PHY 711 Classical Mechanics and Mathematical Methods
10-10:50 AM MWF Olin 103

Plan for Lecture 10:

Continue reading Chapter 3 & 6

1. **Summary & review**
2. **Lagrange's equations with constraints**

PHY 711 Classical Mechanics and Mathematical Methods

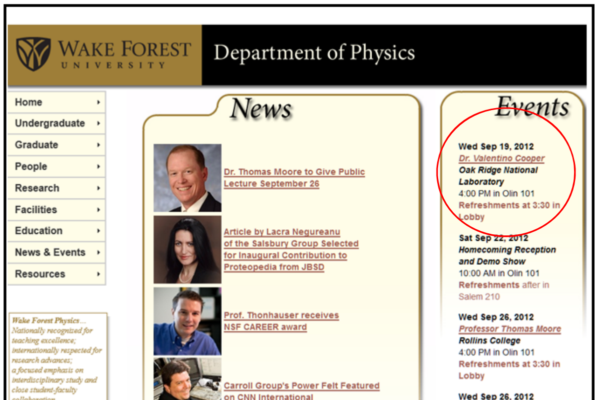
MWF 10 AM-10:50 PM | OPL 103 | http://www.wfu.edu/~natalie/f12phy711/

Instructor: Natalie Holzwarth | Phone:758-5510 | Office:300 OPL | e-mail:natalie@wfu.edu

Course schedule

(Preliminary schedule -- subject to frequent adjustment.)

	Date	F&W Reading	Topic	Assignment
1	Wed, 8/29/2012	Chap. 1	Review of basic principles;Scattering theory	#1
2	Fri, 8/31/2012	Chap. 1	Scattering theory continued	#2
3	Mon, 9/03/2012	Chap. 1	Scattering theory continued	#3
4	Wed, 9/05/2012	Chap. 1 & 2	Scattering theory/Accelerated coordinate frame	#4
5	Fri, 9/07/2012	Chap. 2	Accelerated coordinate frame	#5
6	Mon, 9/10/2012	Chap. 3	Calculus of Variation	#6
7	Wed, 9/12/2012	Chap. 3	Calculus of Variation continued	
8	Fri, 9/14/2012	Chap. 3	Lagrangian	#7
9	Mon, 9/17/2012	Chap. 3 & 6	Lagrangian	#8
10	Wed, 9/19/2012	Chap. 3 & 6	Lagrangian	#9
11	Fri, 9/21/2012	Chap. 3 & 6	Lagrangian	#10



FOREST
SITY

Department of Physics

WFU Physics Colloquium

TITLE: Getting the lead out: A first principles approach to Pb-free piezoelectrics

SPEAKER: Dr. Valentino R. Cooper,

Materials Science and Technology Division,
Oak Ridge National Laboratory

TIME: Wednesday September 19, 2012 at 4:00 PM

PLACE: Room 101 Olin Physical Laboratory

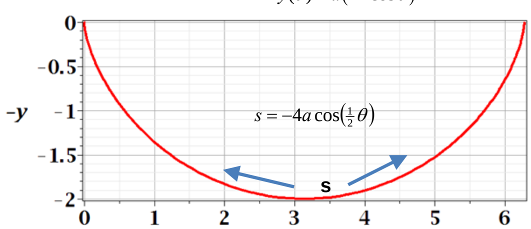
Refreshments will be served at 3:30 PM in the Olin Lounge. All interested persons are cordially invited to attend.

ABSTRACT

The electro-mechanical responses of many ABO_3 perovskite oxides have resulted in their application in a wide range of devices; such as piezoelectric fuel injectors where material responses can be exploited for the precise control of fuel delivery in automotive engines. Unfortunately, many of the oxides which are known to have high piezoelectric responses have unacceptable concentrations of Pb. A relatively recent direction is the exploration of Bi containing perovskites, as Bi can produce very high polarization. Bi's stereochemical activity (resulting in large Born effective charges, Z^*) and small ionic radius (allowing for large ionic displacements) makes it a good alternative to Pb; producing compounds with high

Comment on problem set #6

$$x(\theta) = a(\theta - \sin\theta)$$
$$y(\theta) = a(1 - \cos\theta)$$



Lagrangian for mass traveling along s:

$$L(s(t),\dot{s}(t)) = \tfrac{1}{2}m\dot{s}^2 - mgy = \tfrac{1}{2}m\dot{s}^2 - mg2a\left(1-\left(\frac{s}{4a}\right)^2\right)$$

Lagrangian for mass traveling along s:

$$L(s(t),\dot{s}(t)) = \tfrac{1}{2}m\dot{s}^2 - mgy = \tfrac{1}{2}m\dot{s}^2 - mg2a\left(1-\left(\frac{s}{4a}\right)^2\right)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{s}} - \frac{\partial L}{\partial s} = 0$$

$$\Rightarrow m\ddot{s} = -\frac{mg}{4a}s$$

$$\Rightarrow \ddot{s} = -\frac{g}{4a}s$$

Comments on generalized coordinates:

$$L = L(\{q_\sigma(t)\},\{\dot{q}_\sigma(t)\},t)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_\sigma} - \frac{\partial L}{\partial q_\sigma} = 0$$

Here we have assumed that the generalized coordinates q_σ are independent. Now consider the possibility that the coordinates are related through constraint equations of the form:

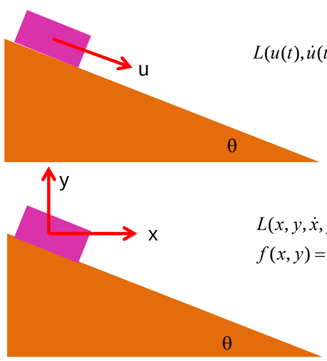
Lagrangian: $L = L(\{q_\sigma(t)\},\{\dot{q}_\sigma(t)\},t)$

Constraints: $f_j = f_j(\{q_\sigma(t)\},t) = 0$

Lagrange multipliers

Modified Euler-Lagrange equations: $$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_\sigma} - \frac{\partial L}{\partial q_\sigma} + \sum_j \lambda_j \frac{\partial f_j}{\partial q_\sigma} = 0$$

Simple example:



$$L(u(t),\dot{u}(t)) = \tfrac{1}{2}m\dot{u}^2 + mgu\sin\theta$$

$$L(x,y,\dot{x},\dot{y}) = \tfrac{1}{2}m(\dot{x}^2+\dot{y}^2) + mgy$$

$$f(x,y) = \sin\theta\, x + \cos\theta\, y = 0$$

Case 1:

$$L(u(t),\dot{u}(t)) = \tfrac{1}{2}m\dot{u}^2 + mgu\sin\theta$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{u}} - \frac{\partial L}{\partial u} = 0 = m\ddot{u} - mg\sin\theta = 0$$

$$\Rightarrow \ddot{u} = g\sin\theta$$

Case 2:

$$L(x,y,\dot{x},\dot{y}) = \tfrac{1}{2}m(\dot{x}^2+\dot{y}^2) + mgy$$

$$f(x,y) = \sin\theta\, x + \cos\theta\, y = 0$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} + \lambda\frac{\partial f}{\partial x} = 0 = m\ddot{x} + \lambda\sin\theta$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{y}} - \frac{\partial L}{\partial y} + \lambda\frac{\partial f}{\partial y} = 0 = m\ddot{y} - mg + \lambda\cos\theta$$

$$\sin\theta\,\ddot{x} + \cos\theta\,\ddot{y} = 0$$

$$\Rightarrow \lambda = mg\cos\theta$$

$$(-\cos\theta\,\ddot{x} + \sin\theta\,\ddot{y}) = g\sin\theta$$

Rational for Lagrange multipliers

Recall Hamilton's principle:

$$S = \int_{t_i}^{t_f} L(\{q_\sigma(t)\}, \{\dot{q}_\sigma(t)\}, t)\,dt$$

$$\delta S = 0 = \int_{t_i}^{t_f} \left(\sum_\sigma \left(\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_\sigma} - \frac{\partial L}{\partial q_\sigma} \right) \delta q_\sigma \right) dt$$

With constraints: $f_j = f_j(\{q_\sigma(t)\}, t) = 0$

Variations δq_σ are no longer independent.

$$\delta f_j = 0 = \sum_\sigma \frac{\partial f_j}{\partial q_\sigma}\delta q_\sigma \quad \text{at each } t$$

$\Rightarrow$ Add 0 to Euler-Lagrange equations in the form:

$$\sum_j \lambda_j \sum_\sigma \frac{\partial f_j}{\partial q_\sigma}\delta q_\sigma$$

9/19/2012 PHY 711 Fall 2012 -- Lecture 10 10

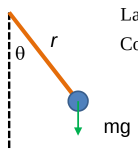
Euler-Lagrange equations with constraints:

Lagrangian: $L = L(\{q_\sigma(t)\}, \{\dot{q}_\sigma(t)\}, t)$

Constraints: $f_j = f_j(\{q_\sigma(t)\}, t) = 0$

Modified Euler-Lagrange equations: $\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_\sigma} - \frac{\partial L}{\partial q_\sigma} + \sum_j \lambda_j \frac{\partial f_j}{\partial q_\sigma} = 0$

Example:



Lagrangian: $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + mgr\cos\theta$

Constraints: $f = r - \ell = 0$

9/19/2012 PHY 711 Fall 2012 -- Lecture 10 11

Example continued:

Lagrangian: $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + mgr\cos\theta$

Constraints: $f = r - \ell = 0$

$$\frac{d}{dt}m\dot{r} - mr\dot{\theta}^2 - mg\cos\theta + \lambda = 0$$

$$\frac{d}{dt}mr^2\dot{\theta} + mgr\sin\theta = 0$$

$$\dot{r} = 0 = \ddot{r} \qquad r = \ell$$

$$\Rightarrow \ddot{\theta} = -\frac{g}{\ell}\sin\theta$$

$$\Rightarrow \lambda = m\ell\dot{\theta}^2 + mg\cos\theta$$

9/19/2012 PHY 711 Fall 2012 -- Lecture 10 12

Another example:

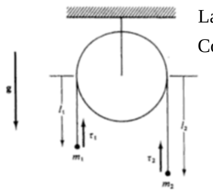


Figure 19.1 Atwood's machine.

$$\text{Lagrangian}: \; L = \tfrac{1}{2} m_1 \dot{\ell}_1^2 + \tfrac{1}{2} m_2 \dot{\ell}_2^2 + m_1 g \ell_1 + m_2 g \ell_2$$

$$\text{Constraints}: \; f = \ell_1 + \ell_2 - \ell = 0$$

$$\frac{d}{dt} m_1 \dot{\ell}_1 - m_1 g + \lambda = 0$$

$$\frac{d}{dt} m_2 \dot{\ell}_2 - m_2 g + \lambda = 0$$

$$\dot{\ell}_1 + \dot{\ell}_2 = 0 = \ddot{\ell}_1 + \ddot{\ell}_2$$

$$\Rightarrow \lambda = \frac{2 m_1 m_2}{m_1 + m_2} g$$

$$\ddot{\ell}_1 = -\ddot{\ell}_2 = \frac{m_1 - m_2}{m_1 + m_2} g$$