

TOPIC: Geometry, Pi, and Ancient Civilizations

NCTM STANDARDS: Geometry, Problem Solving, Connections

GOALS:

- Students will use the Internet to complete a guided Pi WebQuest.
- Students will use geometry to study the ways in which ancient civilizations (in Egypt, Babylonia, and Greece) approximated the constant ratio of the circumference of any circle divided by its diameter, which we today call pi (π). They will also learn about the methods used by people in China and India.
- Students will review geometry concepts including the radius, diameter, circumference, and area of a circle, as well as the perimeter of a polygon.

INTRODUCTION:

Pi is a very important ratio in mathematics, representing the circumference of any circle divided by its diameter. Today, we have historical evidence that many ancient civilizations were familiar with this ratio (although they did not call it pi) and that different civilizations used slightly different approaches for calculating pi with varying degrees of accuracy. So what were the ancient equivalents for pi? Let's find out!

ACTIVITIES:

- Part 1. Introduction. Students use the Internet to complete a guided Pi WebQuest.
- Part 2. Approximating Pi. Students work together on geometric constructions/word problems (Ancient Pi Discovery Worksheet) to find an approximation of pi corresponding to each of three ancient civilizations.

ASSESSMENT/PROJECT:

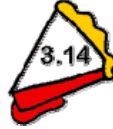
After completing the activities above, students will work with a partner to create a cartoon, poster, song, story, or other approved medium to honor pi and the ancient civilizations we recognize for first using approximations of pi. All final products should also include a definition of pi.

RESOURCES:

Lumpkin, B. (1997). *Geometry activities from many cultures*. Portland: Walch Publishing.

Solli, A. (2005). A chronological history of pi with development activities in problem solving. Retrieved September 21, 2006, from Yale-New Haven Teachers Institute Website: <http://www.cis.yale.edu/ynhti/curriculum/units/1980/7/80.07.11.x.html>

Name: _____



Date: _____

Pi WebQuest

Answer each of the following questions about pi after visiting the websites which have been listed to help you with your quest.

π Visit www.merriam-webster.com and search for pi in the online dictionary.

1. Pi is the ratio of the _____ of a circle to its _____.
2. What is the value of pi rounded to eight decimal places? _____

π Go to www.facade.com/legacy/amiinpi/ and enter your birthday (MMDDYY).

3. When is your birthday? _____
4. Where in pi does your birthday first occur? _____

π Go to <http://mathforum.org/t2t/faq/faq.pi.html> and read about Pi Day.

5. When is Pi Day celebrated? _____
6. Why was this date chosen? _____

π Look at the formulas listed on the following webpage:

http://www.wef.org/ScienceTechnologyResources/TechnicalInformation/ConversionConstantsChemicalInformationFormulas/basic_math.htm.

7. Record the two formulas which involve pi. _____
8. Why *don't* the other formulas on this page involve pi? _____

π Read the article found at <http://www.roanoke.com/news/roanoke/wb/wb/xp-69304> about Gaurav Raja, a high school boy who broke the North American and United States records for memorizing and reciting digits of pi.

9. How many digits of pi did Gaurav memorize and recite? _____

π Read about rational and irrational numbers at

<http://mathforum.org/dr.math/faq/faq.integers.html>.

10. Is pi a rational or an irrational number? _____

Provide a convincing argument for your answer.

Name(s): _____

Ancient Pi Discovery Worksheet

Work through the problems below in order to find an approximation of pi corresponding to each of the following five ancient civilizations.

_____ 1) Ancient Egyptians (1850 BC) used sophisticated mathematical concepts in their daily lives. For example, mathematics was very important for the Egyptians as they worked to build pyramids (which are recognized today for their incredible precision).

An Egyptian scribe named Ahmes (also known as Ahmose) is famous for his mathematical work, especially his work on the Rhind Mathematical Papyrus, a set of complex arithmetic and geometry problems. Ahmes is credited for writing the oldest known text which approximates pi, and the Egyptians will forever be remembered for their rule for finding the area of a circle.

According to Ahmes, the area of a circle with a diameter of 9 units is the same as the area of a square with a side of 8 units. Using our modern formulas for area, find the Egyptian approximation of pi.

_____ 2) In Old Babylonia (1800 BC), clay tablets were used for recording mathematics. Much like the ancient Egyptians, the people in Babylonia (for location, think modern-day Iraq) used complex mathematics to understand and enrich their lives. The Babylonians had an advanced number system, and they used mathematical concepts in the digging and maintenance of canals used for irrigation and transportation.

Use the following to calculate an Old Babylonian approximation of pi (and note that this method *may* have been used in Babylonia):

- Use a compass to draw a circle with a radius of 6 cm.
- Record the length of the diameter.
- Keep your compass open at a width of 6 cm and mark off **six** consecutive arcs on the circle. Connect the points on the circle to create an inscribed **hexagon**.
- Divide the perimeter of the hexagon that you drew by the diameter of your circle.

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_____ 3) Archimedes of Syracuse is one of the most well-known mathematicians of ancient Greece (240 BC) and of all time. Some of his important mathematical contributions are related to circles and an approximation for the ratio of the circumference to the diameter of a circle – pi! Other important math concepts attributed to ancient Greece deal with topics in geometry, conic sections, number theory, logical deduction, and formal proofs.

Archimedes began his approximation of pi by stating that the area of a circle must be somewhere in between the areas of its inscribed and circumscribed polygons. Consider a circle with a radius of 1 unit. Use this radius measure to calculate the area of an inscribed hexagon (inside the circle). **Hint: Use your knowledge of special triangles!** This is your lower bound. Now, calculate the area of a circumscribed hexagon (outside the circle). This is your upper bound. Now state the range in which the area of our circle must lie. Why is this range also a range for pi? **Hint: Don't forget the radius measure!** Keep in mind that this procedure can be repeated using polygons with more sides than a hexagon for a more precise answer.

* In China (260 AD), Liu Hui approximated pi ($\pi \approx 3.14159$) by repeating Archimedes' procedure using a polygon with 3072 sides. Can you imagine?!

* In India (500 AD), Aryabhata approximated pi ($\pi \approx 3.1416$) and may have been one of the first people to recognize that pi is an irrational number.

Student Project Guide

Your assignment is to work with a partner to create a cartoon, poster, song, story, or other approved medium to honor pi and the ancient civilizations we recognize for first using approximations of pi. All final products should also include a definition of pi.

Example: Create a poster which models a timeline for the development of pi throughout history.

You will be graded using the following rubric:

	High Level	Acceptable Level	Low Level
Accuracy	5 4 Students communicate their knowledge of pi and its history clearly/effectively and make few or no errors in content.	3 2 Students do not make incorrect statements about pi or its history but fail to convey a thorough understanding.	1 0 Students fail to define pi accurately and/or make blatantly incorrect statements about its historical significance.
Creativity	5 4 Students use an approved medium to capture an audience's attention and convey important information.	3 2 Students fail to capture an audience's full attention due to distracting use of medium or lack of overall direction.	1 0 Students struggle to apply their medium to the task at hand, use an unapproved medium, and/or fail to focus on the topic itself.
Overall Presentation	5 4 Students use their chosen medium as a means by which to express knowledge and understanding of pi and its history, meanwhile honoring the work of ancient civilizations.	3 2 Students are somewhat clear and effective in their attempts to honor pi and the mathematical works/culture of the studied ancient civilizations.	1 0 Students are not clear and/or effective in their overall effort to define and honor pi and its history through a creative medium. Work is not neat or accurate.

TEACHER NOTES/SOLUTIONS:

Part 1 (Pi WebQuest) Answers:

1. **circumference; diameter**
2. **3.14159265**
3. **answers will vary**
4. **answers will vary**
5. **March 14th (3/14)**
6. **3.14 is a common approximation of pi**
7. **area of a circle = πr^2 and volume of a cylinder = $\pi r^2 h$**
8. **the other formulas involve non-circular shapes**
9. **10,980**
10. **irrational; it cannot be expressed as a ratio of two integers, it has no exact decimal equivalent, etc.**

Part 2 (Ancient Pi Discovery Worksheet) Answers:

1. $8^2 = \pi*(9/2)^2 \rightarrow 64 = \pi*(4.5)^2 \rightarrow 3.160493827 \approx \pi$
2. $(6*6)/(6*2) = 3 \rightarrow 3 \approx \pi$
3. **area inscribed hexagon = lower bound = $6*(1/2)*b*h = 6*(1/2)*1*(\sqrt{3}/2) \approx 2.598$**
area circumscribed hexagon = upper bound = $6*(1/2)*(2/\sqrt{3})*1 \approx 3.464$
since $r = 1$ and area circle = πr^2 , we get area circle = π

*Note: See <http://www.ima.umn.edu/~arnold/graphics.html> for pictures!