August 24, 2010

PHY 741 – Problem Set #1

Read Chapter 1 in **Mahan**; homework is due Monday, August 30, 2010.

For a system described by the probability amplitude $\psi(x)$, we can define the square modulus of the *variance* of a Hermitian operator $\mathcal{A}$ as

$$|\Delta\mathcal{A}|^2 \equiv \langle\psi|\mathcal{A}^2|\psi\rangle - \left(\langle\psi|\mathcal{A}|\psi\rangle\right)^2 .$$

In class we showed that for the 3 Hermitian operators $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ with the commutation relations

$$[\mathcal{A},\mathcal{B}] = i\mathcal{C},$$

the variances satisfy the inequality

$$\Delta\mathcal{A}\Delta\mathcal{B} \geq \frac{1}{2}\langle\psi|\mathcal{C}|\psi\rangle. \tag{1}$$

For this Homework, choose

$$\mathcal{A} = x, \quad \text{and} \quad \mathcal{B} = p \equiv -i\hbar\frac{\partial}{\partial x}.$$

1. What is the operator $\mathcal{C}$ for this case?

2. For each of the following probability amplitudes, evaluate the left and right hand sides of Eq. (1) and check the validity of the inequality.

   (a)

   $$\psi(x) = \frac{1}{\sqrt{a\sqrt{2\pi}}}\mathrm{e}^{ik_0x - x^2/(4a^2)}.$$

   In this expression a is a length parameter and k_0 is a positive parameter with the dimensions of 1/length.

   (b)

   $$\psi(x) = \begin{cases} \sqrt{\frac{630}{a^9}}x^2(a-x)^2 & \text{for } 0 \leq x \leq a \\ 0 & \text{otherwise.} \end{cases}$$

   In this expression a is another length parameter.