

**PHY 711 Classical Mechanics and  
Mathematical Methods**  
10-10:50 AM MWF Olin 103

**Plan for Lecture 10:**  
**Continue reading Chapter 3 & 6**

- 1. Summary & review**
- 2. Lagrange's equations with constraints**

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**PHY 711 Classical Mechanics and Mathematical Methods**  
MWF 10 AM-10:50 PM OPL 103 <http://www.wfu.edu/~natalie/12phy711/>  
Instructor: [Natalie Holzwarth](mailto:natalie@wfu.edu) Phone:758-5510 Office:300 OPL e-mail:natalie@wfu.edu

**Course schedule**  
(Preliminary schedule -- subject to frequent adjustment.)

Date	F&W Reading	Topic	Assignment
1 Wed, 8/29/2012	Chap. 1	Review of basic principles; Scattering theory	#1
2 Fri, 9/31/2012	Chap. 1	Scattering theory continued	#2
3 Mon, 9/03/2012	Chap. 1	Scattering theory continued	#3
4 Wed, 9/05/2012	Chap. 1 & 2	Scattering theory; Accelerated coordinate frame	#4
5 Fri, 9/07/2012	Chap. 2	Accelerated coordinate frame	#5
6 Mon, 9/10/2012	Chap. 3	Calculus of Variation	#6
7 Wed, 9/12/2012	Chap. 3	Calculus of Variation continued	#6
8 Fri, 9/14/2012	Chap. 3	Lagrangian	#7
9 Mon, 9/17/2012	Chap. 3 & 6	Lagrangian	#8
10 Wed, 9/19/2012	Chap. 3 & 6	Lagrangian	#9
11 Fri, 9/21/2012	Chap. 3 & 6	Lagrangian	#10

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**WAKE FOREST UNIVERSITY** Department of Physics

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**Wake Forest Physics...**  
Nationally recognized for teaching excellence. Internationally respected for research advances. A focused emphasis on interdisciplinary study and close student-faculty collaboration.

**News**

**Dr. Thomas Moore to Give Public Lecture September 26**

**Article by Lara Neureanu of the Salisbury Group Selected for Inaugural Contribution to Proteopedia from JBSO**

**Prof. Thomhauser receives NSF CAREER award**

**Carroll Group's Power Felt Featured on CNN International**

**Events**

**Wed Sep 19, 2012**  
**Dr. Valentino Casper**  
**Oak Ridge National Laboratory**  
4:00 PM in Olin 101  
Refreshments at 3:30 in Lobby

**Sat Sep 22, 2012**  
**Homecoming Reception and Demo Show**  
10:00 AM in Olin 101  
Refreshments after in Salem 210

**Wed Sep 26, 2012**  
**Professor Thomas Moore**  
**Rollins College**  
4:00 PM in Olin 101  
Refreshments at 3:30 in Lobby

**Wed Sep 26, 2012**  
**Research at Wake**

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FOREST Department of Physics

WFU Physics Colloquium

**TITLE:** Getting the lead out: A first principles approach to Pb-free piezoelectrics

**SPEAKER:** Dr. Valentino R. Cooper,  
Materials Science and Technology Division,  
Oak Ridge National Laboratory

**TIME:** Wednesday September 19, 2012 at 4:00 PM

**PLACE:** Room 101 Olin Physical Laboratory

Refreshments will be served at 3:30 PM in the Olin Lounge. All interested persons are cordially invited to attend.

**ABSTRACT**

The electro-mechanical responses of many  $ABO_3$  perovskite oxides have resulted in their application in a wide range of devices, such as piezoelectric fuel injectors where material responses can be exploited for the precise control of fuel delivery in automotive engines. Unfortunately, many of the oxides which are known to have high piezoelectric responses have unacceptable concentrations of Pb. A relatively recent direction is the exploration of Bi containing perovskites, as Bi can produce very high polarization. Bi's stereochemical activity (resulting in large Bi lone pair effective charges,  $Z'$ ) and small ionic radius (allowing for large ionic displacements) makes it a good alternative to Pb, producing compounds with high

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Comment on problem set #6

$$x(\theta) = a(\theta - \sin \theta)$$

$$y(\theta) = a(1 - \cos \theta)$$

Lagrangian for mass traveling along  $s$  :

$$L(s(t), \dot{s}(t)) = \frac{1}{2} m \dot{s}^2 - mgy = \frac{1}{2} m \dot{s}^2 - mg2a \left( 1 - \left( \frac{s}{4a} \right)^2 \right)$$

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Lagrangian for mass traveling along  $s$  :

$$L(s(t), \dot{s}(t)) = \frac{1}{2} m \dot{s}^2 - mgy = \frac{1}{2} m \dot{s}^2 - mg2a \left( 1 - \left( \frac{s}{4a} \right)^2 \right)$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{s}} - \frac{\partial L}{\partial s} = 0$$

$$\Rightarrow m\ddot{s} = -\frac{mg}{4a} s$$

$$\Rightarrow \ddot{s} = -\frac{g}{4a} s$$

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Comments on generalized coordinates:

$$L = L(\{q_\sigma(t)\}, \{\dot{q}_\sigma(t)\}, t)$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_\sigma} - \frac{\partial L}{\partial q_\sigma} = 0$$

Here we have assumed that the generalized coordinates  $q_\sigma$  are independent. Now consider the possibility that the coordinates are related through constraint equations of the form:

Lagrangian:  $L = L(\{q_\sigma(t)\}, \{\dot{q}_\sigma(t)\}, t)$

Constraints:  $f_j = f_j(\{q_\sigma(t)\}, t) = 0$

Lagrange multipliers

Modified Euler-Lagrange equations:  $\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_\sigma} - \frac{\partial L}{\partial q_\sigma} + \sum_j \lambda_j \frac{\partial f_j}{\partial q_\sigma} = 0$

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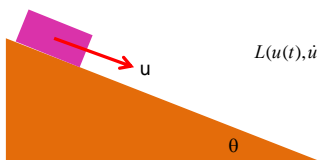
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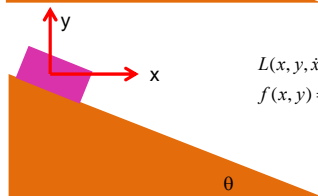
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Simple example:



$$L(u(t), \dot{u}(t)) = \frac{1}{2} m \dot{u}^2 + m g u \sin \theta$$



$$L(x, y, \dot{x}, \dot{y}) = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2) + m g y$$

$$f(x, y) = \sin \theta x + \cos \theta y = 0$$

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Case 1:

$$L(u(t), \dot{u}(t)) = \frac{1}{2} m \dot{u}^2 + m g u \sin \theta$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{u}} - \frac{\partial L}{\partial u} = 0 = m \ddot{u} - m g \sin \theta = 0$$

$$\text{Case 2: } \Rightarrow \ddot{u} = g \sin \theta$$

$$L(x, y, \dot{x}, \dot{y}) = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2) + m g y$$

$$f(x, y) = \sin \theta x + \cos \theta y = 0$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} + \lambda \frac{\partial f}{\partial x} = 0 = m \ddot{x} + \lambda \sin \theta$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{y}} - \frac{\partial L}{\partial y} + \lambda \frac{\partial f}{\partial y} = 0 = m \ddot{y} - m g + \lambda \cos \theta$$

$$\sin \theta \ddot{x} + \cos \theta \ddot{y} = 0$$

$$\Rightarrow \lambda = m g \cos \theta$$

$$(-\cos \theta \ddot{x} + \sin \theta \ddot{y}) = g \sin \theta$$

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Rational for Lagrange multipliers

Recall Hamilton's principle :

$$S = \int_{t_i}^{t_f} L(\{q_\sigma(t)\}, \{\dot{q}_\sigma(t)\}, t) dt$$

$$\delta S = 0 = \int_{t_i}^{t_f} \left( \sum_\sigma \left( \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_\sigma} - \frac{\partial L}{\partial q_\sigma} \right) \delta q_\sigma \right) dt$$

With constraints:  $f_j = f_j(\{q_\sigma(t)\}, t) = 0$

Variations  $\delta q_\sigma$  are no longer independent.

$$\delta f_j = 0 = \sum_\sigma \frac{\partial f_j}{\partial q_\sigma} \delta q_\sigma \quad \text{at each } t$$

⇒ Add 0 to Euler - Lagrange equations in the form :

$$\sum_j \lambda_j \sum_\sigma \frac{\partial f_j}{\partial q_\sigma} \delta q_\sigma$$

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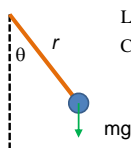
Euler-Lagrange equations with constraints:

Lagrangian:  $L = L(\{q_\sigma(t)\}, \{\dot{q}_\sigma(t)\}, t)$

Constraints:  $f_j = f_j(\{q_\sigma(t)\}, t) = 0$

Modified Euler - Lagrange equations:  $\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_\sigma} - \frac{\partial L}{\partial q_\sigma} + \sum_j \lambda_j \frac{\partial f_j}{\partial q_\sigma} = 0$

Example:



Lagrangian:  $L = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2) + mgr \cos \theta$

Constraints:  $f = r - \ell = 0$

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Example continued:

Lagrangian:  $L = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2) + mgr \cos \theta$

Constraints:  $f = r - \ell = 0$

$$\frac{d}{dt} m\dot{r} - mr\dot{\theta}^2 - mg \cos \theta + \lambda = 0$$

$$\frac{d}{dt} mr^2\dot{\theta} + mgr \sin \theta = 0$$

$$\dot{r} = 0 = \ddot{r} \quad r = \ell$$

$$\Rightarrow \ddot{\theta} = -\frac{g}{\ell} \sin \theta$$

$$\Rightarrow \lambda = m\ell \dot{\theta}^2 + mg \cos \theta$$

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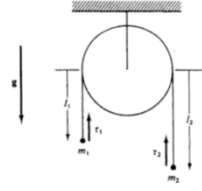
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Another example:



Lagrangian:  $L = \frac{1}{2}m_1\dot{\ell}_1^2 + \frac{1}{2}m_2\dot{\ell}_2^2 + m_1g\ell_1 + m_2g\ell_2$   
 Constraints:  $f = \ell_1 + \ell_2 - \ell = 0$

$$\frac{d}{dt}m_1\dot{\ell}_1 - m_1g + \lambda = 0$$

$$\frac{d}{dt}m_2\dot{\ell}_2 - m_2g + \lambda = 0$$

$$\dot{\ell}_1 + \dot{\ell}_2 = 0 = \ddot{\ell}_1 + \ddot{\ell}_2$$

$$\Rightarrow \lambda = \frac{2m_1m_2}{m_1 + m_2}g$$

$$\ddot{\ell}_1 = -\ddot{\ell}_2 = \frac{m_1 - m_2}{m_1 + m_2}g$$

Figure 19.1 Atwood's machine.

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