

SENATE ELECTIONS WITH INDEPENDENT CANDIDATES

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ABSTRACT

Assuming strict two-party competition, policy balancing models of the US senate imply that senators from the same state will often be from opposite parties and have great ideological divergence. We analyze the effect of independent candidates on these implications. Our theoretical model implies the two state senators will generally not be from opposite parties and will be closer in ideological space than if they were elected under strict two-party competition. Empirical analysis of senate composition from 1991 to 2002 supports the theory.

KEY WORDS • independent candidate • policy balancing • US senate

1. Introduction

Spatial modeling is a standard method for analyzing candidate competition. The simple two-party general election model originally conceived by Hotelling (1929) and popularized by Downs (1957) has been extended to include primary elections (Wittman, 1990; Owen and Grofman, 1996), independent candidates (Palfrey, 1984; Osborne, 1993) and policy balancing across offices (Alesina and Rosenthal, 1995, 2000; Fiorina, 1996). In this article, we develop a model that incorporates all of these features to analyze U.S. senate elections.

Policy balancing is particularly suited to senate elections, because two politicians represent the same state and serve over lapping terms. Models of the senate with policy balancing and two-party competition predict that a state may have two ideologically divergent senators representing different parties in office at the same time (Alesina et al., 1991; Heckelman, 2000, 2004). In contrast, our main theoretical result is that the presence of an independent candidate generally implies that a state will have two senators from the *same* party in office at the same time. Although the senators will still hold different positions, the degree of divergence will not be as great as results from strict two-party competition. We

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test the theory by examining senate congressional sessions from 1991 to 2002. The empirical results are consistent with the notion that independent candidates have the expected effects on senate elections.

It is important to distinguish between policy balancing and strategic voting. Both require sophistication beyond simply comparing candidate positions to the voters' own bliss points. Policy balancing requires voters to calculate the average position of the sitting senator and a candidate, and compare this average to their own bliss points. There are two opportunities for voters to behave strategically in our model. Voters in the primary can behave strategically by not voting for their own preferred candidate who creates the best balance, but rather another candidate more likely to win the general election. (Such a candidate would still be preferable to the other party's nominee.) The entry of the independent into the general election allows for another type of strategic voting since there are now more than two candidates from which to choose. We first consider policy balancing with sincere voting and then policy balancing with strategic voting in the primary elections. Strategic voting tends to give the senate a more central composition.

2. Senate Elections with Policy Balancing

Our model is based on the features of the U.S. senate. There are two senators that represent a given state, and only one is elected at a time. There are two main political parties: the Democrats and the Republicans. Candidates must first win a primary election and then a general election to become a senator. In primary elections, voters can only vote for a candidate from their own party. Because we are mainly interested in the effects of the independent on the general election, we assume that there are two candidates in each primary election, and thus the party median voters are decisive (Wittman, 1990).

To simplify the exposition, we assume that there is a unit mass of voters with bliss points uniformly distributed on the interval $[0, 1]$. As shown in Appendix 2, we can relax this assumption without significantly changing the results as long as the bliss point distribution is symmetric, its tails are 'sufficiently fat', and the endpoints are 'sufficiently close' to the state median. The state median voter is located at $1/2$. The Democratic party is comprised of voters on the interval $[0, 1/2]$ and the Republican party is comprised of voters on the interval $(1/2, 1]$. Thus the party medians are $1/4$ and $3/4$ respectively, and the position of $1/2$ represents the common boundary. Following Wittman (1990), Owen and Grofman (1996), and Heckelman (2000, 2004), we limit Democratic candidates to the interval $[0, 1/2]$ and Republican candidates to the interval $[1/2, 1]$. This prevents potential criss-crossing. Using Alesina et al.'s (1991) terminology, we refer to the sitting senator (the one not up for re-election) as the 'anchor'. Let $A \in [0, 1]$ be the location of the anchor.

Following Alesina et al. (1991) and Heckelman (2000, 2004) we assume that voters wish to balance the policy positions of their two senators. Let $C \in [0, 1]$ be the location of a candidate for a senate seat. Voters do not simply evaluate the position of the candidate by himself, but rather consider the average of the position of the candidate and the anchor. For any given value of A , this balancing creates a new distribution, called the preferred position for a candidate. Let this distribution be denoted by \mathcal{W} . To determine \mathcal{W} , consider an arbitrary voter with bliss point $b \in [0, 1]$. The voter's preferred position for a candidate is found by solving

$$(P) \min_C \left[\frac{1}{2}(C + A) - b \right]^2 \text{ such that } 0 \leq C \leq 1.$$

The solution to this problem is

$$C^* = \begin{cases} 0 & \text{for } b \leq A/2 \\ 2b - A & \text{for } A/2 < b < 1/2 + A/2 \\ 1 & \text{for } b \geq 1/2 + A/2. \end{cases}$$

Thus \mathcal{W} has mass $A/2$ located at zero; mass $1/2 - A/2$ located at one, and the remaining mass of $1/2$ is uniformly distributed on $(0, 1)$.

An important feature of our model is that the voters do not anticipate (or otherwise take into account) the entry of the independent into the general election when voting in the primary election. Alternatively, one might consider a leader–leader–follower framework similar to Palfrey (1984) in which the entry of the independent candidate is anticipated. We cannot apply Palfrey's results to our model, however, because policy balancing leads to an asymmetric distribution of voters' bliss points. In fact, our conjecture is that an equilibrium of the type described by Palfrey does not exist in our model.

3. Sincere Voting

In this section we assume that voters are sincere. In other words, they vote for the candidate who is the closest to their preferred position. Let (D, R) be the location of Democratic and Republican primary winners. Because the party medians are decisive, (D, R) is determined by substituting $1/4$ and $3/4$ for b into problem (P). This yields

$$(D, R) = \begin{cases} (0, 3/2 - A) & \text{for } A \in [1/2, 1] \\ (1/2 - A, 1) & \text{for } A \in [0, 1/2]. \end{cases} \quad (1)$$

We model independent entry as a sequential game (Palfrey, 1984; Osborne, 1993). Because voters do not anticipate the entry of the independent candidate into the general election, we can use (1) to describe the results of the primary election. Given the location of the primary winners, the independent then announces her position Z in the general election.

Following the traditional approach to independent entry (Palfrey, 1984; Osborne, 1993; Hug, 1995) we assume the independent's objective is to maximize her vote share, given (D, R) . As shown by the following proposition, the independent will generally choose an endpoint location. The proof, and all subsequent proofs, are in Appendix 1.

PROPOSITION 1: *Suppose that voters are sincere and an independent enters the general election. Then the vote share maximizing location for the independent is*

$$Z = \begin{cases} 0 & \text{for } A \in (1/2, 1] \\ 1 & \text{for } A \in [0, 1/2) \\ \in [0, 1] & \text{for } A = 1/2. \end{cases}$$

Proposition 1 shows that when the anchor senator is a Republican (but not on the common boundary), the independent selects the left endpoint, which is the same position as the Democratic candidate. When the anchor senator is a Democrat (but not on the common boundary), the independent selects the right endpoint, which is the same location as the Republican candidate. When the anchor senator is on the common boundary between the parties, any location yields equal vote share for the independent.

Given the behavior of the independent, we can now specify who wins the election. It will not be the independent.¹

PROPOSITION 2: *Suppose that voters are sincere and an independent enters the general election. Then the winning senator and the anchor will be equidistant from one of the party medians. Furthermore, if $A \neq 1/2$, then the winning candidate will be from the same party as the anchor senator. If $A = 1/2$ then the Republican or Democratic candidate wins the election with equal ex ante probability.*

Several examples illustrate the implications of Proposition 1 and 2. Assume at first that the anchor is a Republican, but is not on the common boundary (so that $A > 1/2$). From (1), the Democratic candidate is at the left endpoint but the Republican candidate is at $3/2 - A$ which is strictly to the left of the right endpoint. Because of policy balancing, the majority of voters would prefer the Democrat's position. Indeed, if the independent does not enter the election, then the Democratic candidate would win. But if the independent candidate enters, by Proposition 1 the independent adopts the same position as the Democrat, that is, the left endpoint. The independent and the Democrat thus split what would have been the Democrat's vote share if the independent had not entered the race.

1. Shepsle (1991) questions whether independent candidates would enter elections they cannot win. Osborne (1993) suggests these candidates may treat a lost-cause election as an investment in future elections. Palfrey (1984) states it is rational for vote-maximizing candidates to enter the race.

Consequently, the Republican wins the election. The resulting senate has the anchor at A and the newly elected Republican at $3/2 - A$. This yields a balance that is optimal for the Republican party median. Now consider the case in which the anchor is a Republican, but is in fact on the common boundary (so that $A = 1/2$). In this case, the Democratic candidate is at the left endpoint and the Republican candidate is at the right endpoint. The independent is indifferent between any location, but the actual location selected determines the winner of the election. Assuming that all locations are equally likely to be selected, the Republican and Democrat have an equal ex ante probability of winning. If the Republican does indeed win, then the resulting balance is optimal for the Republican party median. If the Democrat wins, then the resulting balance is optimal for the Democratic party median.

Most studies of three-candidate elections predict unstable equilibrium and persistent cycling (Cox, 1987; Osborne, 1993). These results hold when candidates can costlessly change their announced positions to attract more votes. In our model, the positions of the primary winners are fixed, and it turns out that independent candidates, despite the freedom to choose any position, cannot win the general election. In Palfrey's (1984) model, the party candidates select positions equidistant from the state median and the independent candidate cannot win the election. The balancing model developed here differs from Palfrey in that voters' candidate preferences are not the same as their bliss points, and consequently are not symmetrically distributed along the line interval. Moreover, the party candidates do not anticipate the entry of the independent. As shown in (1), we find that the party nominees are not equidistant, and yet it is still true that an independent cannot win.

4. Strategic Voting

In this section, we allow a limited form of strategic voting. Although empirical evidence suggests strategic voting is only of minor concern (Southwell, 1991; Cherry and Kroll, 2003), it remains a natural theoretical consideration (Owen and Grofman, 1996; Swank, 2001; Heckelman, 2004).

One opportunity for strategic voting in our model occurs because there are multiple elections. Voters can behave strategically by considering not only the positions of the candidates in the primary election, but also the expected competition in the subsequent general election. In general, voters have an incentive to be strategic if altering their vote would make themselves better off by having a more preferred candidate win. If voters are unable to coordinate their strategies, then this will only occur when there is an expected tie between candidates.² The

2. In a probabilistic model, strategic behavior can occur as long as voters improve the expected probability of a favored outcome occurring. Since our model is deterministic, voters must be able to

Nash equilibrium in a two-candidate election predicts both candidates will converge to the same location (barring spatial limitations). The primaries in our model meet this condition³ and thus ties will always occur.⁴ As discussed earlier, the party medians are decisive in the primaries. With sincere voting, we determined a party median's preferred candidate as a function of the anchor's position. With strategic voting, we must account for both the anchor's position as well as the position of the other party's candidate to determine a party median's preferred candidate.

The second opportunity for strategic voting occurs in the general election because, after the unanticipated entry of the independent candidate, there are multiple candidates. In the general election, ties may or may not occur depending on the particular positions chosen by each candidate. Given candidate positions, in the event of an expected tie, voters who would otherwise prefer the least popular candidate will instead vote for the other candidate who is closer to their preferred position.

We analyze the case in which voters are strategic in the primary. We do not analyze the case in which voters are strategic in the general election because it is difficult to characterize the equilibrium under the assumptions of our model (there is a continuous distribution of voters and voters have asymmetric preferences). Such an analysis would require a different framework which we leave for future research.

The analysis of strategic voting in the primary follows the structure of the previous analysis. We identify the locations of the primary winners (D, R) and then determine the vote maximizing location for the independent given these locations. Let $\epsilon > 0$ be a small number. Heckelman (2004) shows the unique Nash equilibrium described by Table 1. We give an intuitive discussion of his result here.⁵

By symmetry, we need only consider values for $A \leq 1/2$. Suppose for the moment that $A < 1/4$. Then the Democratic candidate will lose the general election for any combination of (D, R) .⁶ Thus there is no motive for strategic voting in either primary, and the voters vote sincerely. As before, (D, R) is determined by (1). Now suppose $A \in [1/4, 1/2)$. According to Table 1, the Nash equilibrium

actually change the election winner to be better off; otherwise there is no advantage to voting in a non-sincere manner.

3. We also therefore assume candidates will correctly surmise if voters will behave sincerely or strategically.

4. Whatever method is used to break the tie is irrelevant in our model since the same policy position is chosen regardless of which individual candidate wins the party primary.

5. The equilibrium described by Heckelman is actually an ϵ -equilibrium. A proof of this is available from the authors.

6. Consider, for example, $A = 1/4 - \epsilon$. The state median voter's preference is located at $2(1/2) - 1/4 + \epsilon = 3/4 + \epsilon$. Therefore any Republican candidate will be closer to the state median than any Democratic candidate.

Table 1. Strategic Voting Nash Equilibrium

Range for A	(D, R)
$A \in (3/4, 1]$	$(0, 3/2 - A)$
$A \in (1/2, 3/4]$	$(3/2 - 2A + \varepsilon, 1/2)$
$A = 1/2$	$(1/2, 1/2)$
$A \in [1/4, 1/2)$	$(1/2, 3/2 - 2A - \varepsilon)$
$A \in [0, 1/4)$	$(1/2 - A, 1)$

is $(D, R) = (1/2, 3/2 - 2A - \varepsilon)$. In this equilibrium the Democrat receives vote share of $1/2 - \varepsilon/4$ and the Republican receives vote share equal to $1/2 + \varepsilon/4$. By constraint, the Democrat cannot move more to the right, and by moving to the left loses vote share. If the Republican moves to the left, he moves further from the Republican median's preferred candidate position. If the Republican moves to the right, the Democrat might win the election. Thus neither candidate has an incentive to move which entails a Nash equilibrium. The last case we need consider is $A = \frac{1}{2}$. Here, the state median desires an exact clone to win forcing both parties to adopt this position which is equivalent to the standard Median Voter Theorem result.

Given the location of the party candidates as shown in Table 1, the independent selects the vote-maximizing location as specified by the following Proposition.

PROPOSITION 3: *Suppose that voters are strategic in the primaries but sincere in the general election and an independent enters the general election. Then the vote-maximizing location for the independent is*

$$Z = \begin{cases} 0 & \text{for } A \in (3/4, 1] \\ 3/2 - 2A & \text{for } A \in (1/2, 3/4] \\ 1/2 \pm \varepsilon & \text{for } A = 1/2 \\ 3/2 - 2A & \text{for } A \in [1/4, 1/2) \\ 1 & \text{for } A \in [0, 1/4) \end{cases}$$

When the anchor senator is between a party endpoint and that party's median, the independent enters at the endpoint of the other party. When the anchor senator is between the party median and the state median, the independent enters just to the endpoint side of the other party's candidate. If the anchor senator is on the common boundary, the independent is indifferent between entering just to the right or left of the state median.

Given the behavior of the independent, we can now specify who wins the election. With strategic behavior in the primaries, it is possible for the independent to win, but only for the specific value of $A = 1/2$.

PROPOSITION 4: *Suppose that voters are strategic in the primaries but sincere in the general election and an independent enters the general election. For*

$A \neq 1/2$, the winning candidate will be from the same party as the anchor. For $A = 1/2$, the independent candidate wins the election.

Comparing the results in this section with the previous one reveals that the main difference between the strategic case and the sincere case is that the strategic case tends to make the senate have a more central composition. When the anchor senator is more extreme than either of the party medians, the strategic case collapses back to the sincere case. Otherwise, all candidates will hold positions more centrist than they would in the sincere case.⁷

5. Summarizing the Effect of an Independent

To isolate the effects of an independent candidate on senate elections, we first describe previous games of two-party senate elections and then compare to our results.

Consider first the model as described in Sections 2 and 3. Voters are sincere in the primary, but now assume that there is no independent candidate. The results of the election for any value of A are determined by Heckelman (2000). For $A \in (\frac{1}{2}, 1]$, we have $D=0$ is the election winner whereas for $A \in [0, \frac{1}{2})$ we have $R=1$ is the election winner. For $A = \frac{1}{2}$, either $D=0$ or $R=1$ wins by random decision of the indifferent median voter. Now suppose that voters are strategic in the primary, but again, there is no independent candidate. Heckelman (2004) describes the results in this case. For extreme values of A , the results are the same as with sincere voting. For $A \in [1/4, 1/2)$ we have $R = 3/2 - 2A - \epsilon$ wins the election. For $A \in (1/2, 3/4]$ we have $D = 3/2 - 2A + \epsilon$ wins the election. For $A = 1/2$, either $D = 1/2$ or $R = 1/2$ wins by random decision of the indifferent median voter.

Comparing these results to Propositions 2 and 4 reveals two strong theoretical predictions, regardless of whether the voters behave sincerely or strategically in the primary elections. The unexpected entry of an independent candidate into the general election (1) generally leads to less ideological divergence between the two senators, and (2) generally implies that the two senators will be from the same party, rather than from different parties. In the next section, we test the predictions with data from actual senate elections.

7. The one possible exception to this is when $A = 1/2$. While both the Democrat and Republican will definitely hold positions more centrist than if voters were sincere in the primaries, the independent chooses a position just to the left or right of the state median when voters are strategic but would randomly choose any position (other than the endpoints) if voters were sincere. Thus, it is theoretically possible for the independent to randomly choose the state median position and thus be more centrist than in the strategic case, but randomly choosing this exact location has probability of zero.

6. Empirical Analysis

To test our theoretical predictions, we need to determine whether an independent candidate entered a given senate election. Vote shares for all senate elections are taken from the Office of the Clerk of the U.S. House of Representatives website.⁸ We find that 145 of the 199 elections (73%) which took place in our sample, to be detailed later, included an independent party candidate.

We also distinguish between open primary states and closed primary states because Grofman and Brunell (2001) and Heckelman (2004) have shown that the type of primary is an important determinant of senate election outcomes. In a closed primary system, primary voters are clearly delineated and cannot vote in the opposition party's primary. Our theoretical model is akin to a closed primary system. Grofman and Brunell (2001) and Heckelman (2004) find that voters in closed primary states are more likely to elect senators from opposite parties and of greater ideological divergence compared to voters in open primary states, but they do not factor in the degree of third-party competition in their empirical formulations. We use Bott's (1990) classification scheme, presented in the Appendix to Grofman and Brunell (2001), to determine whether a state has an open or closed primary system.⁹ Louisiana is the only state with purely non-partisan elections. There are no primaries and senators are not identified by party in the general election. We therefore do not include Louisiana in our sample.

Following our theoretical predictions, we use two dependent variables to measure the divergence between senators from the same state. The first variable measures ideological divergence and the second measures party divergence. Our empirical representation takes the form of:

$$Y = \kappa + \alpha X + \beta V + \gamma XV + \omega,$$

where Y = a measure of the divergence between senators from the same state; X = a dummy variable for the presence of an independent in the most recent election; V = a dummy variable for closed primary; and ω is a random error term. We are particularly interested in the sign of $\alpha + \gamma$. The mean level of divergence in closed primary states without an independent is equal to $\kappa + \beta$, whereas the mean level of divergence in closed primary states with an independent is equal to $\kappa + \alpha + \beta + \gamma$. Because our model predicts less divergence following an election with an independent candidate (in these closed primary states), we expect $\alpha + \gamma < 0$.

We first describe the determination of, and estimation results for, the ideological divergence dependent variable. Ideological positioning is typically proxied by interest group scores, which rate each senator every year on how often he or

8. <http://clerk.house.gov/members/electionInfo/elections.html>.

9. Indiana was mistakenly left off their list but is included here as an open state (Bott, 1990).

she voted for or against the interest group's favored position on particular pending legislation. The most common interest group scores utilized by academics are those from the Americans for Democratic Action (ADA) and American Conservative Union (ACU). Both groups rate senators on a 0–100 score which corresponds to the unit line interval in our theoretical model. We use the ACU scores because they hold two distinct advantages over the ADA. First, ADA scores strictly measure the percentage of votes in favor of the ADA's position, treating nonvoting or absences the same as a vote against, whereas ACU considers only actual votes in favor or against, treating nonvoting or absences as neutral. Second, Poole and Rosenthal (1997) have shown ADA scores are subject to 'folding' in that the highest score does not actually represent the most extreme position. This also underscores a slight problem in using differences in interest group scores as a proxy for ideological divergence. The two senators could vote opposite each other every time, and still favor the interest group's position half the time. Then both senators would be given identical ratings of 50 which properly classifies both as moderates, but also as having no divergence. Thus, differences in the interest group ratings represent a lower bound on ideological divergence.

Our sample covers the six Congressional sessions of 1991–2 through 2001–2, so that each state has four or five elections (beginning in either 1990 or 1988) from which to observe an effect.¹⁰ Because senators serve overlapping six-year terms, each election will determine the pair of sitting senators of that state for either the next or next two Congressional sessions. Our unit of observation is the difference in the state's two senators' interest group rating averaged for the two years of each particular congressional session. This leaves us with 293 observations, as determined by 199 distinct elections. Differences in ACU ratings for each state's two senators theoretically range from 0 to 100. While no calculated differences topped out at 100, 13 of the 293 observations had the minimal value of 0. Therefore, we estimated the coefficients by Tobit analysis, with censored data at zero.

Estimated coefficients are presented in the first column of Table 2. The sum of the coefficients ($\alpha + \gamma$) on the independent dummy and the interaction term is negative as predicted and a Wald test confirms the statistical significance of this finding. In states with closed primaries, senators elected when challenged by independents are less ideologically divergent from the anchor than senators elected under strict two-party competition. This supports the first theoretical prediction.

We now turn to the party divergence dependent variable. This variable equals 1 if both a Republican and Democrat are currently in office, and 0 if either party

10. Georgia held a special non-partisan election in 2000, using a blanket ballot system similar to Louisiana, in which candidates were not identified by party, and no primary elections were held prior to this election. Thus, we do not include the 2001–2 interest group scores for Georgia in the sample.

Table 2. Estimation Results

Dependent Variable	Difference in Senators' Interest Group Ratings	Senators from Different parties (= 1) or Same Party (= 0)
Constant (κ)	26.04* (4.31)	-0.43* (0.18)
Independent (α)	2.87 (5.09)	0.18 (0.21)
Closed Primary (β)	23.48* (7.75)	0.64* (0.31)
Independent X Closed Primary (γ)	-19.85* (8.96)	-0.75* (0.36)
Wald test statistic ($\alpha + \gamma < 0$)	5.32*	3.73*
Mean, dependent variable	31.80	0.39
Regression standard error	30.80	0.49

Standard error in parenthesis: * designates $p \leq .05$.

controls both senate seats. To best compare the results, we match the sample observations to the ideological divergence variable. Of the 293 congressional session observations under consideration, 115 (39%) are simultaneously split between the two parties for a given state. Probit estimates are in the last column of Table 2. The results largely conform to the ideological divergence results. We again find $\alpha + \gamma$ is negative and is statistically significant, indicating here that the winner of a senate election who faced third-party competition is less likely to be of a different party than the anchor compared to the winner of a senate election without any independent entry. This result supports the second theoretical prediction.¹¹

One caveat to our results is that we only consider whether or not there is an independent in the election. We do not directly analyze the location of the independent. A stronger test of the theory would determine if the independent's location was nearer the candidate from the opposite party as the anchor. Unfortunately, the equivalent of ACU scores for unelected candidates is not readily available.

11. Our model only considers the impact of entry by a single independent, which is the common approach in the spatial literature. Empirically, there are often multiple independents, and our theoretical results might differ depending on how many independents are modeled. For our sample, we find that only 46 of the elections had exactly one independent party candidate on the ballot. If we limited our sample to strict two-party vs three-party candidate elections to directly match the model assumptions (discarding all Congressional sessions where the most recent winner came from an election with more than three potential candidates), the sample is limited to only 145 total observations, and several states are not included at all. Utilizing the smaller sample still results in the summation of $\alpha + \gamma$ being negative for both regressions, but is not statistically significant for either.

7. Conclusion

A standard insight from the policy-balancing literature is that senators from the same state will often be from opposite parties and ideologically divergent. Our results show that an independent candidate entering the election alters this conclusion. In our model with independent entry, policy balancing does not usually lead to different parties holding office at the same time, and the two elected senators are not as divergent. With sincere voting in the primary, only when the anchor senator is on the boundary between the parties is it possible for the senators to be from opposite parties. With strategic voting in the primary, it is not possible for senators to be from opposite parties. In the case of independent victory, the independent will essentially match the position of the anchor senator. Thus, we find that independent entry into the general election will tend to result in the election of the candidate who is of the same party as the anchor, and closer ideologically who would have been elected without independent entry. Our empirical tests covering congressional sessions from 1991–2 through 2001–2 are consistent with these predictions.

Appendix 1

Proof of Proposition 1: Suppose that $A \in (1/2, 1]$. Then $(D, R) = (0, 3/2 - A)$. Before the independent enters, the indifferent voter's preferred position \tilde{b} is halfway between D and R . We have $\tilde{b} = 3/4 - A/2$. The vote share for D , V_D , is the mass from $[0, \tilde{b}]$:

$$V_D = A/2 + 1/2(3/4 - A/2) = 3/8 + A/4. \quad (2)$$

Likewise, the vote share for R , V_R , is the mass from $(\tilde{b}, 1]$:

$$V_R = 1/2 - A/2 + 1/2(1 - (3/4 - A/2)) = 5/8 - A/4. \quad (3)$$

Table A1 presents the range of possible choices for Z and the corresponding vote shares.

Given the permissible values of A , $Z=0$ yields the largest vote share. For $A=1/2$, all the cases in Table A1 (except the last one, which is no longer feasible since $R=1$) yield the same vote share of $1/4$.

The results for $A \in [0, 1/2)$ follow by logic similar to the case $A \in (1/2, 1]$.

Proof of Proposition 2: Again consider first the case $A \in (1/2, 1]$. From proof of Proposition 1, independent selects $Z=0$. Thus $V_Z = V_D = 3/16 + A/8$, leaving $V_R = 5/8 - A/4$ for $R=3/2 - A$. Thus $V_R > V_D$ so R wins. Note that

Table A1. Vote shares under sincere voting.

Z	Vote Share	Explanation
$Z = D = 0$	$3/16 + A/8$	One half of (2)
$0 < Z < R$	$1/2(1/2(3/2 - A)) = 3/8 - A/4$	One half of mass between (0, R)
$Z = R$	$5/16 - A/8$	One half of (3)
$Z = R + \epsilon$	$1/4 - \epsilon/4$	Mass at one plus mass between ($R + \epsilon/2$, 1)

$(R + A)/2 = 3/4$ which is the location of the Republican median. By similar logic, if $A \in [0, 1/2)$ then $D = 1/2 - A$ wins. In this case, $(D + A)/2 = 1/4$ which is the location of the Democratic median. Finally, if $A = 1/2$ then either $R = 1$ or $D = 0$ wins with equal probability, and $(R + A)/2 = 3/4$, and $(D + A)/2 = 1/4$.

Proof of Proposition 3: For $A \in [0, 1/4)$, no Democratic candidate can win the general election. For $A \in (3/4, 1]$, no Republican candidate can win the general election. So there is no motive for strategic voting in these cases, and the results follow from Proposition 1.

Now consider the case $A \in (1/2, 3/4]$. From Table 1, we have $(D, R) = (3/2 - 2A + \epsilon, 1/2)$. Before the independent enters, the vote shares are $V_D = 1/2 + \epsilon/4$ and $V_R = 1/2 - \epsilon/4$. Table A2 presents the range of possible choices for Z and the corresponding vote shares. The independent gets the greatest vote share by selecting $Z = D - \epsilon = 3/2 - 2A$. The result for $A \in [1/4, 1/2)$ follows a similar logic and yields the result that $Z = R + \epsilon = 3/2 - 2A$.

Finally, consider the case $A = 1/2$. Since in this special case voters' candidate preferences are symmetrically distributed and $(D, R) = (1/2, 1/2)$, Cox's (1987) results can be applied. The optimal location for the independent is plus or minus ϵ from $1/2$. The independent candidate gets vote share $1/2 - \epsilon/4$ while the other candidates split the remaining vote share. Thus the independent candidate wins.

Proof of Proposition 4: For $A = 1/2$ we have $(D, R) = (1/2, 1/2)$ and $Z = 1/2 \pm \epsilon$. The independent candidate gets vote share $1/2 - \epsilon/4$ while the other candidates split the remaining vote share. Thus the independent candidate wins.

For $A \in (1/2, 3/4]$ we have $(D, R) = (3/2 - 2A + \epsilon, 1/2)$ and $Z = 3/2 - 2A$. Because the independent located just to the left of the Democrat, the Democrat gets the smallest vote share. We have $V_Z = 3/4 - A/2 + \epsilon/4$ and $V_R = 1/2 - \epsilon/4$. For this range of A , it follows that $V_R > V_Z$.

For $A \in (3/4, 1]$ the results follow from Proposition 2 because the strategic case is the same as the sincere case.

Table A2. Vote shares under strategic voting.

Z	Vote Share	Explanation
$Z = D - \varepsilon$	$3/4 - A/2 + \varepsilon/4$	Mass at zero plus mass from $(0, D - \varepsilon/2)$
$Z = D$	$1/4 + \varepsilon/8$	One half of V_D
$D < Z < R$	less than $1/4$	One half of mass between D and R
$Z = R$	$1/4 - \varepsilon/8$	One half of V_R
$Z = R + \varepsilon$	$3/4 - A/2 - \varepsilon/4$	Mass at one plus mass from $(1/2 + \varepsilon/2, 1)$

The cases for $A \leq 1/2$ follow by a similar logic.

Appendix 2: The Distribution of Voters

We now consider the effect of relaxing our assumption that bliss points are uniformly distributed. Suppose we have a symmetric distribution of bliss points in which the tails are ‘sufficiently fat’ and the endpoints are ‘sufficiently close’ to the state median. To describe these conditions more formally, it is useful to consider a range of bliss points on the interval $[-E, E]$. Let F be the cumulative distribution function. Let m denote the location of the Republican party median (in other words $F(m) = 3/4$.) Let ϕ be the location in the Republican party such that $2/3$ of voters are to the left (so that $F(\phi) = 2/3$). Assume that the bliss point distribution is continuous, symmetric, non-increasing on $[0, E]$ and non-decreasing on $[-E, 0]$. Also assume that $E < 4\phi$ and $m < 2\phi$.

Consider an example in which $E = 1$. Let the density function for bliss points be described by

$$f(x) = \begin{cases} 0 & \text{for } x < -1 \\ h + (1 - 2h)(x + 1) & \text{for } -1 \leq x \leq 0 \\ (1 - h) - (1 - 2h)x & \text{for } 0 \leq x \leq 1 \\ 0 & \text{for } x > 1. \end{cases}$$

Note that if $h = 1/2$, then the distribution is uniform. It is easy to verify that the new assumptions are satisfied for $\frac{1}{2} \geq h > \frac{5}{18}$.

The proofs of the analogs to Propositions 1 to 4 are more tedious under the new assumptions, so we merely summarize the results.¹² Consider first the sincere voting case. In Proposition 1, when the anchor is located exactly at the state median position, then any location between the two candidates maximizes the independent’s vote share. Under the new set of assumptions, when the anchor is located ‘near’ the state median, then any location between the two candidates

12. The proofs of the propositions are available upon request from the authors.

maximizes the independent's vote share. Otherwise, the results of Propositions 1 and 2 carry over directly to the more general case. For example, given that the anchor is far enough to the right of the state median, the independent selects the same position as the Democratic candidate, and hence the Republican candidate wins the election.

The results for the strategic case (Propositions 3 and 4), are essentially the same under the new set of assumptions. For extreme values of the anchor, the strategic case collapses into the sincere case. For values of the anchor closer to the state median, the independent candidate enters, say, just to the left of the Democratic candidate, and the Republican candidate wins the election. When the anchor is exactly at the state median, the independent enters just to the left or the right of the anchor, and wins the election.

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