

PHY 711 Classical Mechanics and Mathematical Methods

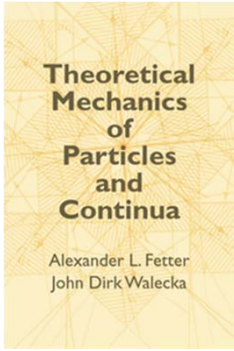
10-10:50 AM MWF Olin 103

Plan for Lecture 1:

1. Welcome & overview
2. Class structure & announcements
3. Introduction to Maple software
4. Chapter 1 – scattering theory

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Textbook:



**SIGNIFICANT NAMES IN MECHANICS
AND MATHEMATICAL PHYSICS**

Isaac Newton (1642-1727)
 Daniel Bernoulli (1700-1782)
 Leonhard Euler (1707-1783)
 Jean Le Rond d'Alembert (1717-1783)
 Joseph Louis Lagrange (1736-1813)
 Pierre Simon de Laplace (1749-1827)
 Adrien Marie Legendre (1752-1833)
 Jean Baptiste Joseph Fourier (1768-1830)
 Karl Friedrich Gauss (1777-1855)
 Siméon-Denis Poisson (1781-1842)
 Friedrich Wilhelm Bessel (1784-1846)
 Augustin-Louis Cauchy (1789-1857)
 George Green (1793-1841)
 Carl Gustav Jacob Jacobi (1804-1851)
 William Rowan Hamilton (1805-1865)
 Joseph Liouville (1809-1882)
 George Gabriel Stokes (1819-1903)
 Hermann Ludwig Ferdinand Helmholtz (1821-1894)
 Gustav Robert Kirchhoff (1824-1887)
 William Thomson (Lord Kelvin) (1824-1907)
 Georg Friedrich Bernhard Riemann (1826-1866)
 John William Strutt (Lord Rayleigh) (1842-1919)

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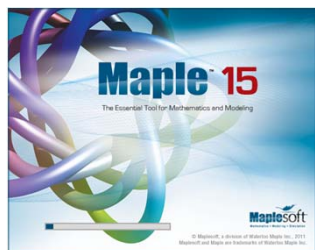
**Fall 2012 Schedule
for N. A. W. Holzwarth**

	Monday	Tuesday	Wednesday	Thursday	Friday
8:00-9:00	Lecture Preparation/ Office Hours		Lecture Preparation/ Office Hours		Lecture Preparation/ Office Hours
9:00-10:00	General Physics I PHY113	Lecture Preparation/ Office Hours	General Physics I PHY113	Lecture Preparation/ Office Hours	General Physics I PHY113
10:00-11:00	Classical Mech PHY711		Classical Mech PHY711		Classical Mech PHY711
11:00-12:30	Office Hours	Physics Research	Office Hours	Physics Research	Office Hours
12:30-2:00	Condensed Matter Theory Journal Club		Physics Research		Physics Research
2:00-3:30					
3:30-5:00	Physics Research		Physics Colloquium		CEES -- Renewable Energy Research

Travel dates:
• Oct. 17, 2012 Duquesne University

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Introduction to algebraic manipulation software



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http://www.wfu.edu/~natalie/f12phy711/lecturenote/maple_example.mw

The screenshot shows the Maple 15 software interface. The main window displays several mathematical operations and their results:

- $f := x \rightarrow \exp(-x^2);$ (C1)
- $g := x \rightarrow \int f(u) du, u=0..x;$ (C2)
- $g(x);$ (C3)
- $\text{evalf}(g(2.0));$ (C4)
- $\text{plot}(f(x), g(x), x=0..10);$ (C5)

The results shown are:

- $f := x \rightarrow e^{-x^2}$
- $g := x \rightarrow \int_0^x f(u) du$
- $\frac{1}{2} \sqrt{\pi} \text{erf}(x)$
- 0.8820813908

A plot is shown at the bottom, displaying the function $f(x) = e^{-x^2}$ and its integral $g(x) = \int_0^x e^{-u^2} du$ over the interval $x \in [0, 10]$. The plot shows $f(x)$ as a bell-shaped curve and $g(x)$ as a curve that starts at the origin and levels off towards a horizontal asymptote.

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Scattering theory:

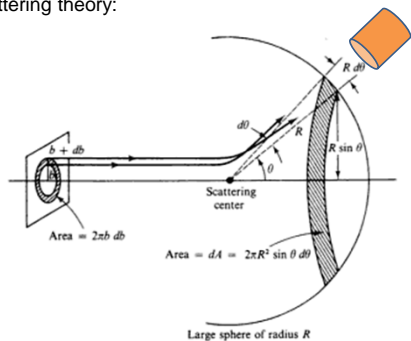


Figure 5.5 The scattering problem and relation of cross section to impact parameter.

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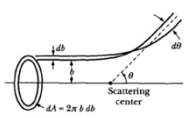
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Differential cross section

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\text{Number of detected particles at } \theta \text{ per target particle}}{\text{Number of incident particles per unit area}}$$

= Area of incident beam that is scattered into detector at angle θ



$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{2\pi b db}{2\pi \sin\theta d\theta} = \frac{b}{\sin\theta} \left|\frac{db}{d\theta}\right|$$

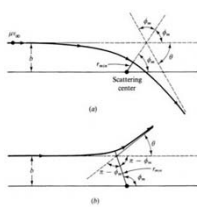
Figure from Marion & Thorton, Classical Dynamics

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Differential cross section

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{2\pi b db}{2\pi \sin\theta d\theta} = \frac{b}{\sin\theta} \left|\frac{db}{d\theta}\right|$$

How can we find $b(\theta)$?



Note that :

- $\ell = \mu v_\infty b$
- μ = reduced mass
- v_∞ = velocity at large separation

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Conservation of energy in the center of mass frame:

$$E = \frac{1}{2} \mu \left(\frac{d\mathbf{r}}{dt}\right)^2 + V(r)$$

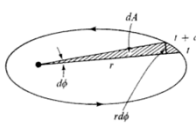
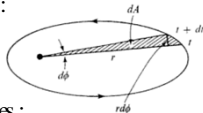
$$= \frac{1}{2} \mu \left(\frac{dr}{dt}\right)^2 + \frac{\ell^2}{2\mu r^2} + V(r)$$


Figure 3.2 The areal velocity in a central field.

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Conservation of angular momentum:

$$\ell = \mu r^2 \left(\frac{d\phi}{dt} \right)$$



Transformation of trajectory variables :

$$r(t) \Leftrightarrow r(\phi)$$

$$\frac{dr}{dt} = \frac{dr}{d\phi} \frac{d\phi}{dt} = \frac{dr}{d\phi} \frac{\ell}{\mu r^2}$$

$$\Rightarrow E = \frac{1}{2} \mu \left(\frac{dr}{dt} \right)^2 + \frac{\ell^2}{2\mu r^2} + V(r)$$

$$= \frac{1}{2} \mu \left(\frac{dr}{d\phi} \frac{\ell}{\mu r^2} \right)^2 + \frac{\ell^2}{2\mu r^2} + V(r)$$

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$$\Rightarrow E = \frac{1}{2} \mu \left(\frac{dr}{dt} \right)^2 + \frac{\ell^2}{2\mu r^2} + V(r)$$

$$= \frac{1}{2} \mu \left(\frac{dr}{d\phi} \frac{\ell}{\mu r^2} \right)^2 + \frac{\ell^2}{2\mu r^2} + V(r)$$

Solving for $r(\phi) \Leftrightarrow \phi(r)$

$$\left(\frac{dr}{d\phi} \right)^2 = \left(\frac{2\mu r^4}{\ell^2} \right) \left(E - \frac{\ell^2}{2\mu r^2} - V(r) \right)$$

$$d\phi = dr \left[\frac{\ell / r^2}{\sqrt{2\mu \left(E - \frac{\ell^2}{2\mu r^2} - V(r) \right)}} \right]$$

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$$d\phi = dr \left[\frac{\ell / r^2}{\sqrt{2\mu \left(E - \frac{\ell^2}{2\mu r^2} - V(r) \right)}} \right]$$

Further simplification at large separation:

$$\ell = \mu v_{\infty} b$$

$$E = \frac{1}{2} \mu v_{\infty}^2$$

$$\Rightarrow \ell = \sqrt{2\mu E} b$$

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When the dust clears :

$$d\phi = dr \left(\frac{\ell / r^2}{\sqrt{2\mu \left(E - \frac{\ell^2}{2\mu r^2} - V(r) \right)}} \right)$$

$$d\phi = dr \left(\frac{b / r^2}{\sqrt{1 - \frac{b^2}{r^2} - \frac{V(r)}{E}}} \right)$$

$\Rightarrow \phi(b, E)$

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