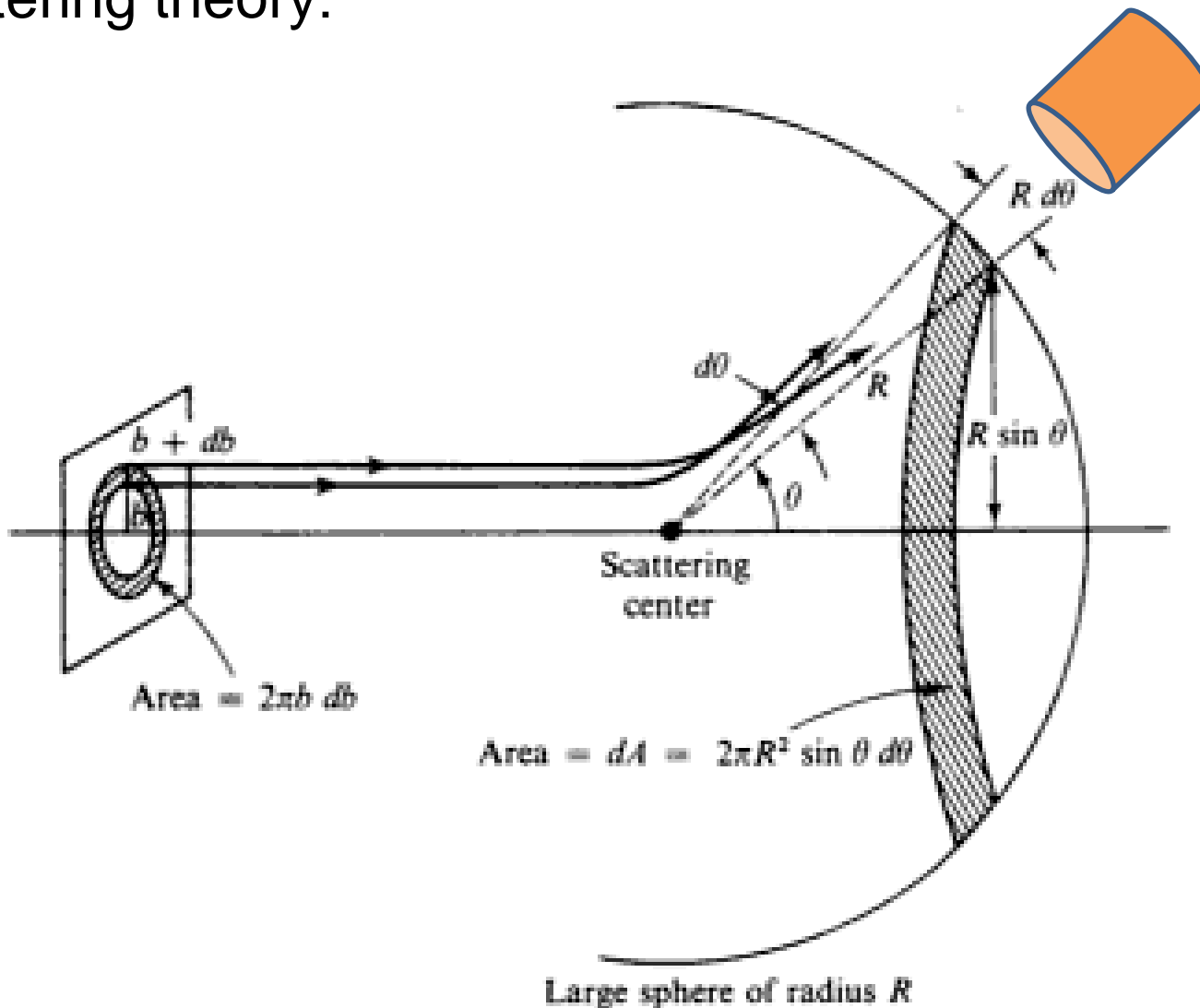
> plot({Y(t, 0.6), Y(t, 0.8), Y(t, 0.99)}, t = 0 .. 5);

```

The plot shows three curves: a blue curve (Y(t, 0.6)), a green curve (Y(t, 0.8)), and a red curve (Y(t, 0.99)). The x-axis ranges from 0 to 5, and the y-axis ranges from 0 to 10. The curves are piecewise linear with sharp jumps at certain points.

At the bottom right of the window, the status bar displays: Memory: 0.74M Time: 0.09s Text Mode.

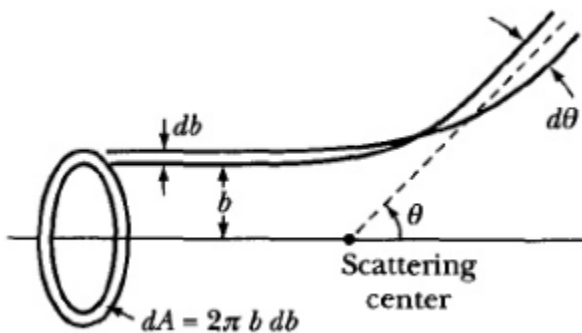
# Scattering theory:



**Figure 5.5** The scattering problem and relation of cross section to impact parameter.

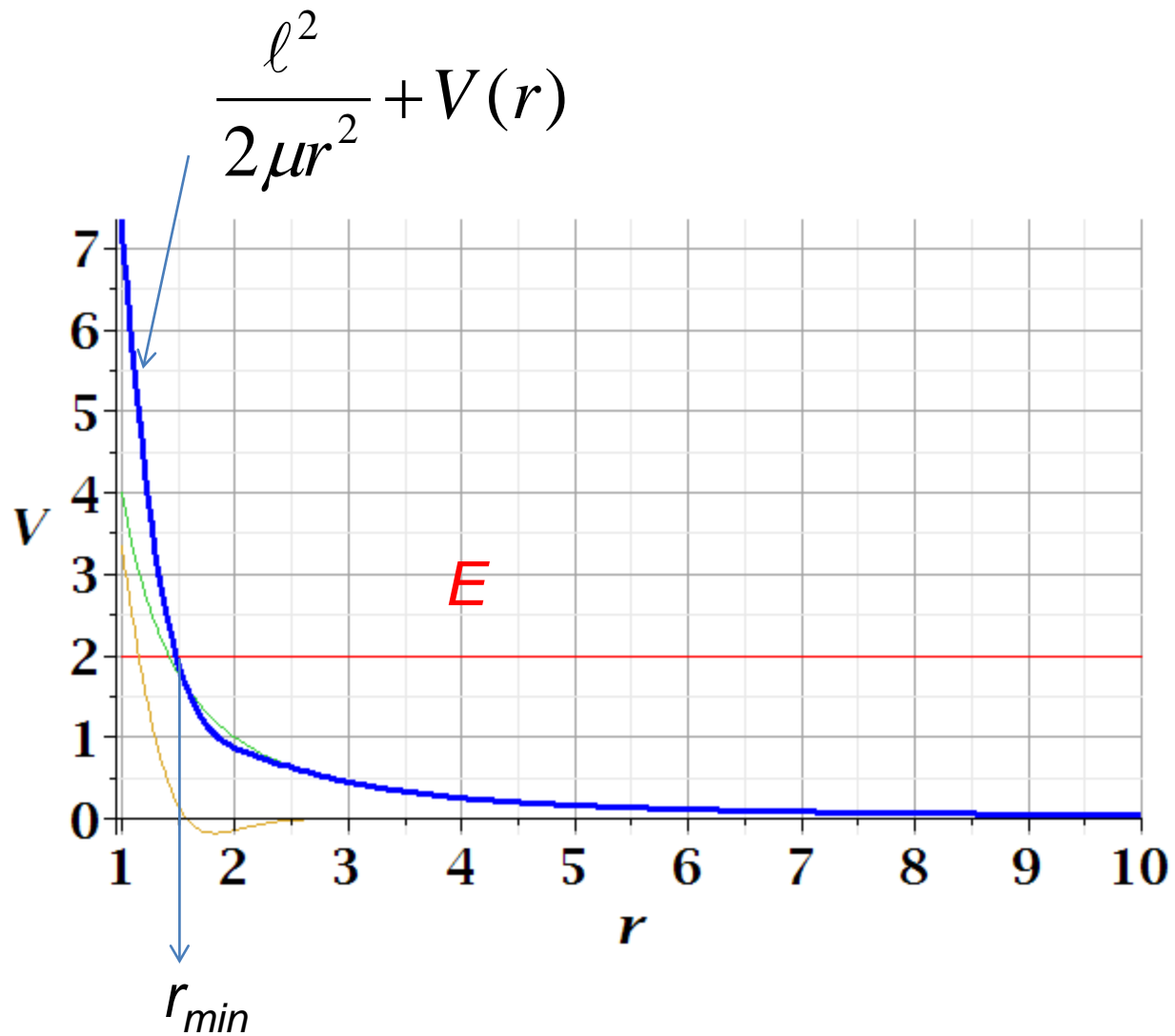
## Differential cross section

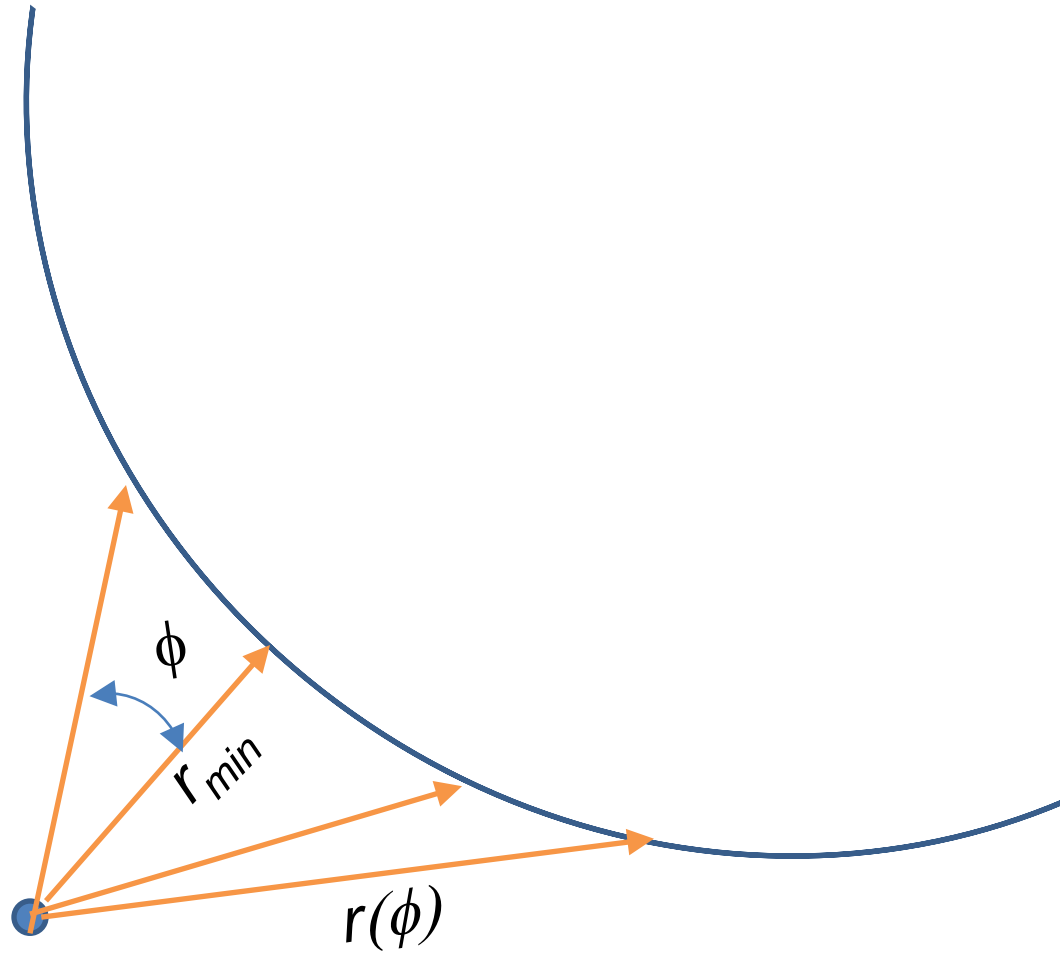
$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\text{Number of detected particles at } \theta \text{ per target particle}}{\text{Number of incident particles per unit area}}$$
$$= \text{Area of incident beam that is scattered into detector at angle } \theta$$



$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{2\pi b db}{2\pi \sin\theta d\theta} = \frac{b}{\sin\theta} \left|\frac{db}{d\theta}\right|$$

Figure from Marion & Thorton, Classical Dynamics





Conservation of angular momentum :

$$\ell = \mu r^2 \left( \frac{d\phi}{dt} \right)$$

Transformation of trajectory variables :

$$r(t) \Leftrightarrow r(\phi)$$

$$\frac{dr}{dt} = \frac{dr}{d\phi} \frac{d\phi}{dt} = \frac{dr}{d\phi} \frac{\ell}{\mu r^2}$$

Conservation of energy in the center of mass frame :

$$\begin{aligned} E &= \frac{1}{2} \mu \left( \frac{dr}{dt} \right)^2 + \frac{\ell^2}{2\mu r^2} + V(r) \\ &= \frac{1}{2} \mu \left( \frac{dr}{d\phi} \frac{\ell}{\mu r^2} \right)^2 + \frac{\ell^2}{2\mu r^2} + V(r) \end{aligned}$$



$$\begin{aligned} \Rightarrow E &= \frac{1}{2} \mu \left( \frac{dr}{dt} \right)^2 + \frac{\ell^2}{2\mu r^2} + V(r) \\ &= \frac{1}{2} \mu \left( \frac{dr}{d\phi} \frac{\ell}{\mu r^2} \right)^2 + \frac{\ell^2}{2\mu r^2} + V(r) \end{aligned}$$

Solving for  $r(\phi) \Leftrightarrow \phi(r)$

$$\left( \frac{dr}{d\phi} \right)^2 = \left( \frac{2\mu r^4}{\ell^2} \right) \left( E - \frac{\ell^2}{2\mu r^2} - V(r) \right)$$

$$d\phi = dr \left( \frac{\ell / r^2}{\sqrt{2\mu \left( E - \frac{\ell^2}{2\mu r^2} - V(r) \right)}} \right)$$

$$d\phi = dr \left( \frac{\ell / r^2}{\sqrt{2\mu \left( E - \frac{\ell^2}{2\mu r^2} - V(r) \right)}} \right)$$

Further simplification at large separation :

$$\ell = \mu v_{\infty} b$$

$$E = \frac{1}{2} \mu v_{\infty}^2$$

$$\Rightarrow \ell = \sqrt{2\mu E} b$$

When the dust clears :

$$d\phi = dr \left( \frac{\ell / r^2}{\sqrt{2\mu \left( E - \frac{\ell^2}{2\mu r^2} - V(r) \right)}} \right)$$

$$d\phi = dr \left( \frac{b / r^2}{\sqrt{1 - \frac{b^2}{r^2} - \frac{V(r)}{E}}} \right)$$

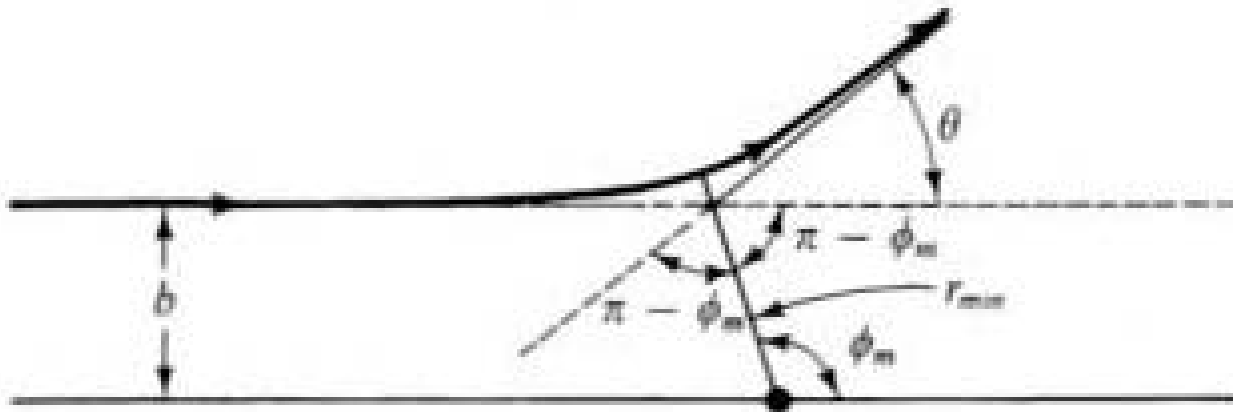
$$\Rightarrow \phi(b, E)$$

$$\int_0^{\phi_{\max}} d\phi = \int_{r_{\min}}^{\infty} dr \left( \frac{b/r^2}{\sqrt{1 - \frac{b^2}{r^2} - \frac{V(r)}{E}}} \right)$$

where :

$$1 - \frac{b^2}{r_{\min}^2} - \frac{V(r_{\min})}{E} = 0$$

Relationship between  $\phi_{\max}$  and  $\theta$ :



$$2(\pi - \phi_{\max}) + \theta = \pi$$

$$\Rightarrow \phi_{\max} = \frac{\pi}{2} - \frac{\theta}{2}$$

$$\phi_{\max} = \frac{\pi}{2} - \frac{\theta}{2} = \int_{r_{\min}}^{\infty} dr \left( \frac{b/r^2}{\sqrt{1 - \frac{b^2}{r^2} - \frac{V(r)}{E}}} \right)$$

$$\theta = \pi - 2b \int_{r_{\min}}^{\infty} dr \left( \frac{1/r^2}{\sqrt{1 - \frac{b^2}{r^2} - \frac{V(r)}{E}}} \right)$$

$$\theta = \pi - 2b \int_0^{1/r_{\min}} du \left( \frac{1}{\sqrt{1 - b^2 u^2 - \frac{V(1/u)}{E}}} \right)$$

Scattering angle equation :

$$\theta = \pi - 2b \int_0^{1/r_{\min}} du \left( \frac{1}{\sqrt{1 - b^2 u^2 - \frac{V(1/u)}{E}}} \right)$$

Rutherford scattering example:

$$\frac{V(r)}{E} \equiv \frac{\kappa}{r} \quad \frac{1}{r_{\min}} = \frac{1}{b} \left( -\frac{\kappa}{2b} + \sqrt{\left(\frac{\kappa}{2b}\right)^2 + 1} \right)$$

$$\theta = \pi - 2b \int_0^{1/r_{\min}} du \left( \frac{1}{\sqrt{1 - b^2 u^2 - \kappa u}} \right) = 2 \sin^{-1} \left( \frac{1}{\sqrt{(2b/\kappa)^2 + 1}} \right)$$

## Rutherford scattering continued :

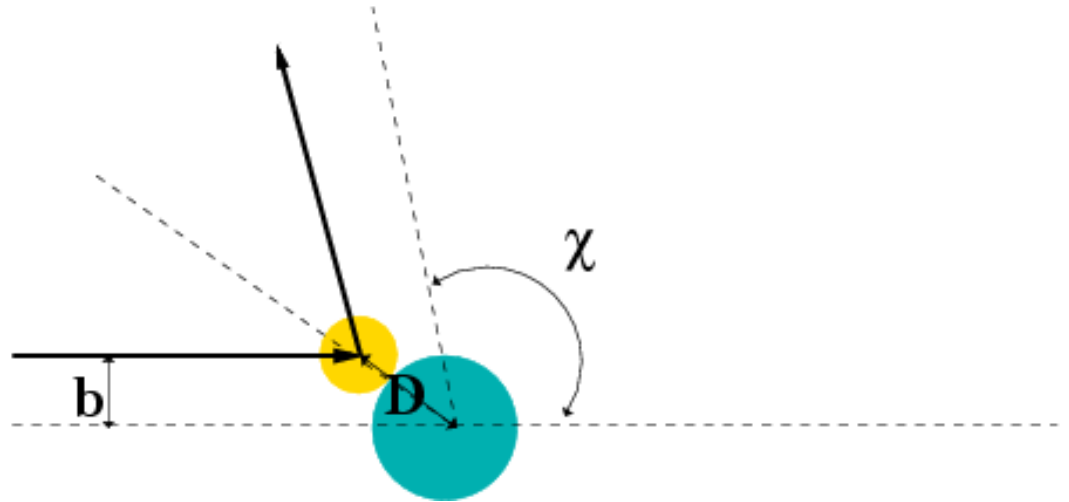
$$\theta = 2 \sin^{-1} \left( \frac{1}{\sqrt{(2b/\kappa)^2 + 1}} \right)$$

$$\frac{2b}{\kappa} = \left| \frac{\cos(\theta/2)}{\sin(\theta/2)} \right|$$

$$\left( \frac{d\sigma}{d\Omega} \right) = \frac{b}{\sin\theta} \left| \frac{db}{d\theta} \right| = \frac{\kappa^2}{16} \frac{1}{\sin^4(\theta/2)}$$



# Hard sphere scattering



For your homework you will show that

$$b = D \cos\left(\frac{\chi}{2}\right)$$

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{b}{\sin\chi} \left|\frac{db}{d\chi}\right| = \frac{D^2}{4}$$