

Authentic Activities for Connecting Mathematics to the Real World

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<http://www.wfu.edu/~mccoy/mprojects.pdf>

Projects can be an important part of a Standards-based mathematics class. A project may be defined as an extended instructional activity that involves using **problem solving** and **reasoning** to **connect** mathematics in a real-world context and to **represent** and **communicate** understandings.

Contents

1. Choosing a GREEN Car
2. Poverty
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8. Collapsing Cubes
9. Cube Coloring
10. Law of Cosines in Football
11. Wheel of Fortune
12. Remarkable Mathematicians

Project 1. Choosing a GREEN Car.

Choosing a GREEN Car

INTRODUCTION: Environmental issues are important to everyone living on the earth today. Global warming, air pollution, and use of non-renewable resources are all sources of concern. This project uses online government information to compare cars on the following variables: annual petroleum consumption, greenhouse gas emissions, and air pollution score. Greenhouse gases refer to emissions that cause global climate change. Air pollution refers to vehicle emissions that are harmful to human health and/or cause smog.

NCTM STANDARDS: Data Analysis & Probability, Communication, Connections, Representation, Problem Solving

MATERIALS: Computers with Internet connection and spreadsheet software

GOALS: Students will collect online data, represent it in a table and a graph, and analyze outcomes to draw conclusions.

ACTIVITIES:

- Working in small groups, students will select five different cars to research.
- Using the <<http://www.fueleconomy.gov>> website, they will find each car and record the annual petroleum consumption, greenhouse gas emissions, and air pollution score. See Worksheet.
- Students will record the data in a spreadsheet and construct a line graph to represent it visually.
- Each group will submit a report, including their table, graph, and conclusions about choice of car.

ASSESSMENT: Group reports will be graded using attached rubric. Each individual student will submit a reflection about the data-driven decision process and their thoughts about “going green.”

Choosing a GREEN Car

1. Go to: <http://www.fueleconomy.gov>
2. Click on "Find and Compare Cars" or "Hybrids, etc."
3. Select a "Model Year" and then a "Make"
4. Select a car. Click "Compare."

www.fueleconomy.gov

Find and Compare Cars | Gas Mileage Tips | Gasoline Prices | Your MPG Will Vary | Why is Fuel Economy Important? | Your MPG | Hybrids, Diesels, Alt Fuels, Etc. | Tax Incentives | Extreme MPG

U.S. Department of Energy | Print the Fuel Economy Guide | U.S. Environmental Protection Agency

Find a Car

Compare Side-by-Side

Search by Class

Search by Make

Search by MPG

Cars that don't need gasoline

Best and Worst MPG

2006 Saturn Ion

Use Your Gas Prices & Annual Miles

Switch to Metric units



Compare side-by-side

→

Estimated New EPA MPG

REGULAR GASOLINE

26

Combined

23

City

32

Hwy

MPG ratings for this vehicle have been revised

Compare to Official EPA Window Sticker MPG

MPG Estimates from Drivers Like You

Average based on 5 vehicles.

30.4

Lo 25 → Hi 34

View Individual Estimates

Disclaimer

Energy Impact Score

Annual Petroleum Consumption
(1 barrel=42 gallons)

🇺🇸 - Imported

🇺🇸 - Domestic



13.2 barrels/year

Environment

Worst Best

16.2 3.5

7.1 tons/year

Air Pollution Score

Best

0 6 10



▶ Show Scores for California and Northeast States

▶ Show Detailed Air Pollution Information

More about emissions....

- ◆ [What's the difference between air pollution and greenhouse gas emissions?](#)
- ◆ [Want more info? See EPA's Green Vehicle Guide](#)

5. Record data in table.

Choosing a GREEN Car

See <http://www.fueleconomy.gov>

Car	Annual Petroleum Consumption	Greenhouse Gas Emissions	Air Pollution Score

Notes:

Annual Petroleum Consumption indicates the number of barrels of petroleum used by the vehicle each year. This includes fuel burned by the vehicle as well as petroleum used in producing, refining and delivering it. Using fewer barrels is better.

The **Greenhouse Gas Emission** rating shows the annual amount of greenhouse gases emitted by the selected vehicle in relation to the lowest and highest amounts emitted by other vehicles. Greenhouse gases refer to emissions that cause global climate change. A lower score is better.

The **Air Pollution Score** represents the amount of health-damaging and smog-forming airborne pollutants the vehicle emits. Air pollution refers to vehicle emissions that are harmful to human health and/or cause smog. Scoring ranges from 0 (worst) to 10 (best). A higher score is better.

Choosing a GREEN Car – Scoring Rubric

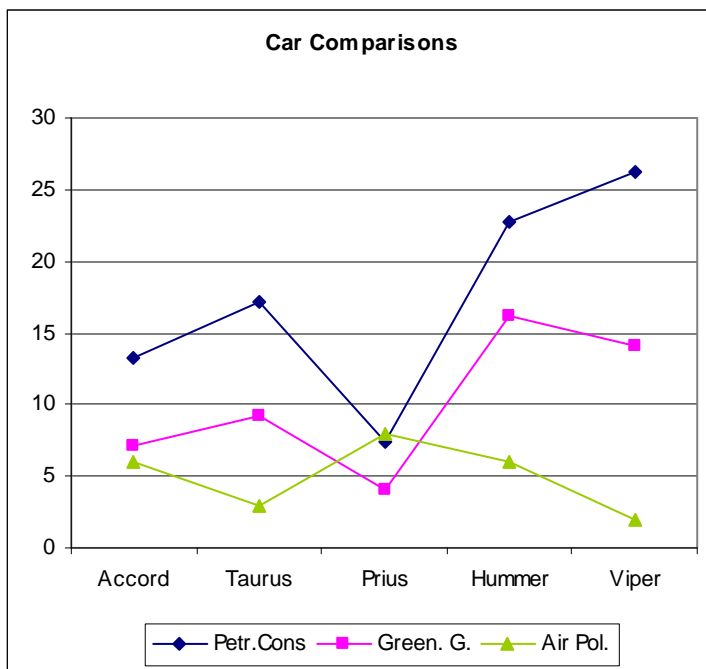
	Excellent 5 4	Satisfactory 3 2	Below Expectations 1 0
Internet Research _____	Students effectively navigated Internet and easily gathered information from electronic database.	Students navigated Internet and gathered information from electronic database.	Students required assistance to navigate Internet and gathered information from electronic database.
Data Table _____	Data are presented accurately and neatly in table.	Data are presented mostly accurately and neatly in table.	Data table contains several inaccuracies and/or is not neat.
Graph _____	Graph is appropriate and accurately represents data. Graph includes title, legend, and labels.	Graph is appropriate and mostly accurately represents data. Graph includes title, legend, and labels.	Graph may not appropriately represent data and has inaccuracies and/or is missing titles, legend, or labels.
Analysis and Conclusions _____	Students carefully analyzed the information collected and drew appropriate conclusions supported by evidence. Mathematical reasoning was evident.	Students' conclusions could be supported by stronger evidence. Level of analysis could have been deeper. Mathematical reasoning was mostly adequate.	Students' conclusions simply involved restating information. Conclusions were not supported by evidence. Mathematical reasoning was not evident.
Individual Reflection _____	The reflection responds to the group process and to thoughts on "Going Green." The paper is well written and organized with complete sentences and no mechanics problems.	The reflection responds somewhat to the group process and to thoughts on "Going Green." The paper is well written and organized with complete sentences and few mechanics problems.	The reflection does not clearly respond to the group process and to thoughts on "Going Green." The paper is poorly written and not well organized with several mechanics problems.

Total _____

Comments:

Sample Solution:

Car	Annual Petroleum Consumption	Greenhouse Gas Emissions	Air Pollution Score
Accord	13.2	7.1	6
Taurus	17.1	9.2	3
Prius	7.4	4.0	8
Hummer	22.8	16.2	6
Viper	26.3	14.1	2



Project 2. Poverty.

Poverty

The activities in these projects use all five of the NCTM process standards. The context is a real-world connection in a problem-solving format. Analysis involves reasoning with multiple representations of data and communication of conclusions.

Project I. What Is Poverty?

The reality of poverty is introduced and students are asked to consider whether they could live below the poverty level. They are asked to create a budget and to relate their own life to that of a family below the poverty line. (Any math course 6-12)

Project II. Who Are the Poor?

Students are given two tables from the U.S. Census Bureau and asked to analyze this data, to create graphs, and summarize their conclusions. The tables give the poverty level in the 50 states and the District of Columbia, and demographic characteristics of people in poverty such as age, race, and education. (Discrete Mathematics, General Mathematics, Statistics)

Project III. How is poverty related to School Achievement?

Students collect online data for poverty level and achievement level for a random sample of school districts in their state. They construct a scatterplot and a regression equation, and describe this relationship. (Algebra I, Algebra II, Statistics)

Project IV. How Has Poverty Changed?

Students are introduced to the Gini Coefficient to quantify the inequality of income across the population. Data is examined for the past 30 years and students are asked to explain results. (Pre-Calculus, Statistics)

Project IV. What Can Be Done?

Students are given a number of sources to learn more about poverty. (All)

I. What is Poverty?

- A. About the Hurricane Katrina disaster in New Orleans, Senator Barack Obama said

Whoever was in charge of planning was so detached from the realities of inner city life in New Orleans ... that they couldn't conceive of the notion that every American couldn't "load up their family in an SUV, fill it up with \$100 worth of gasoline, stick some bottled water in the trunk and use a credit card to check into a hotel on safe ground."
(Newsweek, Sept. 19, 2005)

- B. Food, shelter, clothing, health care, transportation - these are only the beginnings of the basic necessities of modern American living. Each year, the federal government calculates the minimum amount of money required by families to meet these basic needs. The resulting calculation is what is commonly referred to as the "poverty line." For 2004, the government set the poverty guidelines as follows.

Table I. 2004 Poverty Line

Size of Family Unit	Weighted Average Thresholds
One person	9,645
Two persons	12,334
Three persons	15,067
Four persons	19,307
Five persons	22,831
Six persons	25,788
Seven persons	29,236
Eight persons	32,641
Nine persons or more	39,048

SOURCE: U.S. Census Bureau.

- C. View the documentary "**Tour Poverty USA**" from the United States Conference of Catholic Bishops at

<http://www.usccb.org/cchd/povertyusa/tour.htm>

- Do you think your family could live with this income?
- How much would it bother you to give up many of the "extras?"
- Create a budget estimating what you spend for these "extras" in a normal year.
- How does your life relate to that of a family below the poverty line?

II. Who Are the Poor?

- A.** Table II-A gives the percentages and ranks of people below the poverty level in the 50 states and the District of Columbia. Select the ten states which are highest in percent of people below the poverty level. Using a spreadsheet (or a graphing calculator or graphing by hand), construct a line graph of states (horizontal axis) and percentage of people below the poverty level (vertical axis). Then on the same graph, plot the percentages of children below the poverty level. Label the graphs of total people and of children. What can you say about these variables?
- B.** Still using Table II-A, repeat the exercise in Part A above for the bottom-ranked ten states (lowest in poverty level). What can you say about these variables? What can you say about the comparison of the top and bottom ten states?
- C.** Table II-B gives percentages of several subgroups of people who are very poor (income at less than 50% of the poverty level). Select one characteristic, and construct a bar graph comparing the percentages of those subgroups who are "very poor." What can you say about your results? Repeat for two other characteristics. Write a summary of your conclusions.

Table II-A. Percent of People Below Poverty Level - 2004

State	Percent of People Below Poverty Level	Rank for People Below Poverty Level	Percent of Children below Poverty Level	Rank for Children Below Poverty Level
United States	13.1		18.4	
Alabama	16.1	9	23.3	8
Alaska	8.2	49	11.2	48
Arizona	14.2	16	20.3	17
Arkansas	17.9	5	25.9	5
California	13.3	20	18.9	21
Colorado	11.1	32	14.5	34
Connecticut	7.6	50	10.5	50
Delaware	9.9	41	13.8	38
District of Columbia	18.9	4	33.9	1
Florida	12.2	27	17.7	24
Georgia	14.8	13	21.3	12
Hawaii	10.6	38	14.4	35
Idaho	14.5	14	19.6	18
Illinois	11.9	29	16.8	28
Indiana	10.8	36	14.8	32
Iowa	9.9	41	12.4	44
Kansas	10.5	39	12.5	42
Kentucky	17.4	7	25.0	6
Louisiana	19.4	2	30.0	3
Maine	12.3	25	17.1	27
Maryland	8.8	46	11.4	47
Massachusetts	9.2	44	12.5	42
Michigan	12.3	25	17.6	25
Minnesota	8.3	48	10.7	49
Mississippi	21.6	1	31.0	2
Missouri	11.8	30	16.2	30
Montana	14.2	16	19.2	19
Nebraska	11.0	33	13.1	40
Nevada	12.6	23	18.8	22
New Hampshire	7.6	50	9.7	51
New Jersey	8.5	47	11.8	45
New Mexico	19.3	3	27.7	4
New York	14.2	16	20.7	15
North Carolina	15.2	12	21.9	11
North Dakota	12.1	28	15.5	31
Ohio	12.5	24	18.3	23
Oklahoma	15.3	11	20.7	15
Oregon	14.1	19	19.1	20
Pennsylvania	11.7	31	16.8	28
Rhode Island	12.8	22	21.0	14
South Carolina	15.7	10	22.8	10
South Dakota	11.0	33	14.8	32
Tennessee	14.5	14	21.1	13
Texas	16.6	8	22.9	9
Utah	10.9	35	13.3	39
Vermont	9.0	45	11.7	46
Virginia	9.5	43	12.9	41
Washington	13.1	21	17.2	26
West Virginia	17.9	5	24.4	7
Wisconsin	10.7	37	14.0	36
Wyoming	10.3	40	14.0	36

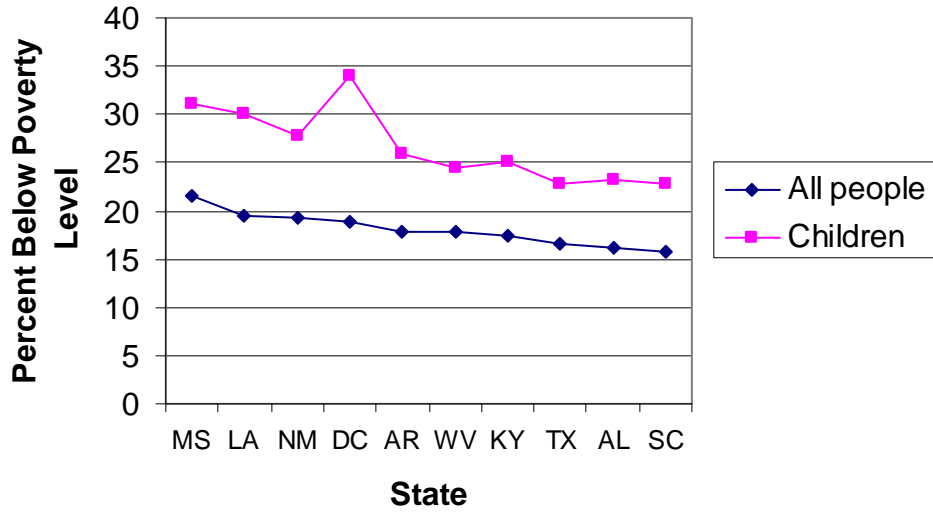
Source: U.S. Census Bureau, 2004 American Community Survey

**Table II-B. Selected Characteristics of People at Specified Levels of Poverty
2004**

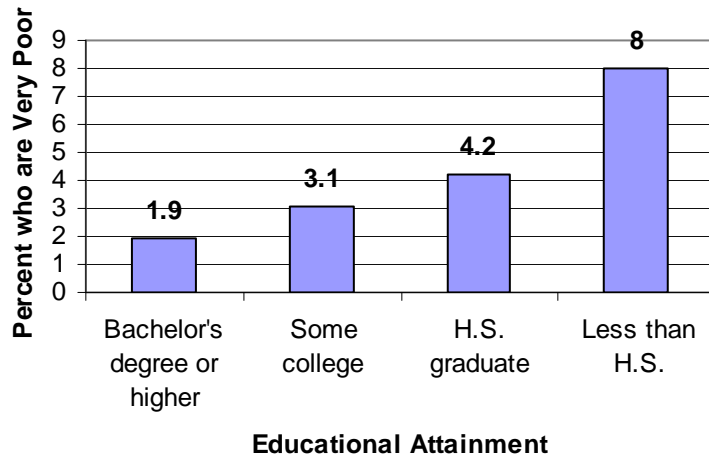
Subject	All People for Whom Poverty Status is Determined	People at Less than 50% of the Poverty Level
Population for whom poverty status is determined	284,577,956	5.7%
GENDER		
Male	139,214,726	5.0%
Female	145,363,230	6.4%
AGE		
Under 18 years	71,810,759	8.3%
18 to 64 years	178,561,896	5.3%
65 years and over	34,205,301	2.1%
RACE OR ETHNICITY		
White	215,298,360	4.3%
Black or African American	34,576,665	12.6%
American Indian and Alaska Native	2,137,754	11.4%
Asian	12,076,732	5.6%
Native Hawaiian and Other Pacific Islander	401,425	8.6%
Some other race	14,733,087	8.2%
Hispanic or Latino origin	40,219,766	8.6%
HOUSEHOLD TYPE		
In married-couple family households	180,844,134	1.9%
In other households	103,733,822	12.2%
EDUCATIONAL ATTAINMENT		
(Population 25 years and over)	186,534,177	
Less than high school graduate	29,976,049	8.0%
High school graduate (includes equivalency)	55,055,121	4.2%
Some college or associate's degree	51,091,603	3.1%
Bachelor's degree or higher	50,411,404	1.9%
CITIZENSHIP STATUS		
Native	250,346,192	5.6%
Foreign born	34,231,764	6.6%
DISABILITY STATUS		
With disability	37,771,428	7.2%
No disability	226,399,338	5.1%
WORK STATUS		
(Population 16 to 64 years)	186,589,012	
Worked full-time, year-round	88,904,929	0.3%
Worked less than full-time, year-round	59,720,615	6.8%
Did not work	37,963,468	15.2%
HOUSING		
In owner-occupied housing units	198,711,099	2.5%
In renter-occupied housing units	85,866,857	13.0%

Source: U.S. Census Bureau, 2004 American Community Survey

Percent Below Poverty Level 2004 Lowest Ten States



Education and Poverty - 2004



III. How is poverty related to School Achievement?

Data are available online that give poverty levels of each school district in the country. Similarly, as part of NCLB, each state publishes achievement data for each school and school district. Students can collect data from these sites, or the teacher can collect the data and hand out tables for student use in the project.

Assignment

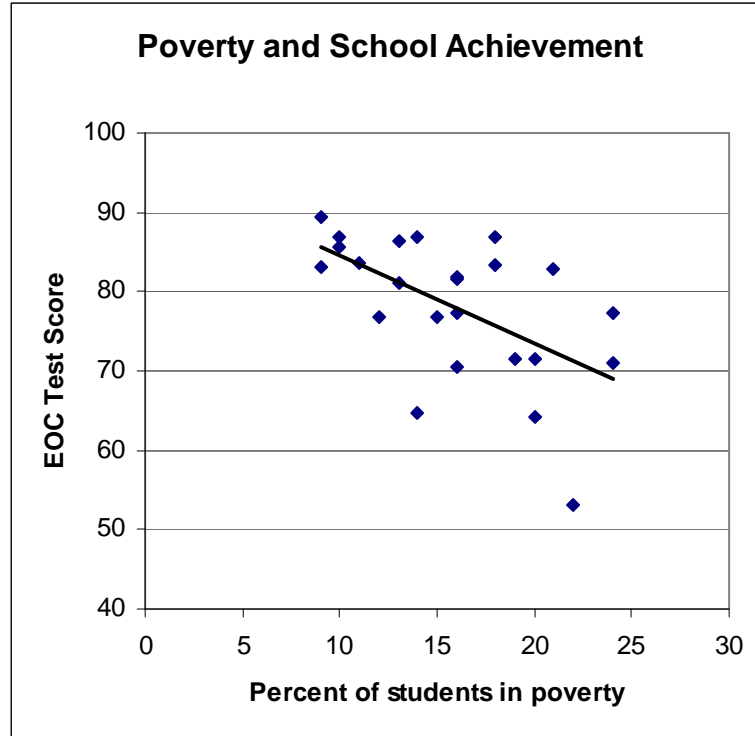
- Randomly select 10-20 counties or school districts in your state. List in a spreadsheet.
- Go to <http://www.publicschoolreview.com/> and obtain the "Percent of students in poverty" in each district. Enter this information on the spreadsheet.
- Go to your state's NCLB report site and obtain the achievement scores for these districts. Depending on the data reported, you may have to select which achievement measure you will use. The location of this data for North Carolina is <http://www.ncreportcards.org/> and the measure used in this example is the average End-of-Course Scores for all high school tests. Enter this data into the spreadsheet.
- Calculate a correlation coefficient to describe the relationship.
- Construct a scatterplot and a regression equation.

Questions

1. What can you say about the relationship between poverty and school achievement?
2. Why is the slope negative?
3. Do you think you would obtain different results with a different random sample? (Different groups could select different school districts for the sample and compare.)
4. Calculate the Coefficient of Determination (r^2). What can you say about your sample based on this statistic?
5. If a school district had 15% of its students in poverty, what would you predict the mean EOC score would be?
6. What do these data suggest about NCLB?

For 20 randomly selected school districts in North Carolina, 2003-2004:

% in Poverty	Mean EOC
18	86.8
16	81.8
13	86.3
14	86.8
11	83.5
21	82.8
16	70.5
9	89.3
12	76.8
15	76.7
9	83.2
20	71.6
10	87.0
10	85.7
24	71.0
18	83.3
16	77.2
19	71.5
16	81.5
24	77.2
20	64.1
14	64.7
22	53.1



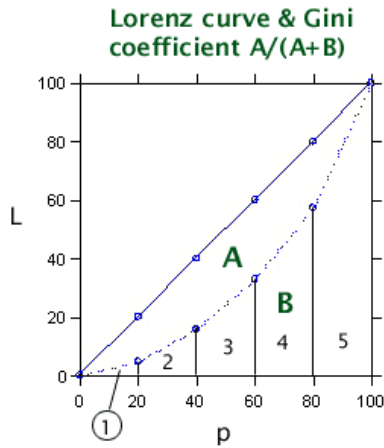
$r = -.56$

$r^2 = .32$

$y = -1.1x + 95.5$

IV. How Has Poverty Changed?

In economics, one measure of inequality is called the **Gini Coefficient**. This statistic allows us to quantify the distribution of income across a population. The Gini Coefficient ranges from 0 to 1, where 1 is perfect inequality (one part of the population has all the income and the rest have none) and 0 is perfect equality (all in the population have equal shares). The general shape of the graph is



By definition, the **Gini Coefficient** is the ratio of the area between the 45 degree equality line and the **Lorenz Curve** which is the graph of our population (P) and income (L) data. This area represents the amount of inequality. So, in this graph the **Gini Coefficient** is equal to Area A/(Area A + Area B). The closer the Lorenz curve is to the line of perfect equality, the smaller the Gini coefficient, and the less the inequality.

Since we don't know the equation of the curve, we will use the **Trapezoid Rule** to approximate the area (B) under the curve by finding the areas of the polygons:

Polygon 1 is a triangle, $((b * h)/2)$

Polygons 2-5 are trapezoids, $(h * (b1 + b2)/2)$.

Note: the bases are vertical lines.

(Area A + Area B) will always be $(100*100)/2$

Area B will be (Area 1 + Area 2 + Area 3 + Area 4 + Area 5)

Area A will be (Area A + Area B) – Area B

The Gini Coefficient can also be represented by the following formula

$$G = \left| 1 - \sum_{k=1}^{k=n} (X_k - X_{k-1})(Y_k + Y_{k-1}) \right| \quad \text{where } k = \text{the number of partitions.}$$

We will compare data in 10 year increments from 1970 to 2000.

Share of Aggregate Income Received by Households

Year	Lowest fifth	Second fifth	Third fifth	Fourth fifth	Highest fifth
2000	3.6	8.9	14.8	23.0	49.8
1990	3.8	9.6	15.9	24.0	46.6
1980	4.2	10.2	16.8	24.7	44.1
1970	4.1	10.8	17.4	24.5	43.3

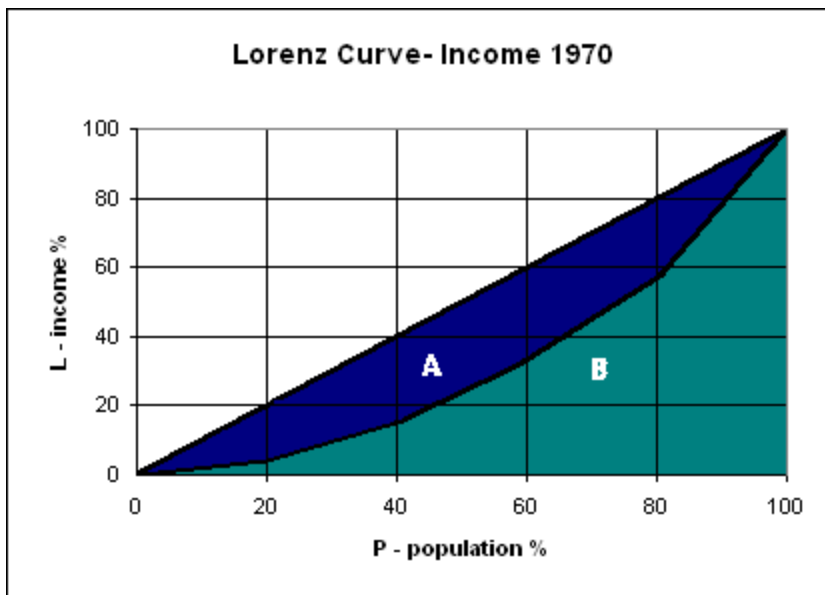
Source: U.S. Census Bureau

For the year 1970:

First, we will arrange the data in a working table and calculate **P** = the cumulative share of the population (%) and **L** = cumulative share of income (%) for each fifth of our population year. (Spreadsheet is recommended, but it could be done by hand or with calculator.)

Gini 1970

Income Category	Share of Total Income %	P %	L %
Bottom Fifth	4.1	20	4.1
2 nd Fifth	10.8	40	14.9
3 rd Fifth	17.4	60	32.3
4 th Fifth	24.5	80	56.8
Top Fifth	43.3	100	100.1



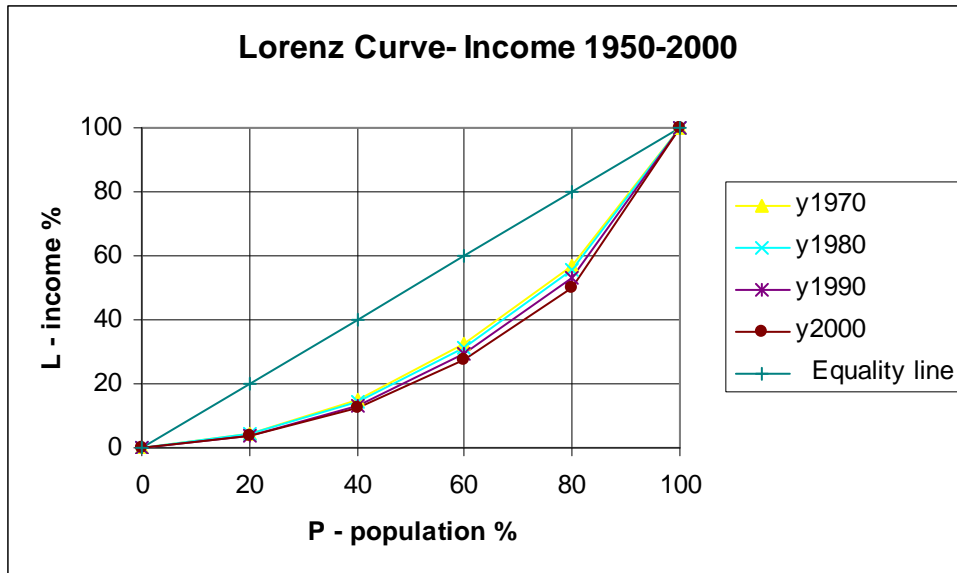
The next task is to calculate.

Area A + Area B	$100 \times 100 / 2$	5000
Area 1 = $((b \times h) / 2)$	$20 \times 4.1 / 2$	41
Area 2 = $(h \times (b_1 + b_2) / 2)$	$20 \times (4.1 + 14.9) / 2$	190
Area 3 = $(h \times (b_1 + b_2) / 2)$	$20 \times (14.9 + 32.3) / 2$	472
Area 4 = $(h \times (b_1 + b_2) / 2)$	$20 \times (32.3 + 56.8) / 2$	891
Area 5 = $(h \times (b_1 + b_2) / 2)$	$20 \times (56.8 + 100.1) / 2$	1569
Total Area B = (Area 1 + Area 2 + Area 3 + Area 4 + Area 5)		3163
Area A = (Area A + Area B) – Area B	$5000 - 3162$	1837
Gini Coefficient = $(\text{Area A} / (\text{Area A} + \text{Area B}))$	$1838 / 5000$	0.367

Complete the graphs and calculations for the Gini Coefficients for 1970, 1980, 1990, and 2000. (You may either put them on the same graph or create separate graphs.) Make a table of the Gini Coefficients for all the years. Look at the graphs and the Gini Coefficients. What can you say about your results? Is there a trend? What conclusions can you draw?

y70		y80		y90		y00	
P	L	P	L	P	L	P	L
20	4.1	20	4.2	20	3.8	20	3.6
40	14.9	40	14.4	40	13.4	40	12.5
60	32.3	60	31.2	60	29.3	60	27.3
80	56.8	80	55.9	80	53.3	80	50.3
100	100.1	100	100	100	99.9	100	100.1

	y1970	y1980	y1990	y2000
A + B	5000	5000	5000	5000
A1	41	42	38	36
A2	190	186	172	161
A3	472	456	427	398
A4	891	871	826	776
A5	1568	1559	1533	1504
Ar B	3163	3114	2996	2875
Ar A	1837	1886	2004	2125
Gini	0.367	0.377	0.401	0.425



V. What Can Be Done? Learn More. Care More. Do More.

- Children's Defense Fund
 - <http://www.childrensdefense.org/>
- Educate to End Poverty in America
 - <http://www.usccb.org/cchd/povertyusa/involved.shtml>
- Fight Hate and Promote Tolerance
 - <http://www.tolerance.org/>
- How to End Poverty (Time Magazine)
 - <http://www.time.com/time/covers/1101050314/>
- National Center for Children in Poverty (Columbia University)
 - <http://www.nccp.org/>
- Race, Poverty, and Katrina (NPR)
 - <http://www.npr.org/templates/story/story.php?storyId=4829446>
- U.S. Census Bureau Poverty Data
 - <http://www.census.gov/hhes/www/poverty/poverty.html>

Project 3. Native American Games of Chance.

Choose one of the four Native American games described below. Play the game several times, and then explain the probability involved. Make sure that you address the sample space, theoretical and experimental probability. Present your findings to the class.

Game 1. Hubbub (Wa'lade hama'gan).

Place six two-sided “dice” (lima beans painted on one side) in a basket and toss.

If you toss 6-alike, you get 3 points.

If you toss 5-alike, you get 1 point.

If you toss less than five alike, you get no points and pass turn to your opponent

Play until one player reaches 50 points.

Game 2. Moccasin Game.

The objective of the game is for one player to guess which of 4 or more moccasins the token has been hidden under. One player hides a small token under a moccasin while the second player looks away. The second player then attempts to guess which moccasin the token is under. The guess is indicated by touching the moccasin with a stick.

Game 3. Stick Game.

Hold four sticks (popsicle sticks or tongue depressors, decorated on one side) in one hand, and let them fall to the ground or the table. Take turns. Play 10 rounds.

Scoring:

- All four up 5 points
- Three up and one down 2 points
- Two up and two down 1 point
- One up and three down 2 points
- All four down 5 points

Game 4. Ashbii. (ash been)

The game is played with three painted sticks and a basket. The first is called the “tsi’i (zeen) head.” It is painted completely black on one side and half black on the other. The second stick is called “nezhi’ (nezshi), and is painted half red on one side and half black on the other side. The third stick is called “tqelli” (zelli) and is painted all red on one side and all black on the other.

Players alternate throwing the sticks. The winner is who first scores 25 points.

Scoring (The highest one score for each throw):

- 1 point for all black or half black.
- 2 points for all red or half red.
- 3 points for crossing (all black and all red, all black and half red, all red and half black, half red and half black).
- 5 points for crossing all red and half red.

Teacher Resources for Project 1. Native American Games of Chance.

These activities provide an appropriate multicultural context in which students can experiment and experience probability concepts.

Game 1. Hubbub (Wa'lade hama'gan).

Wa'lade hama'gan (Bowl and Dice) is one of many similar games played by early Native American tribes. The Penobscot Nation in New England called it Hubbub because of the chanting that accompanied the tossing of the dice. The dice were traditionally made from carved bone or antler, animal teeth, or small stones. They were engraved, burned, polished, or painted to distinguish the sides.

Game 2. Moccasin Game.

The Moccasin Game and its many variations was very popular in many Native American tribes. It was used to sharpen wit and thinking skills and test a person's ability to fool their opponent. The hand games usually involved gambling and were played to "hand game music" involving drums. More specific and complex "rules" existed and varied among tribes.

Game 3. Stick Game.

Native American peoples played a number of stick games with varied rules. The sticks can be painted on one side with Native American patterns.

Game 4. Ashbii. (ash been)

Ashbii (ash been) is a game played by Navajo women and children. The game was observed in the town of Chin Lee Arizona on the reservation there. The Navajo women sometimes played this game while sitting under a buffalo hide which had been staked up for drying. Each game will require three sticks:

The first is called the "tsi'i (zeen) head." It is painted completely black on one side and half black on the other.



The second stick is called "nezhi' (nezshi), and is painted half red on one side and half black on the other side.



The third stick is called "tqelli" (zelli) and is painted all red on one side and all black on the other.



Culin, S. (1992). *Games of the North American Indians. Volume 1: Games of chance.*

Lincoln, NE: University of Nebraska Press.

Krause, M. C. (2000). *Multicultural mathematics materials. 2nd Edition.* Reston, VA: NCTM.

Zaslavsky, C. (2002). Native American games and activities. In J. E. Hankes & G. R. Fast (Eds.) *Changing the faces of mathematics: Perspectives on Indigenous people of North America* (151-155), Reston, VA: NCTM

III. BARBIE



Measure the Barbie doll and determine her real life measures (Select a height).

Draw your life-size Barbie and discuss her influence on young girls.

<i>Proportion:</i>	$\frac{\textit{Barbie height}}{\textit{Real height}} = \frac{\textit{Barbie measurement}}{\textit{Real measurement}}$
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2-D Measurements of the Doll

Real Life

Height	_____	_____
Width at shoulders	_____	_____
Inseam	_____	_____
Length of Head	_____	_____
Length of Arm	_____	_____
Width of Thigh	_____	_____
Width of Stomach	_____	_____
Length of Foot	_____	_____
Others:	_____	_____
	_____	_____
	_____	_____

Teacher Resources for Project 2. Cars, Critters, and Barbie.

Known-Scale Toys that work well for this activity are model cars of various sizes. The scale is usually given on the box (for example, 1:24). A variety of different sizes provide an interesting context. Students may also bring their own models from home, provided that they still have the box or information giving the scale. Students measure the cars and use the given scale to estimate the size of the real car. Once this part of the project is completed, students may go to websites of car manufacturers to find actual sizes and check their work.

Unknown-Scale Toys may be any type of animal models. Small zoo or farm animals are inexpensive and appropriate for this activity. These typically do not include the scale. So this time students work in reverse. They look up the animal in a reference book and obtain its average length, width, or height. Then they measure the toy, and calculate the scale.

Barbie is a favorite activity of male and female students from elementary school through college levels and beyond to inservice teachers. The model Barbie is sketched on large butcher-paper, and students may use colored markers to complete her picture. This is a valuable lesson for many reasons, including the fact that the life-size Barbie is somewhat grotesque. The proportions of the doll do not translate well to real life size, and this is apparent to students.

This project should be discussed fully and individual students should be asked to write about their mathematical and social findings.

Project 5. Exploring Fractals.

Koch Snowflake

This figure is called a Koch Snowflake, named after the Swedish mathematician Niels Fabian Helge von Koch (1870-1924).

1. Starting with an equilateral triangle ($n=0$), trisect each side.
2. Build smaller equilateral triangles on the exterior, with two vertices on the trisection points, to form a six-pointed star ($n=1$).
3. Draw triangles on each of the twelve sides ($n=2$).
4. Complete the next two iterations and derive the general formula.

Step	Side Length	Number of Sides	Perimeter
0	1	3	3
1			
2			
3			
4			
General Formula for n^{th} step			

5. What happens to the perimeter and area of this fractal as n approaches infinity?
6. Make a fractal like the Koch snowflake using a shape other than a triangle.

Sierpinski Gasket

Waclaw Seirpinski (1882-1969) was a Polish mathematician who is best known for his work with fractals and space-filling curves.

1. Start with an equilateral triangle ($n=0$).
2. Connect the midpoints of each side.
3. Remove the center triangle; shade it and imagine that the center is a hole ($n=1$).
4. Connect the midpoints of the three remaining triangles.
5. Repeat until the triangles are too small to continue.

Step	Number of New Holes	Area of One New Hole	Area Removed this Step	Total Area Remaining
0	0	0	0	1
1				
2				
3				
4				
General Formula for n^{th} step				

6. What happens to the area as n approaches infinity?
7. How many points will never be removed?
8. How can we say that the area is zero if an infinite number of points still remain?
9. Draw a Pascal's triangle up to at least the sixteenth row. Circle all the odd numbers. Can you see a relationship to the Sierpinski Gasket?

Teacher Resources for Project 3. Exploring Fractals.

Koch Snowflake



Step	Side Length	Number of Sides	Perimeter
0	1	3	3
1	1/3	4 · 3	3(4/3) ¹
2	(1/3) ²	4 ² · 3	3(4/3) ²
3	(1/3) ³	4 ³ · 3	3(4/3) ³
4	(1/3) ⁴	4 ⁴ · 3	3(4/3) ⁴
General Formula for nth step	(1/3) ⁿ	4 ⁿ · 3	3(4/3) ⁿ

(5.) The perimeter approaches infinity, but the area is finite. If a box was drawn around the shape, the shape would never grow outside that box yet the perimeter would become infinitely large.

Sierpinski Gasket



Step	Number of New Holes	Area of One New Hole	Area Removed this Step	Total Area Remaining
0	0	0	0	1
1	1	1/4	1/4	3/4
2	3	1/16	3/16	9/16
3	9	1/64	9/64	27/64
4	27	1/256	27/256	81/256
General Formula for nth step	3 ⁿ⁻¹ where n>=1	(1/4) ⁿ	3 ⁿ⁻¹ / 4 ⁿ or (1/3) (3/4) ⁿ where n>=1	(3/4) ⁿ

- (6) As n approaches infinity, the area approaches zero.
- (7) Infinitely many points will never be removed. The points at the corners of all triangles will never be removed.
- (8) A point has no area. No matter how many zeros we add, the total is still zero.

Naylor, M. (1999). Exploring fractals in the classroom. *Mathematics Teacher*, 92 (4), 360-366.

<http://library.thinkquest.org/26242/full/fm/fm16.html>

<http://math.rice.edu/~lanius/frac/>

Project 6. The Golden Rectangle in Multicultural Art.

The golden rectangle has sides in the ratio of 1.618 to 1. It was called “golden” because people found its proportions pleasing. It occurs in nature and appears in the art and architecture of many cultures

Select a cultural item to investigate, such as Shoshone beadwork, Navajo blankets, Senegalese wall hangings, or Guatemalan scarves. Locate at least 10 examples of this art on the internet (see suggested links below). Measure dimensions of the non-square rectangles and calculate the ratios of the sides. Is the golden rectangle being used in the sample you studied? Is it used sometimes, often, or always? Decide on a clear representation of your data. Prepare a PowerPoint slide presentation to report your results.

Shoshone Beadwork:

<http://www.windriverhistory.org/exhibits/ShoshoneArt/beadwork/index.html>

<http://www.native-languages.org/beadwork.htm>

Navajo Blankets and Rugs:

http://www.indianterritory.com/pages/brief_history_of_navajo_weaving.htm

<http://www.toh-atin.com/Pages/Weavings/weavings.html>

Senegalese Wall Hangings:

<http://www.africancraft.com/search.php?kwd=kente>

<http://www.du.edu/duma/africloth/map.html>

Guatemalan Scarves:

<http://www.happymangoimports.com/othertextiles.html>

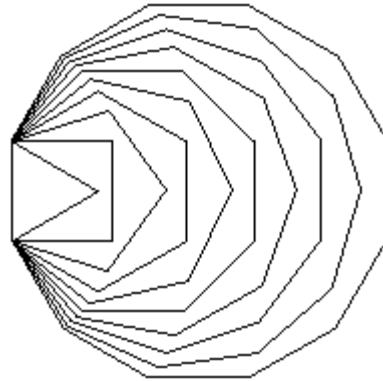
<http://www.guatravellers.com/fabrics.htm>

Note: This project is very flexible and could be used with a variety of nature, art, or architecture media.

Project 7. Tessellations of Regular Polygons.

A **tessellation** is a geometric figure that completely covers a plane surface with no holes and no overlaps. Regular polygons may tessellate in three different ways: regular, semiregular, and demiregular.

Make a set of several copies of each regular polygon (3-12) with equal sides for all. You may construct these, use a template, use computer software, or enlarge the figure at right and cut several copies of each figure.



REGULAR TESSELLATIONS

Using models of the regular polygons (with sides 3 through 12), determine which polygons will tessellate the plane. Each figure will use only one polygon.

Present three representations of the regular tessellations you find:

- (1) A diagram showing the regular tessellation (colors are nice!),
- (2) A “tessellation notation,” for example 8,8,8 means each vertex has three octagons,
- (3) Explain your findings numerically, in terms of angle measures.

SEMIREGULAR TESSELLATIONS

Semiregular tessellations are similar to regular tessellations, except that they contain combinations of the regular polygons. Each vertex point will contain the same combination of multiples of two or three regular polygons. There are eight semiregular tessellations.

Present three representations of the semiregular tessellations you find:

- (1) A diagram showing the semiregular tessellation (colors are nice!),
- (2) A “tessellation notation” for the vertex combination,
- (3) Explain your findings numerically, in terms of angle measures.

DEMIREGULAR TESSELLATIONS

Demiregular tessellations have two or three different vertices which are repeated to create the tessellation. See if you can find the 14 demiregular tessellations and represent them as above.

Teacher Resources for Project 5. Tessellations of Regular Polygons.

Solutions for the all types are presented below. Diagrams consist of those polygons at each vertex point. Numerical justification is based on each vertex = 360 degrees.

REGULAR TESSELLATIONS

- (1) 3,3,3,3,3,3
- (2) 4,4,4,4
- (3) 6,6,6

SEMIREGULAR TESSELLATIONS

- (1) 3,3,3,3,6
- (2) 3,3,3,4,4,
- (3) 3,4,6,4
- (4) 4,8,8
- (5) 3,6,3,6
- (6) 3,3,4,3,4
- (7) 3,12,12
- (8) 4,6,12

DEMIREGULAR TESSELLATIONS

- (1) 3,12,12 • 3,4,3,12 (triangles, squares, dodecagons)
- (2) 3,3,4,3,4 • 3,3,4,12 • 3,4,3,12 (triangles, squares, dodecagons)
- (3) 3,6,3,6 • 3,3,6,6 (triangles, hexagons)
- (4) 3,3,3,3,3 • 3,3,4,12 (triangles, squares, dodecagons)
- (5) 3,4,6,4 • 4,6,12 (triangles, squares, hexagons, dodecagons)
- (6) 3,3,3,3,3 • 3,3,4,12 • 3,3,4,3,4 (triangles, squares, dodecagons)
- (7) 3,3,4,3,4 • 3,4,6,4 (triangles, squares, hexagons)
- (8) 3,3,3,4,4 • 3,3,4,3,4 • 3,4,6,4 (triangles, squares, hexagons)
- (9) 3,3,3,3,3 • 3,3,4,3,4 (#1: triangles, squares)
- (10) 3,3,3,3,3 • 3,3,4,3,4 (#2: triangles, squares)
- (11) 3,3,3,3,3 • 3,3,3,4,4 • 3,3,4,3,4 (#1: triangles, squares)
- (12) 3,3,3,3,3 • 3,3,3,4,4 • 3,3,4,3,4 (#2: triangles, squares)
- (13) 3,4,6,4 • 3,4,4,6 (triangles, squares, hexagons)
- (14) 3,3,3,4,4 • 3,4,6,4 (triangles, squares, hexagons)

Diagrams:

<http://library.thinkquest.org/16661/simple.of.regular.polygons/semiregular.4.html>

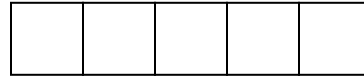
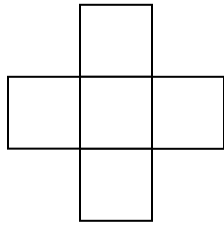
<http://library.thinkquest.org/16661/of.regular.polygons/demiregular.2.html>

Printable Templates:

<http://library.thinkquest.org/16661/templates/index.html>

Project 8. Collapsing Cubes.

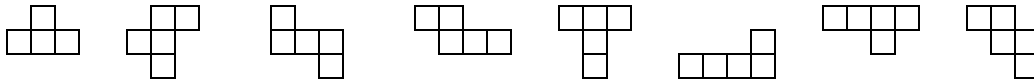
You can begin this investigation by trying to visualize how the sides of a cube can be collapsed into different two-dimensional patterns. First, we'll look at a simpler version of this problem. Suppose we look at a cube without one face—a box without a top. We can see that the box has only five sides and for simplicity, that these sides are all squares. If we unfold the box, we might get the figure on the left below, but not the pattern on the right.



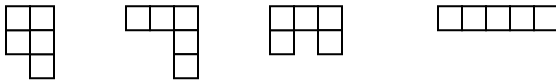
1. Find the other patterns of five squares that can be folded into a box without a top. Be careful not to count the same pattern twice. Said another way: rotations or reflections of a pattern are not counted a second time. Sketch these on graph paper and cut out the patterns. One-inch graph paper is recommended.
2. There are twelve different arrangements of five squares, but only the eight found in Exercise 1 can be folded up into a box without a top. Can you find all twelve? Are the areas of each of these the same? Explain! Are the perimeters of each of these the same? Explain!
3. Now see if you can collapse a regular tetrahedron in the same manner. You will have four equilateral triangles to work with. Again be careful not to count a rotation or a reflection of a combination you already have.
4. A challenging problem is to try to collapse a cube. This problem is very similar to the box-without-a-top problem. The only addition is one more square. Unfortunately, this one square increases the number of combinations to thirty-five. The number of combinations that are solutions is ten. Find as many of them as you can. Also find as many of the six-square patterns as you can. Do these all have the same area? Perimeter?

Teacher Resources for Project 6. Collapsing Cubes.

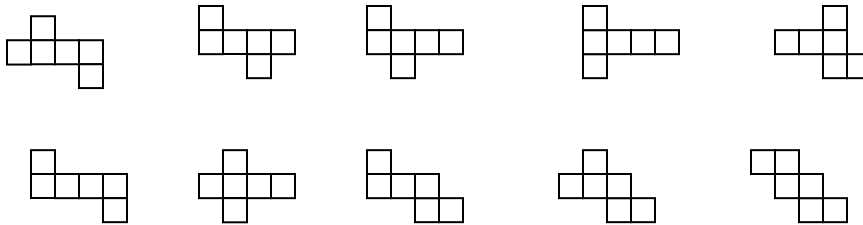
(1) Solutions to the topless box:



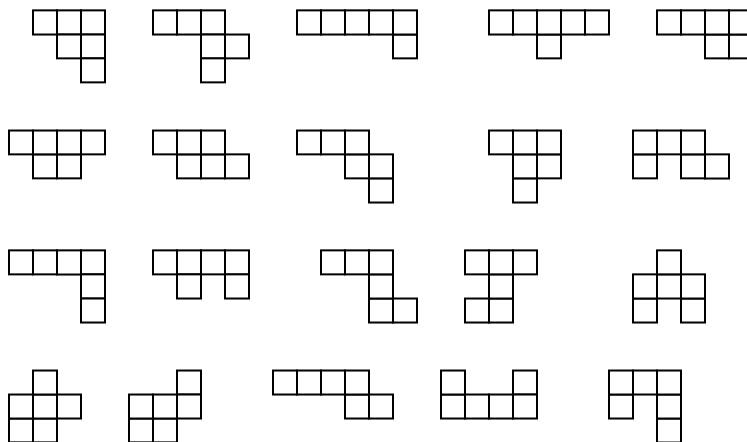
(2) These configurations do not fold into a topless cube. The areas are always the same, but the perimeters differ.



(4) Solutions to the cube.



These configurations cannot be folded into a cube.
All have the same area. The perimeters differ.



Project 9. Cube Coloring.

Investigate what happens when different-sized cubes are constructed from unit cubes, the surface areas are painted, and the large cubes are then disassembled into the original unit cubes. How many of the $1 \times 1 \times 1$ unit cubes are painted on three faces, two faces, one face, and no faces?

After your group has built the cubes of sizes 2, 3, 4, and 5, think about answers to the questions and discuss your strategies. Organize your data into a chart like the one below. Discuss the patterns in the chart. Determine the general expressions for the cube of size n in each column.

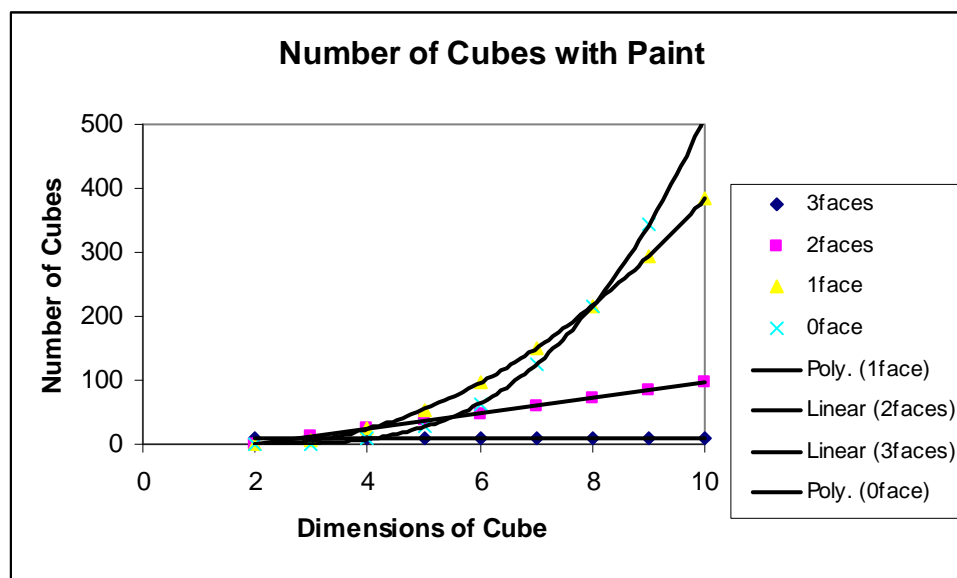
Dimensions	Number of $1 \times 1 \times 1$ cubes needed	3 faces painted	2 faces painted	1 face painted	0 faces painted
$2 \times 2 \times 2$					
$3 \times 3 \times 3$					
$4 \times 4 \times 4$					
$5 \times 5 \times 5$					
.					
.					
$n \times n \times n$					

Use the general expressions to complete each column through size 10. Observe the growth patterns in the different columns and graph these patterns, with the horizontal axis being Dimensions of the cube, and the vertical axis being Number of cubes. You should get a different representation for each number of faces. Determine whether these are constant, linear, quadratic, or cubic.

Teacher Resources for Project 6. Collapsing Cubes.

- The cubes with three faces painted are always the eight corner cubes.
- The cubes with two faces painted occur on the edges between two corners, and the two faces column increases by 12 each time.
- The cubes with one face painted occur as squares on the six faces of the original cube.
- The zero-faces column consists of cubes of numbers that are 2 less than the size of the original cube.

Dimensions	Number of 1 x 1 x 1 cubes needed	3 faces painted	2 faces painted	1 face painted	0 faces painted
2 x 2 x 2	8	8	0	0	0
3 x 3 x 3	27	8	12	6	1
4 x 4 x 4	64	8	24	24	8
5 x 5 x 5	125	8	36	54	27
6 x 6 x 6	216	8	48	96	64
7 x 7 x 7	343	8	60	150	125
8 x 8 x 8	512	8	72	216	216
9 x 9 x 9	729	8	84	294	343
10 x 10 x 10	1000	8	96	384	512
n x n x n	n^3	8	$12(n-2)$	$6(n-2)^2$	$(n-2)^3$



The 0-face graph is cubic. The 1-face graph is quadratic.
 The 2-face graph is linear. The 3-face graph is constant.

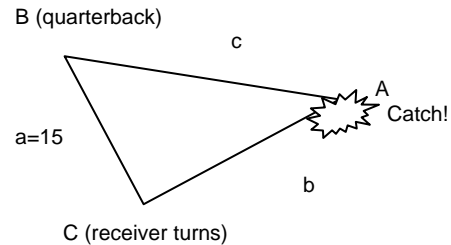
Project 10. Law of Cosines in Football.

The Law of Cosines.

In football, receivers tend to run in triangle patterns in order to receive the ball. Straight lines are preferred to curves so the quarterback can easily predict where the receiver will be. When receivers run, turn, and run, and the ball meets them, triangles are formed.

For example, a receiver may run 15 yards ahead and turn 95 degrees and the run forward again to meet the ball at the given point.

A few formations have been marked by tape on the field. You will use the speed of one of your group members and the speed of the ball when thrown to determine velocity, and thus solve for c , how far ahead of the receiver the quarterback should throw the ball.



Velocity = distance / time

Law of Cosines: $c^2 = a^2 + b^2 - 2ab \cos C$

Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

For each triangle:

1. Measure side a (in feet)
2. Measure angle BCA
3. Have one team member throw the ball 20 yards and someone else catch it while a third person clocks the airtime. Use this distance and time to find the ball velocity (ft/sec)
4. Have a team member run 20 yards, and calculate the velocity (ft/sec)
5. Calculate the relationship between the two velocities and put them in terms of each other (It will be easier to put them in terms of runner's velocity.
6. Now use the Law of Cosines to solve for each b and c . These represent the distance ahead of the receiver that the quarterback must throw the ball and the distance the ball must travel.
7. Write a paragraph explaining how the Law of Cosines applies to football.

Extension: You might take digital photos or videotape your high school football team, and analyze actual game situations in terms of the Law of Cosines.

Teacher Resources for Project 8. Law of Cosines in Football.

Teacher Preparation:

- Students should already be familiar with the Law of Cosines
- Before class, mark off triangles on an open field using string or tape.
- Give each group a tape measure, protractor, and stopwatch.

Activity:

Students will find the velocity of the ball by throwing it and clocking the air time. They will then find the velocity of the receiver by clocking a student's time to run (a) yards, which you will have marked off but they will have measured. Using a proportion, they can label side c in relationship to b. Finally, students will measure angle C with a protractor this having enough information to solve for the unknowns using the Law of Cosines. Students must know how to solve quadratic equations, and will most likely have to use the quadratic formula since these numbers may not be "pretty."

Some Possible Triangles

Triangle #	Side a	Angle BCA	Runner's Velocity (b)	Ball Velocity (c)
1	15 ft.	120°	15 ft/sec	60 ft/sec (4b)
2	30 ft.	80°	20 ft/sec	60 ft/sec (3b)
3	25 ft.	90°	25 ft/sec	50 ft/sec (2b)

Triangle 1: $(4b)^2 = (15)^2 + b^2 - 2(15)(b)(\cos 120^\circ)$

$$15b^2 - 15b - 225 = 0, \text{ so } b = 4.405 \text{ ft.}$$

Triangle 2: $(3b)^2 = (30)^2 + b^2 - (2)(30)(\cos 80^\circ)$

$$8b^2 + 10.44b - 900 = 0, \text{ so } b = 9.97 \text{ ft.}$$

Triangle 3: Using the Law of Cosines

$$(2b)^2 = (25)^2 + b^2 - (2)(25)(\cos 90^\circ)$$

Or, using the Pythagorean Theorem,

$$(25)^2 + b^2 = (2b)^2$$

$$\text{Both, simplify to } 3b^2 - 625 = 0, \text{ so } b = 14.4 \text{ ft.}$$

Project 11. Wheel of Fortune.

You will be watching “Wheel of Fortune” and completing an analysis of the letters used in the show.

Make a list of all the letters of the alphabet and chart the letters used on “Wheel of Fortune” using tally marks. Do not count letters guessed but not found in the phrase.

Find the total for each letter, and the grand total of all letters in the whole show. For each letter, divide the total for that letter by the grand total to find the percent of each letter used.

- (1) Find the top ten letters used (in order).
- (2) Find the bottom five letters used.
- (3) How many vowels are in the top ten?
- (4) Which consonants would be the most useful?
- (5) Which vowel might be the least useful?
- (6) What percentage of all the letters surveyed were vowels?
- (7) If you watched three different days of “Wheel of Fortune,” do you think your results will be similar? Test and see.

See also:

Holbrook, E. L. (1998). Wheel of Fortune for the mathematics classroom. *Mathematics Teaching in the Middle School*, 4 (1), 32-36.

Project 12. Remarkable Mathematicians.

STEP 1. You should select a famous mathematician to study for this project. You might want to browse one or more of the following websites to make your choice:

HUGE database at University of St. Andrews:

<http://www-groups.dcs.st-and.ac.uk/~history/>

Biographies of Women Mathematicians:

<http://www.agnesscott.edu/lriddle/women/women.htm>

African Americans in the Sciences:

<http://www.princeton.edu/~mcbrown/display/faces.html>

STEP 2. Research your mathematician in the databases above, and also on the Internet and in any available reference books.

STEP 3: Prepare a presentation for the class about your mathematician. Cover the following topics:

1. Name
2. Country (or countries)
3. Dates of birth and death
4. Education (college and degree)
5. Awards
6. One interesting fact
7. Another interesting fact
8. Mathematics Specialty
9. One example of this mathematics
10. One sentence summary of why this mathematician is remarkable.

McCoy, L. (2001). Remarkable Women of Mathematics and Science. In Jacobs, J. E., Becker, J. R., & Gilmer, G. F. (Eds.) *Changing the Faces of Mathematics: Perspectives on Gender* (pp. 125-132). Reston, VA: National Council of Teachers of Mathematics.