Graphing Goes Live

Lauren Brooks
Wake Forest University
brooln7@wfu.edu

NCTM STANDARDS: Algebra, Geometry, Communication, Representation

GOALS:

Students will visually demonstrate their knowledge of graphing functions and associated concepts, varying in difficulty dependent upon the class involved. They will review the idea of coordinates, slope, y-intercept, derivative graphs, unit circle measurements and trigonometric graphs.

INTRODUCTION:

The creation of the Cartesian coordinate plane by René Descartes provided an important new concept to discover in the world of mathematics. Bonaventura Cavalieri followed suit soon after with the discovery of polar coordinates, making it easier to represent a function whose radius depends on an angle. With the infinite number of functions that we investigate in the mathematics classroom today, using the outdoors to find a new, exciting way to explain graphing to students can really help them visualize the image and characteristics of different graphs.

ACTIVITIES:

Part 1. Introduction: Teacher reviews how to construct graphs, and depending on subject matter, reintroduces the vocabulary associated with that activity.

Part 2. Student Construction: Students will use their knowledge of graphs to create three dimensional living graphs, with themselves as the coordinates. There will be three extensions below for the different subjects that can be explored.

ASSESSMENT:

At the end of the introduction activity, students will be taken to a previously created coordinate plane, Cartesian or polar, depending on the lesson. They will use their knowledge of slopes, y-intercepts, radiuses, and angles to represent those graphs. They will have to use their knowledge of these concepts in order to complete this activity successfully.

Teacher Notes – Cartesian plane, linear functions

Introduction – Students can often struggle with the abstract concepts of slope and y-intercept. Actually creating a three dimensional graph where they have to walk to create the functions can help them understand the ideas of "rise over run", or "where y=0". This real life example can help them to physically create the graph so they will visually see what you mean by these sayings. We represent linear functions in the form y= mx +b where m is the slope, b is the y-intercept, and –b/m is the x-intercept.

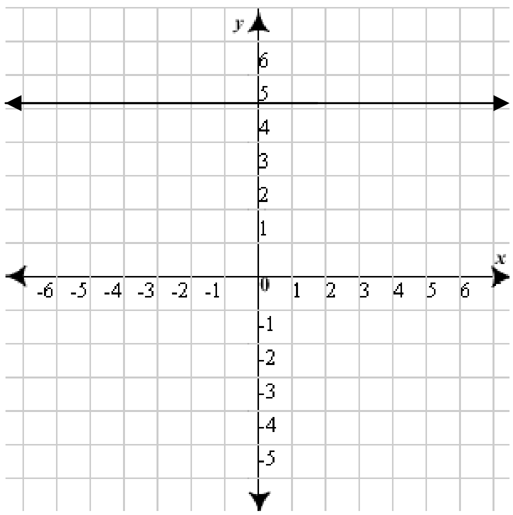
Preparation -
Creating the Coordinate Plane: Using your school parking lot if you have perpendicular spaces, you can easily create a coordinate plane using the middle line as the origin. If a school parking lot is not available, try to find a cement space where you can lay down masking tape to create the graphing plane. I suggest laying down one piece horizontally and then vertically, in alternating orders to assure you are making close to perpendicular and parallel lines. Also, try to make sure that the squares that you are creating are at least a foot apart, so the students will not be too close to each other when creating the graphs.

Choosing the Graphs: Because students will be learning about slope and y-intercept, it is important that there are a variety of slopes and y-intercepts explored during the activity. There should be negative and positive slopes, as well as varying y-intercept amounts. You could also challenge the students to find both x and y intercepts of the graphs if it exists.

Student Participation: Have a group of students create each group, and use as many "ordered pairs" (students) as you'd like to create the line. After creating the line, have another student walk the rise over run to determine the slope. You can vary this by giving students the Y-intercept and slope of the line and having them walk the rise over run from each point, in each direction. Clearly, this needs to be done until an X and Y intercept can be found, and the slope can be determined. Have students hold hands to actually create the line.

Solutions for the Guided Practice –

1. y= 5



X-Intercept: N/A

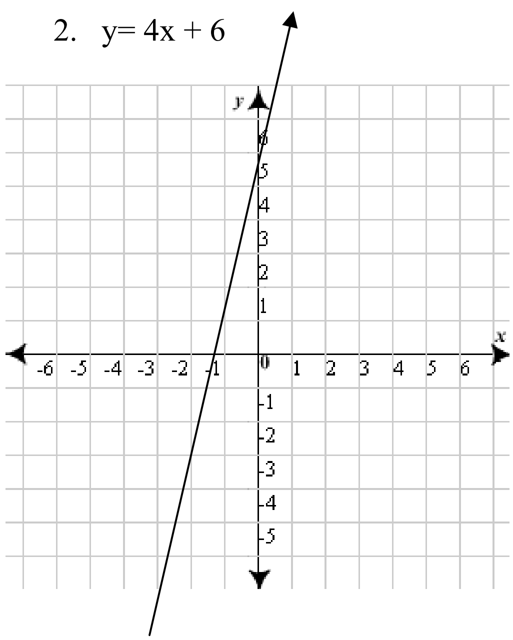
Y-Intercept: (0,5)

Slope: 0

The ordered pairs we used to create the graph:

X	F(X)
-3	5
-1	5
0	5
2	5
4	5

2. y= 4x + 6



X-Intercept: (-3/2,0)

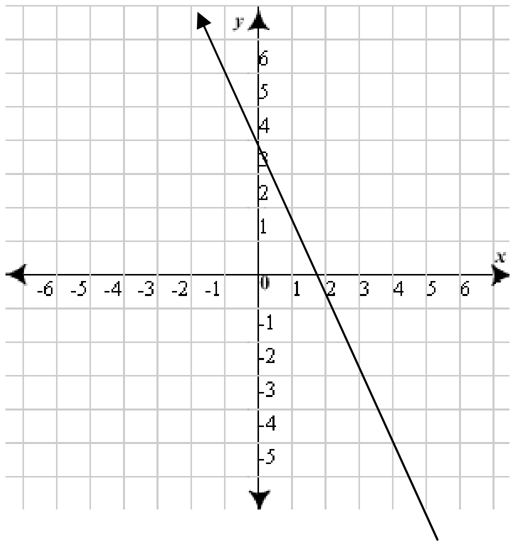
Y-Intercept: (0,6)

Slope: 4/1

The ordered pairs we used to create the graph:

X	F(X)
0	6
-1	2
0	3/2
-2	-2

3. y= -2x + 4



X-Intercept: (2,0)

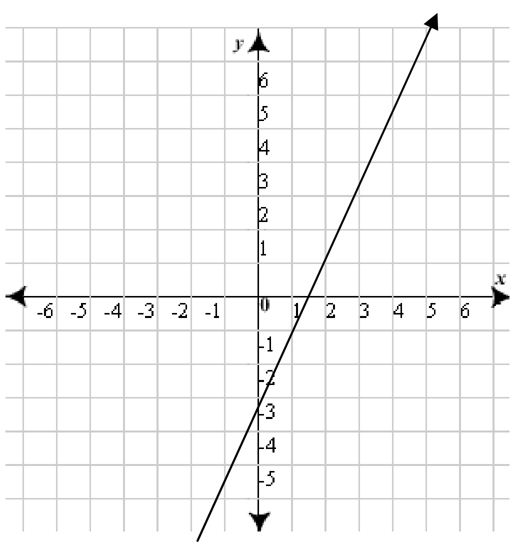
Y-Intercept: (0,4)

Slope: -2/1

The ordered pairs we used to create the graph:

X	F(X)
3	-2
2	0
0	4
-1	6

4. y= 5/2x - 7/2



X-Intercept: (7/5,0)

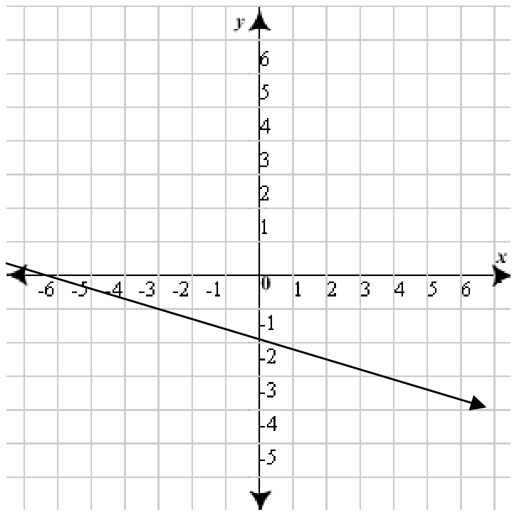
Y-Intercept: (0,-7/2)

Slope: 5/2

The ordered pairs we used to create the graph:

X	F(X)
-1	-6
1	-1
0	7/5
3	4

5. y= (-1/3)x -2



X-Intercept: (-6,0)

Y-Intercept: (0,-2)

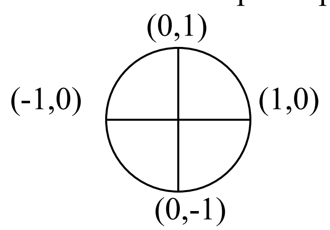
Slope: -1/3

The ordered pairs we used to create the graph:

X	F(X)
-6	0
-3	-1
0	-2
3	-3

Teacher Notes – Trigonometric Functions

Introduction – Students often have trouble understanding the curvature and characteristics of trigonometric functions, especially when the inside function is anything other than X. It is important for them to see the trigonometric functions in different forms so they know how the graph is manipulated when it changes. This activity will help students form these graphs in a fun and creative way. Make sure students understand the concept of the unit circle before participating in this activity as a good introduction to the topic.



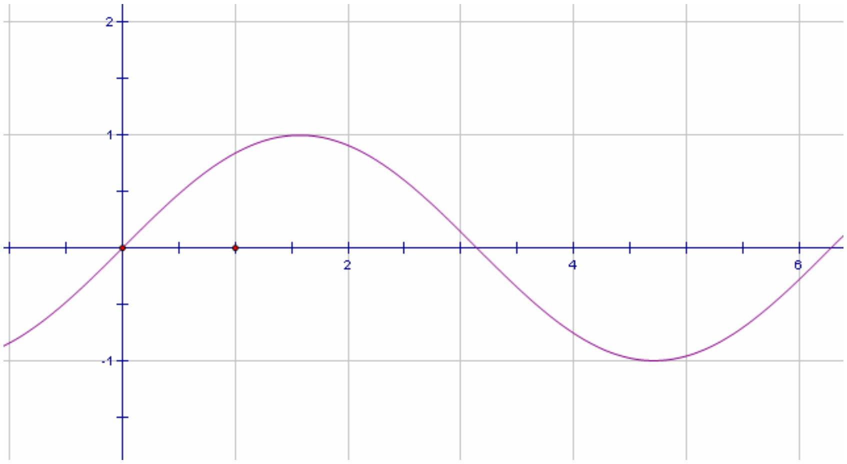
Preparation -
Creating the Coordinate Plane: Using your school parking lot if you have perpendicular spaces, you can easily create a coordinate plane using the middle line as the origin. If a school parking lot is not available, try to find a cement space where you can lay down masking tape to create the graphing plane. I suggest laying down one piece horizontally and then vertically, in alternating orders to assure you are making close to perpendicular and parallel lines. Also, try to make sure that the squares that you are creating are at least a foot apart, so the students will not be too close to each other when creating the graphs. Note that for this activity it will be best if you go ahead and mark out increments of π beforehand.

Choosing the Graphs: Begin using a simple trigonometric function, such as sin(x), and varying the graph to sin(2x). Do the same for cosine functions as well. Have students graph out what they think the new graphs will look like before you help them graph it on your human-sized coordinate plane.

Student Participation: Have a group of students create each graph, and use as many "ordered pairs" (students) as you'd like to create the function. Because there is curvature in the graph, it will be necessary to use as many students as possible. It is best if you give coordinates for students to stand on, and then after having called out as many coordinates as needed, have them connect by holding hands to form the graph. After creating the function, have another student walk the length of the graph to determine when it begins repeating itself to determine the period. You can vary this by giving students the period, range, radius, and a few points and tell them to figure out the function.

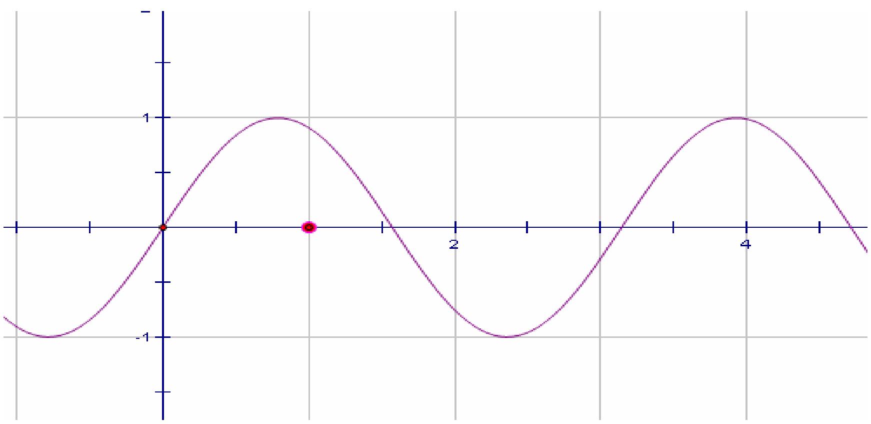
Solutions for the Guided Practice –

1. y= sin(X)



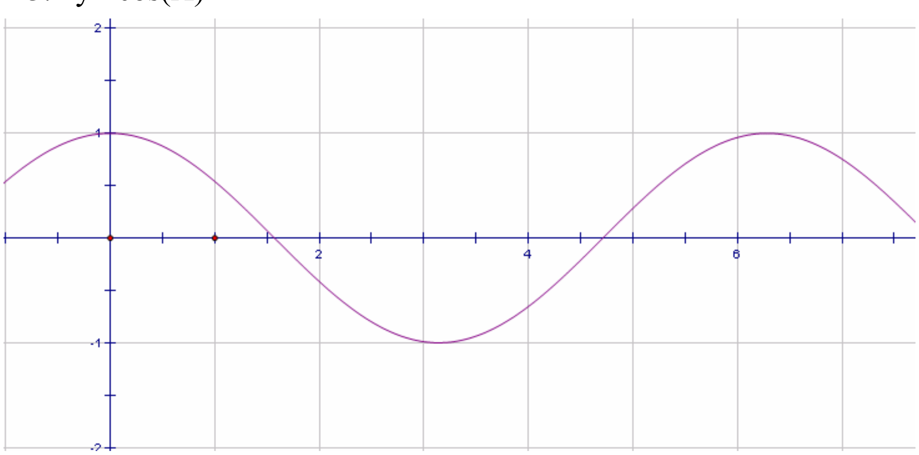
Minimum: -1
Maximum: 1
Range: -1<y<1
Period: 2π

2. y= sin(2X)



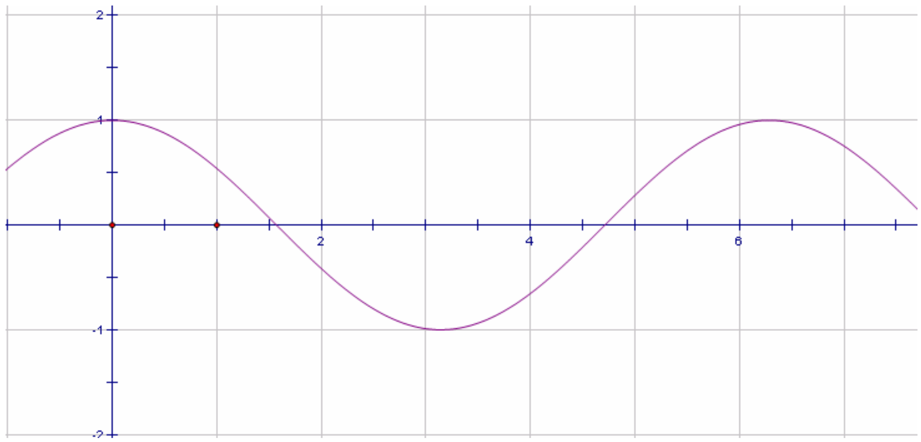
Minimum: -1
Maximum: 1
Range: -1<y<1
Period: π

3. y= cos(X)



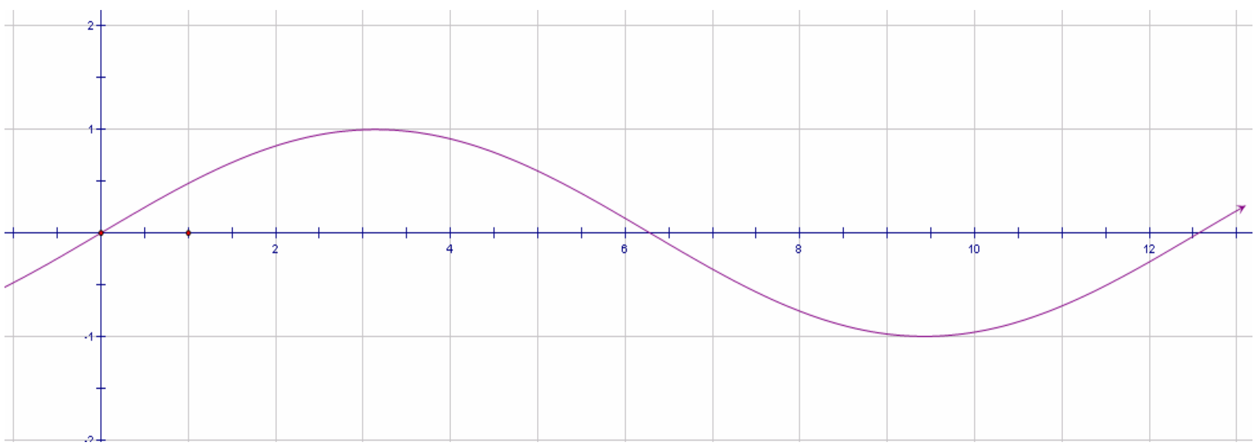
Minimum: -1
Maximum: 1
Range:-1<y<1
Period: 2π

4. y= cos(3X)



Minimum: -1
Maximum: 1
Range:-1<y<1
Period: 2π/3

5. y= sin(1/2X)



Minimum: -1
Maximum: 1
Range:-1<y<1
Period: 4π

Teachers Notes – Polar Coordinates

Introduction – Understanding that there is more than one plane system can often baffle students at the beginning. Learning about polar coordinates is a vital part to upper level mathematics, giving insight to functions in a new setting. Having the knowledge to go back and forth between both Cartesian and polar coordinate planes is a great asset to any student. That conversion is found in the following equations:

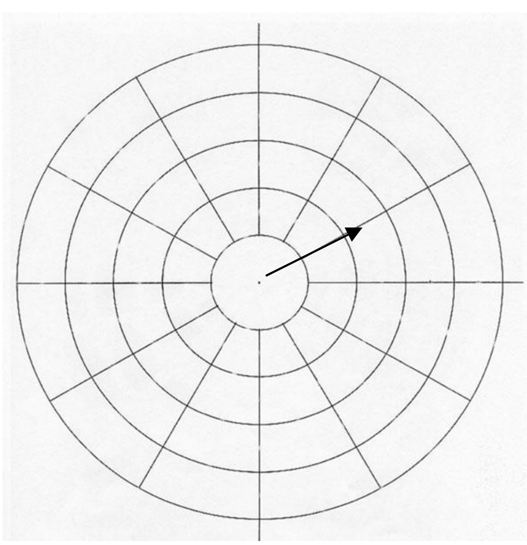
$X = r \cos \theta$
$Y = r \sin \theta$
$R^2 = \sqrt{(x^2+y^2)}$

Preparation:
Creating the Plane: It's not as hard to create a polar plane as one might think. Using masking tape, create an origin that is a set of perpendicular lines. Using a protractor, measure off increments of degrees you will be using around the top half of the graph. I suggest 30°, 60°, 90°, 120°, 150°, and 180°. Once you have marked out those pie pieces, use masking tape to make a radius to and through your origin to the other side of the plane. This way when you create a 30° wedge, you are also creating the 210° at the same time. You will half the work that you actually have to do by this process. Now you should have what appears to be many wedges. To complete the plane, take some sidewalk chalk and draw unit circles around the whole figure as evenly as possible. This completes your polar coordinate plane.

Choosing the Graphs: With polar coordinates there are many ornate graphs that can be represented. Because you are outside and it will be hard to check for accuracy, it is best to stick to simpler graphs as a way to introduce the subject, rather than try to draw an intricate polar rose outdoors, although it can be done.

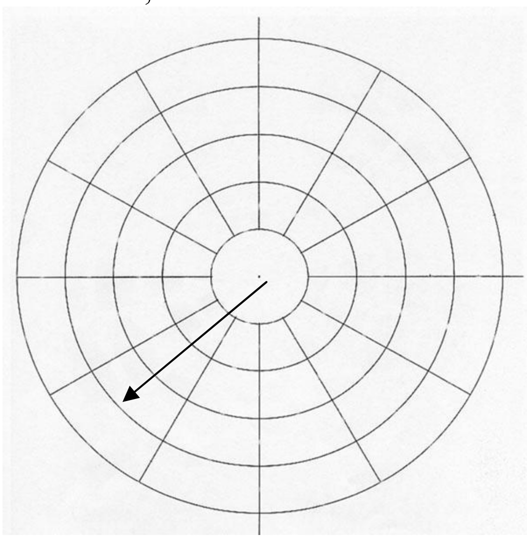
Solutions to the Guided Practice –

1. $r = 1; \theta = \pi/6$



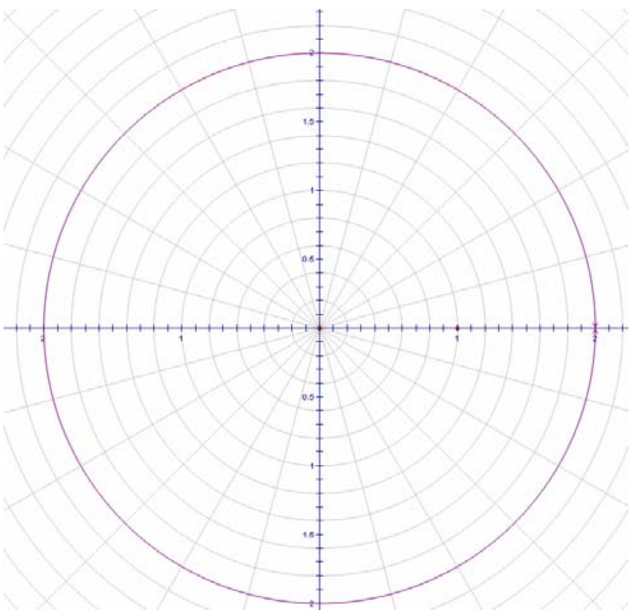
Radius: 1
Theta: $\pi/6$

2. r = 3; θ = 5π/4



Radius: 3
Theta: 5π/4

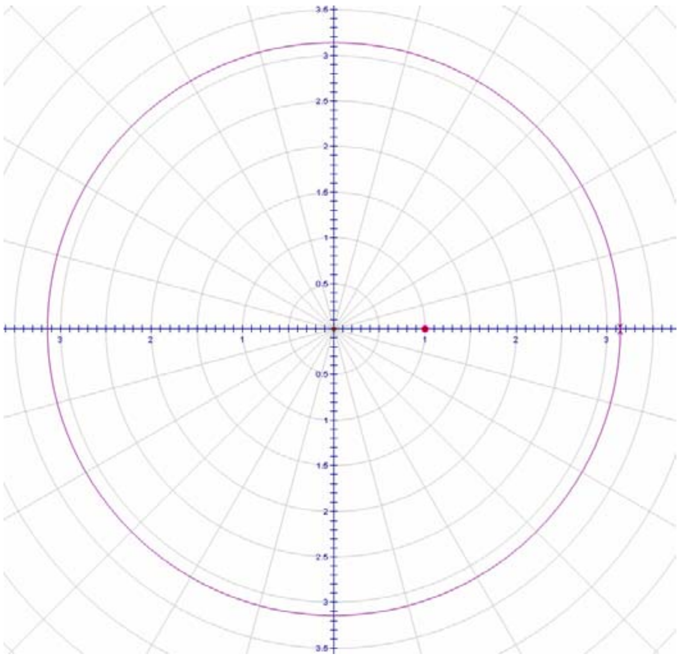
3. r(θ) = 2



Radius: 2
Theta: 2π

R	θ
2	2π
2	π/6
2	3π/2
2	5π/4
2	10π/6

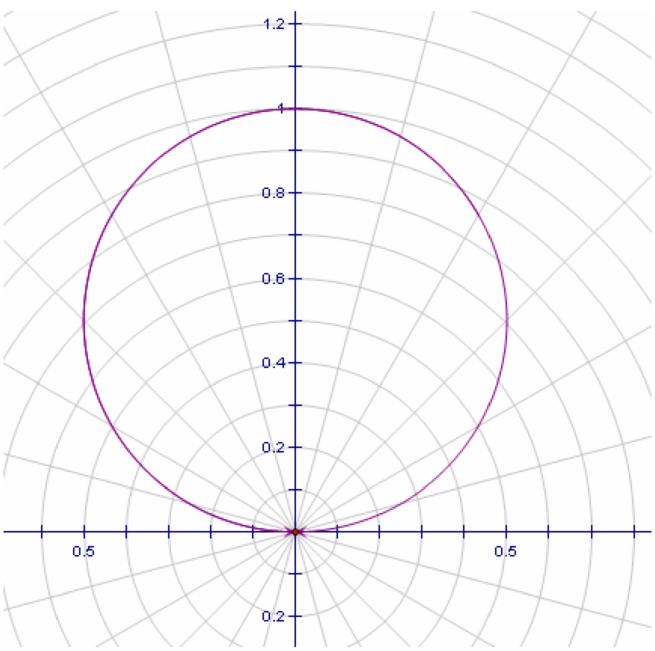
4. $r(\theta) = \pi$



Radius: π
Theta: 2π

R	θ
π	2π
π	π
π	$4\pi/6$
π	$8\pi/3$
π	$7\pi/4$

5. $r(\theta) = \sin(\theta)$



Radius: 1/2
Theta: 2π

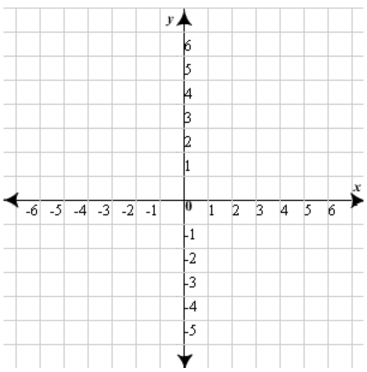
R	θ
0	0
1	$\pi/2$
.5	$\pi/6$
.7	$\pi/4$
.5	$5\pi/6$
.7	$3\pi/4$

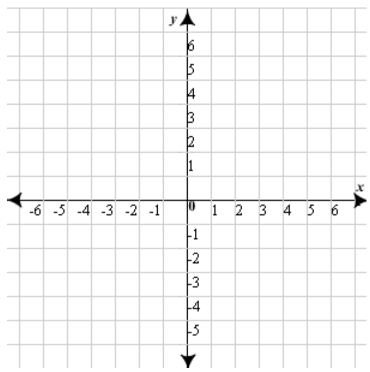
Graphing Goes Live – Linear Functions

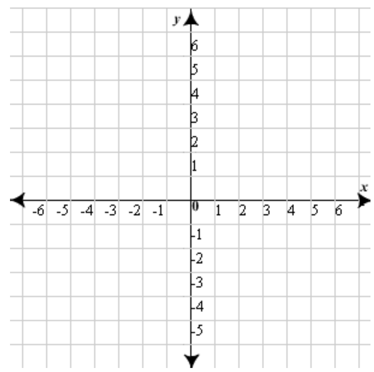
Graphs are a visual way to represent functions. Using the Cartesian coordinate plane, we can describe each function by its individual characteristics. Within each linear function, there is a slope and an intercept. Your job is to investigate each graph and using people as ordered pairs, create the representation in the Cartesian plane that correctly symbolizes each function.

Before creating each graph with other students in your class, graph what you think it will look like below. Then, graph what it actually looks like. Lastly, fill in the table and blanks for each function.

6. y= 8





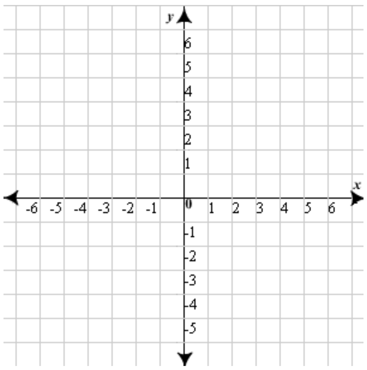


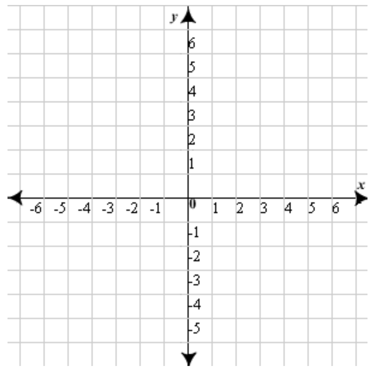
X-Intercept: ______ Y-Intercept ______ Slope:

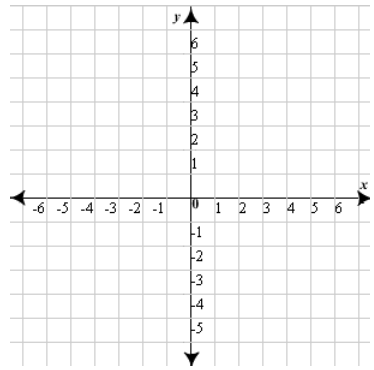
The ordered pairs we used to create the graph:

X	F(X)

7. y= 4x + 6





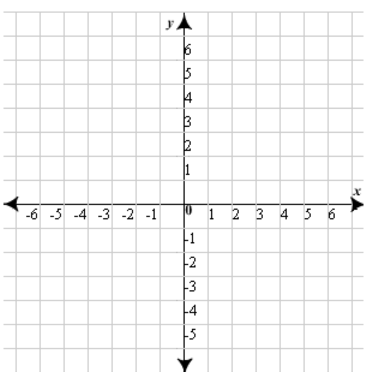


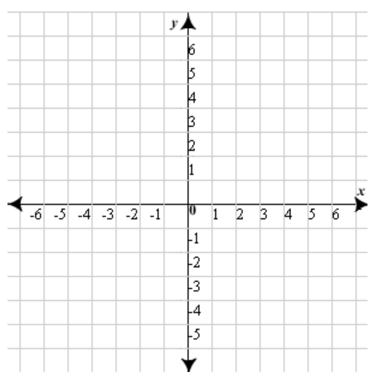
X-Intercept: ______ Y-Intercept ______ Slope:

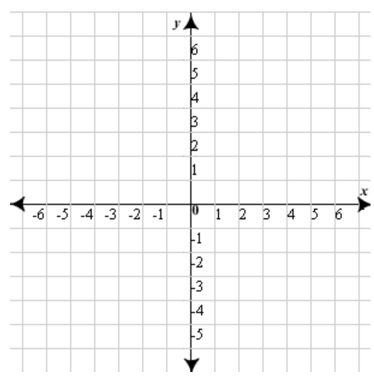
The ordered pairs we used to create the graph:

X	F(X)

8. y= -2x + 4





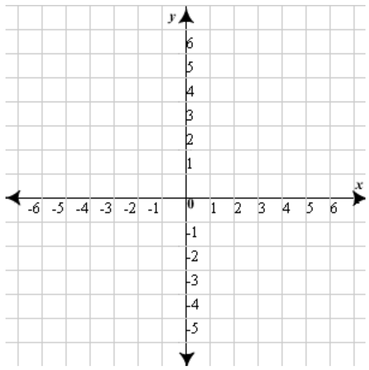
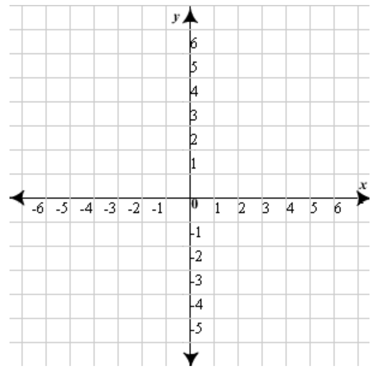
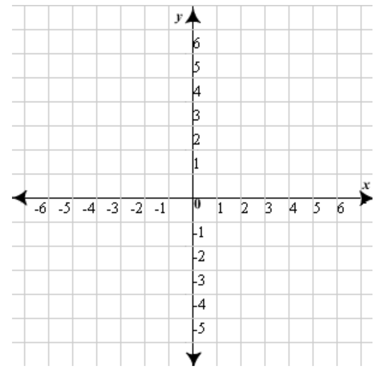


X-Intercept: ______ Y-Intercept ______ Slope:

The ordered pairs we used to create the graph:

X	F(X)

9. y= 5/2x - 7/2

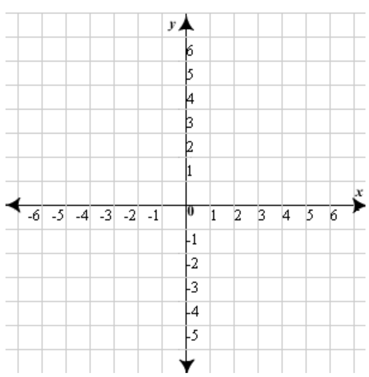
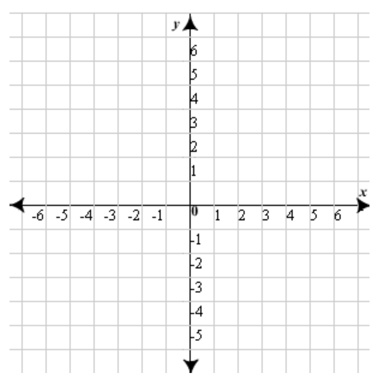
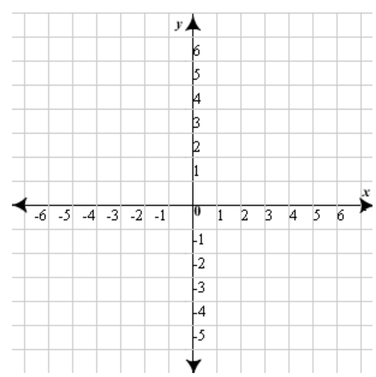




X-Intercept: ______ Y-Intercept ______ Slope:

The ordered pairs we used to create the graph:

X	F(X)

10. y= (-1/3)x -2

X-Intercept: ______ Y-Intercept ______ Slope:

The ordered pairs we used to create the graph:

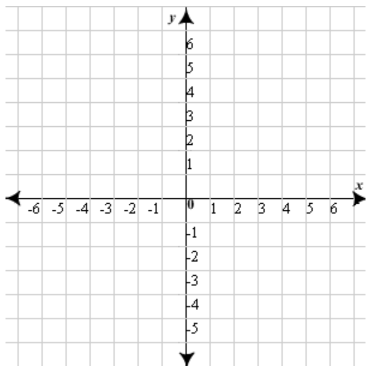
X	F(X)

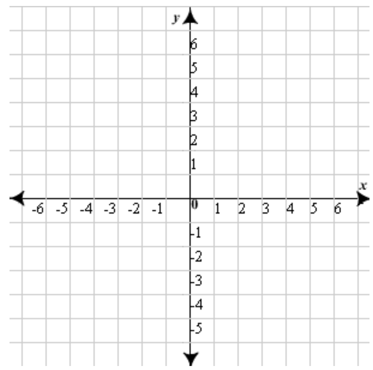
Graphing Goes Live – Trigonometric Functions

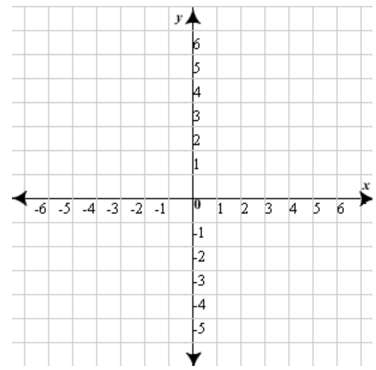
Graphs are a visual way to represent functions. Using the Cartesian plane, we can plot out the graphs of trigonometric functions to see their individual characteristics.

Before creating each graph with other students in your class, graph what you think it will look like below. Then, graph what it actually looks like. Lastly, fill in the blanks for each function.

6. y= sin(X)



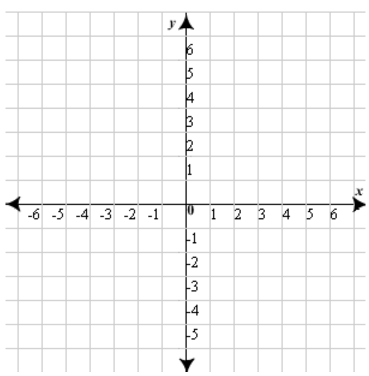


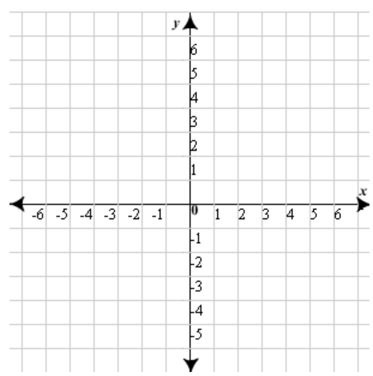


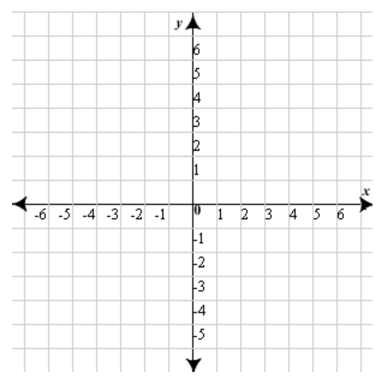
Minimum: Maximum:

Range: Period: Radius:

7. y = sin(2X)



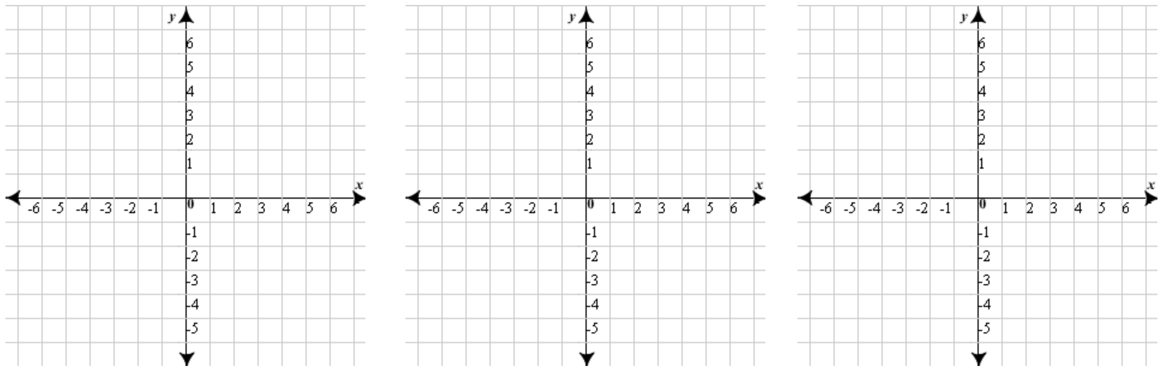




Minimum: Maximum:

Range: Period: Radius:

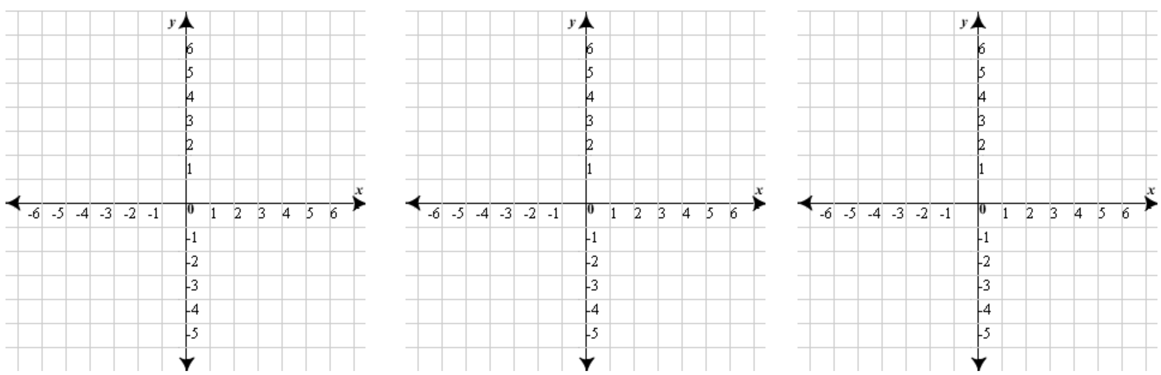
8. y= cos(X)



Minimum: Maximum:

Range: Period: Radius:

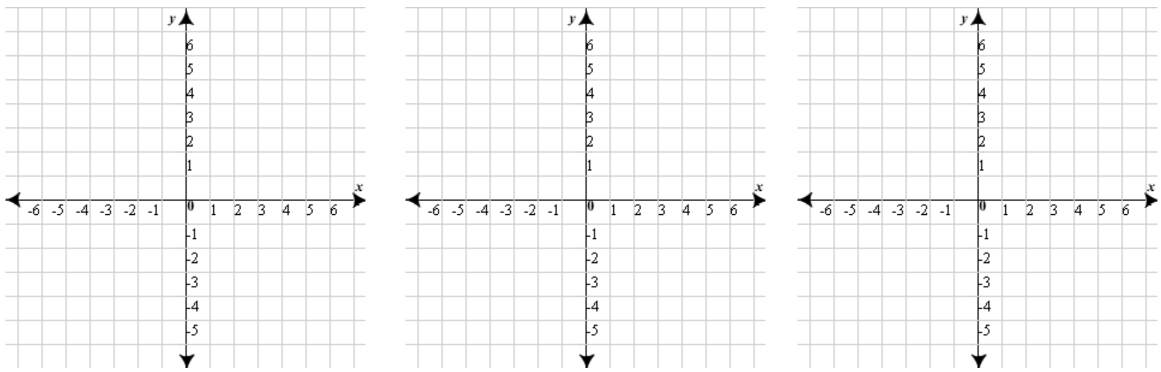
9. y= cos(3X)



Minimum: Maximum:

Range: Period: Radius:

10. y= sin(1/2X)



Minimum: Maximum:

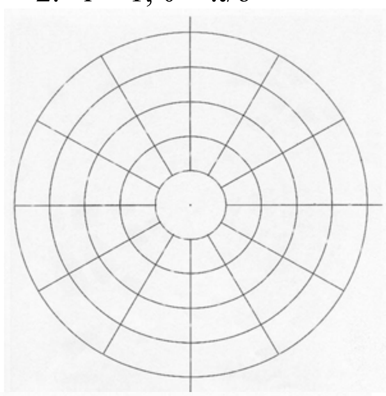
Range: Period: Radius:

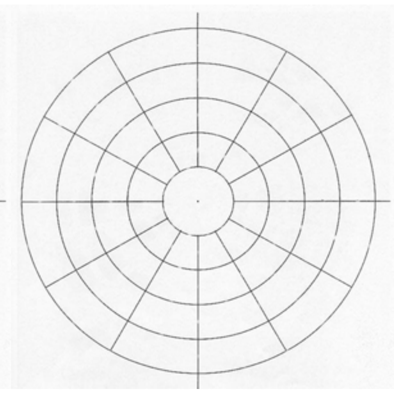
Graphing Goes Live – Polar Coordinates

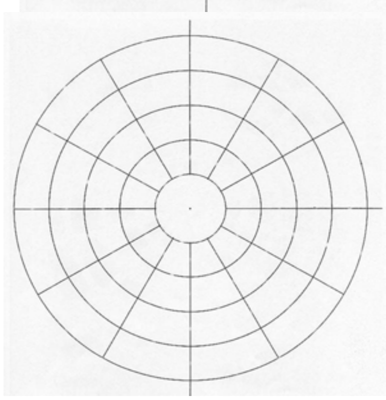
Graphs are a visual way to represent functions. Using the Polar coordinate plane, we can plot out the graphs of different functions to see their individual characteristics.

Before creating each graph with other students in your class, graph what you think it will look like below. Then, graph what it actually looks like. Lastly, fill in the blanks for each function.

2. $r = 1$; $\theta = \pi/6$

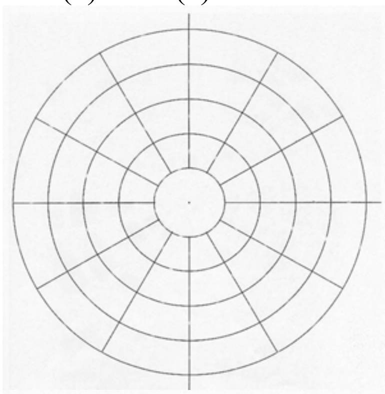
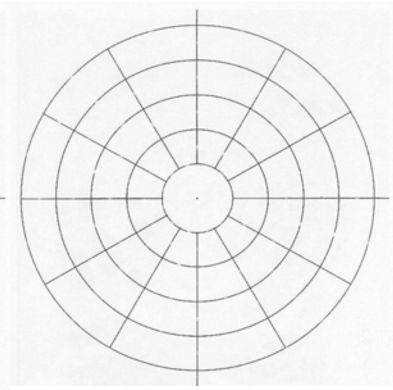
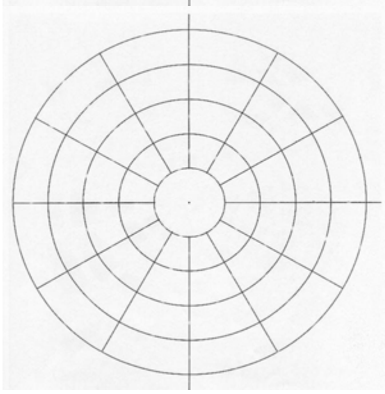






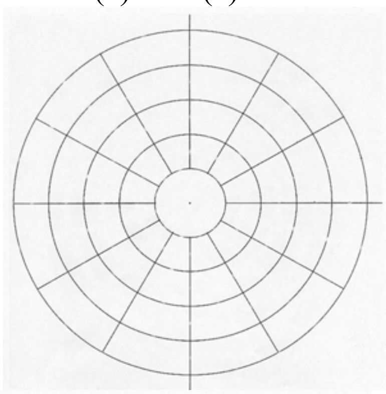
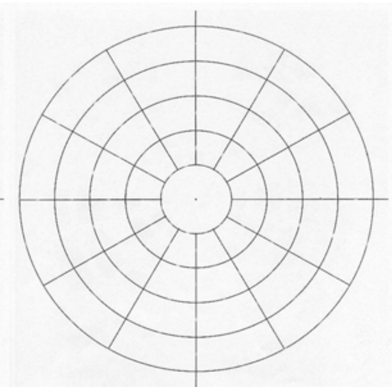
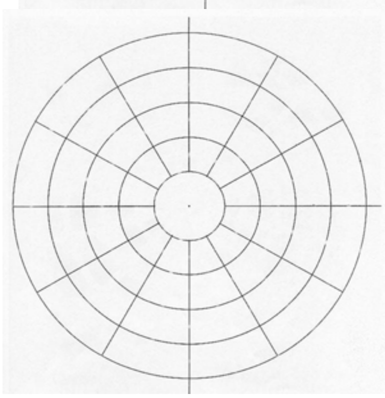
Radius:
Theta:

3. $r(\theta) = \cos(\theta)$

Radius:
Theta:

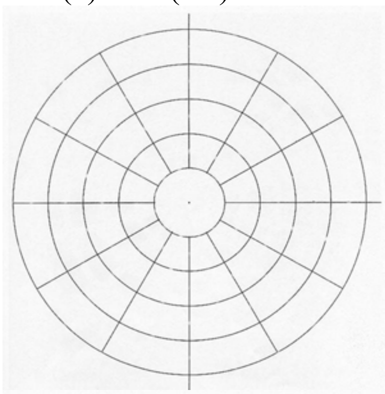
4. $r(\theta) = \sin(\theta)$

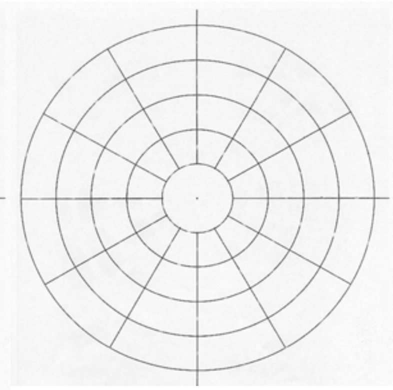




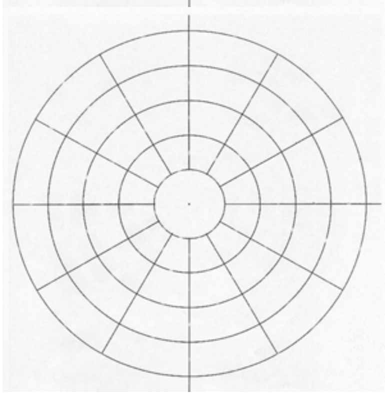
Radius:
Theta:

R	θ

5. $r(\theta) = \sin(2\theta)$



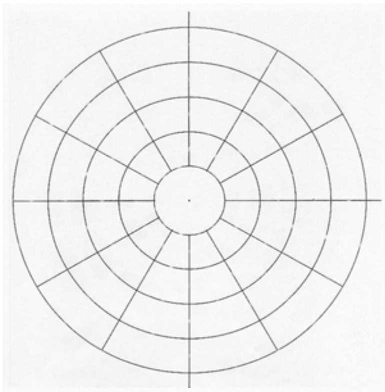


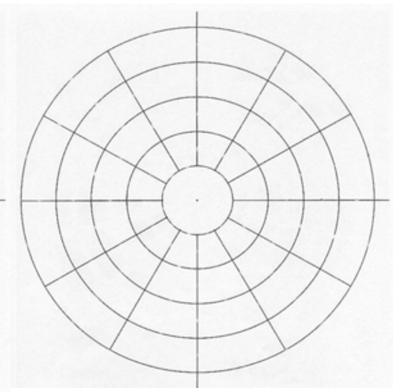


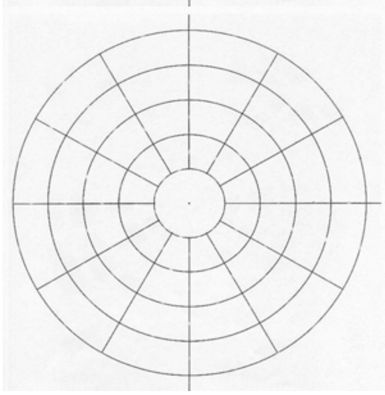
Radius:
Theta:

R	θ

6. $r(\theta) = 2 \sin(2\theta)$







Radius:
Theta:

R	θ