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PHY 741 – Hint for Problem Set #4

In order to solve problem 2.15, the following identities may prove useful. For an operator **A**, a function of **A** may be evaluated be using a Taylor's expansion. For example, for any constant s,

$$e^{s\mathbf{A}} \equiv 1 + \frac{s\mathbf{A}}{1!} + \frac{s^2\mathbf{A}^2}{2!} + \frac{s^3\mathbf{A}^3}{3!} + \ldots .$$

A famous identity involving two operators **A** and **B** can be shown to be equivalent to a series of commutators (see, for example, Merzbacher's text):

$$e^{s\mathbf{A}}\mathbf{B}e^{-s\mathbf{A}} = \mathbf{B} + \frac{s}{1!}[\mathbf{A},\mathbf{B}] + \frac{s^2}{2!}[\mathbf{A},[\mathbf{A},\mathbf{B}]] + \frac{s^3}{3!}[\mathbf{A},[\mathbf{A},[\mathbf{A},\mathbf{B}]]] + \ldots .$$