

Rent-setting in multiple winner rent-seeking contests

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Abstract

In this paper, we analyze a multiple winner rent-seeking contest where the number of winners is set by a self-interested regulator. The winners receive a license to compete in a market. The structure of competition in the market influences the number of winners through the preferences of the regulator. The model indicates that Cournot competitors can be better off than firms that are able to collude on output determination. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

A rent-seeking contest can take the form of interest groups competing to receive benefits from a regulator. Analyses of rent-seeking contests generally assume that a single group will win the contest and receive all of the benefits.¹ This assumption is, however, inconsistent with many regulated markets. For example, local governments often issue more than one license to operate a taxi cab company.

Berry (1993) and Clark and Riis (1996) have analyzed rent-seeking contests where there are multiple winners. In their models, the regulator determines which firms win the contest, but the number of winners is exogenous. If the number of

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¹ See the surveys by Nitzan (1994) and Tollison (1997).

winners is a political decision, these models are an incomplete description of multiple winner rent-seeking contests.

In his survey paper, Nitzan (1994) stresses that it is important to understand how political decisions about the structure of rent-seeking contests are determined. Appelbaum and Katz (1987) analyze such decisions in single winner rent-seeking contests. They view a contest regulator as playing a dual role as rent-setter and rent-seeker. The regulator sets the level of rent that the winning group receives. At the same time, the regulator seeks rent by choosing the level of rent that is consistent with his or her own interests. In the case of multiple winner contests, a regulator also has an opportunity to seek rent by determining the number of winners. This paper analyzes such a multiple winner rent-seeking contest in which the number of winners is determined endogenously by a self-interested regulator.

Our model supposes a market where there is entry regulation, but not price regulation. Symmetric firms seek rent by competing for licenses to operate in the market. Firms that secure licenses then compete in the market by selecting an output level. The structure of competition in the output market and the number of licenses determine the welfare of consumers. For example, firms that receive licenses might collude on their output decisions. Then consumers' welfare is constant with respect to the number of licenses, because the firms act as if they are a monopolist regardless of the number of firms obtaining licenses.

Following Appelbaum and Katz (1987), we assume that the regulator's utility depends on total rent-seeking expenditures of firms and the welfare of consumers. We show that, under weak assumptions, total rent-seeking expenditures of firms are maximized when a single license is awarded. We use this result to derive the preferred number of licenses for the regulator under various assumptions about the structure of competition in the output market. If firms that win licenses are expected to collude on their output decisions, then the preferred choice for the regulator is one license. If, however, the winners are expected to behave as Cournot competitors, then the regulator may allow more than one firm in the market.

A standard result in the industrial organization literature is that, in a symmetric model, colluding firms have more total industry profit than Cournot competitors. This result does not necessarily hold in our model because firms expend resources on rent-seeking, and in addition, the regulator determines the number of firms in the industry based on the firms' expected rent-seeking behavior. We derive a condition on industry profit before rent-seeking that implies industry profit after rent-seeking is minimized when one license is issued. When this condition is satisfied, Cournot firms are at least as well off as their colluding counterparts, because the regulator will not allow more than one license in a market that would be comprised of colluding firms.

Many of our results require restrictions on the function that determines the value of the rent earned by firms that are given licenses. To illustrate the results, we analyze a market characterized by linear demand and constant marginal

production costs. Under these standard conditions, all of the restrictions that we require for our results are satisfied. We also show that as the number of rent-seeking firms becomes large, Cournot competitors become unambiguously better off than colluding firms.

Our analysis complements the literature on market entry games. The key characteristic of our model is the endogenous selection of the number of firms that are allowed to enter the market. This number cannot be determined without specifying the subsequent competition that occurs between firms in the market. One type of market entry game similar to ours focuses on *unregulated* markets with fixed entry costs. In these games, the structure of competition between firms plays a role similar to that in our model (see Dixit and Shapiro, 1986; Nti, 1989, 2000).

2. Rent-setting and the structure of competition

Our model has firms, consumers and a regulator. There are N symmetric risk-neutral firms that compete for k licenses to operate in the market. (We assume that $k \leq N$.) The regulator determines k as well as the allocation of licenses to the firms. A firm that is selected to receive a license earns a “rent”, which exceeds the return in a competitive market. The value of the rent is determined by the number of licenses and the structure of competition. To keep the notation simple, we use $V(k)$ to denote the value of the rent to a firm as a function of k . Firms seek this rent by expending resources to lobby the regulator. The regulator sets the level of rent through the choice of k . The welfare of consumers is determined by the number of licenses and the structure of competition. We use $C(k)$ to denote consumers’ surplus as a function of k .

We adopt a leader–follower framework in which the regulator first announces the number of licenses to be made available. Then, each firm determines the resources to devote to rent-seeking, given the number of available licenses. We have a simple model of complete information. Firms’ reaction functions are known by the regulator when he or she selects the number of licenses to maximize utility. The game is solved recursively by the regulator, given the firms’ reaction functions.

2.1. Firm behavior in rent-seeking

Firms’ rent-seeking decisions are analyzed as a non-cooperative game. Let z be an N dimensional vector of rent-seeking expenditures, where z_i corresponds to the expenditure of the i th firm. Firm i solves the problem of maximizing its expected profit, which we represent as

$$\max P_i(z, k) V(k) - z_i, \quad (1)$$

where $P_i(z, k)$ is the probability that firm i receives one of the licenses as a function of the number of available licenses and the vector of rent-seeking expenditures. Licenses are awarded to firms in a multi-round lottery procedure, as described by Clark and Riis (1996). The probability that a firm receives a license in the first round is equal to its individual expenditure divided by the total rent-seeking expenditures. That is, the first round is a standard contest success function as described by Tullock (1980). The first round winner is then removed from the lottery and the process is repeated for $k - 1$ subsequent rounds. (There is no additional rent-seeking between rounds.) The probability of winning in subsequent rounds is equal to a firm's individual expenditures divided by the sum of expenditures of the remaining firms.

Let the total rent-seeking expenditures in the symmetric Nash equilibrium solution to the rent-seeking game be given by $Z(N, k)$. Under the assumption that the number of licenses is an exogenous parameter, it has been shown by Clark and Riis that

$$Z(N, k) = \begin{cases} V(k) \left[\frac{N - 1}{N} \right] & \text{for } k = 1 \\ V(k) \left[\frac{k(N - 1)}{N} - \sum_{j=1}^{k-1} \frac{k - j}{N - j} \right] & \text{for } k > 1. \end{cases} \tag{2}$$

With a single license, we have Tullock's (1980) familiar result for total rent-seeking expenditures. The summation term in $Z(N, k)$ is a direct consequence of the expression for the probability of winning in a multi-round lottery. Clark and Riis note that it is difficult to state general comparative statics results for k . The sign of the derivative of $Z(N, k)$ with respect to k can vary depending on the specific functional form for $V(k)$.

In our model, k is not an exogenous parameter, but rather a decision variable of the regulator. We can use the Clark and Riis result, however, as a convenient summary of the firms' reaction functions. For any choice of k , total rent-seeking expenditures by firms is given by Eq. (2). Although we have not yet discussed the utility function for the regulator, it will turn out to be useful to have identified the strict global maximum of with respect to k . This may seem at first to be difficult because we cannot determine, in general, the sign of the derivative of $Z(N, k)$ with respect to k . The following proposition shows, however, that we can indeed describe the strict global maximum provided we make one rather weak additional assumption.

Proposition 1. *Suppose that $V(1) \geq V(k)k$. Then $k = 1$ is a strict global maximum of $Z(N, k)$.*

The proof of this proposition and all subsequent proofs are in Appendix A. The assumption in Proposition 1 that $V(1) \geq V(k)k$ appears reasonable for most

market structures. The value $V(1)$ represents the rent earned in a market when only a single firm is permitted to operate. The value $V(k)k$ represents the total rent earned in a market when k firms are permitted to operate. We expect a single firm to generate at least as much rent as the total rent earned by k firms. Even if the k firms are able collude when they make their output decisions, their best course of action is to duplicate the choices that the single firm would make. Any competitive interaction between the k firms makes them less effective at earning rent than the single firm.

Proposition 1 shows that, under a reasonable assumption about rents, rent-seeking expenditures are maximized when the regulator gives only one firm a license to operate in the market.²

2.2. Considerations of the regulator

The regulation literature has often assumed that regulators are benevolent bureaucrats who seek only to maximize the consumers' welfare. Alternatively, bureaucrats and politicians might be represented as wealth-maximizers. Buchanan (1989) proposes a view that is between the two extremes.³

We consider a utility-maximizing regulator who is concerned with consumers' welfare and personal pecuniary interests. The regulator's utility function is given by

$$U(C(k), Z(N, k)) = \alpha C(k) + (1 - \alpha) Z(N, k),$$

where α is a parameter bounded by 0 and 1. As in Appelbaum and Katz (1987), the regulator's utility is an increasing function of both firms' rent-seeking expenditures and consumers' welfare.⁴

Suppose that the regulator's utility function did not depend on $C(k)$. The regulator then derives utility only from rent-seeking expenditures. From Proposition 1, it is clear that such a regulator would not have an incentive to offer more than one license, since rent-seeking expenditures are maximized when the firms compete to capture a single license. Given, however, that we posit that the regulator has at least some concern about consumers' welfare, Proposition 1 is not sufficient to specify the number of licenses chosen by the regulator. We therefore

² This result has also been observed in slightly different contexts by Clark and Riis (1998) and Glazer and Hassin (1988).

³ See Buchanan (1989, p. 31).

⁴ In their model, regulators were concerned with satisfying consumers only to retain a position of power to secure future rents. We allow for the possibility that the regulator has an independent interest in consumers' welfare.

also consider the structure of competition in the market, for it is this structure that determines consumers' welfare.

Let k^* denote the regulator's preferred choice of k . We first consider a market that consists of firms that can collude, so that the regulator expects that licensed firms will subsequently be able to behave like a monopolist when making their output decisions. It follows that consumers' surplus does not increase as the number of firms allowed in the market increases. In our notation, these ideas are represented by

$$V(1) = V(k)k, \quad (3)$$

and

$$C(k) = C(k') \quad \text{for any } k, k'. \quad (4)$$

The next proposition shows that the regulator allows only one license in such a market.

Proposition 2. *Suppose Eqs. (3) and (4) are satisfied. Then $k^* = 1$.*

Now consider a market in which there is some competitive interaction between firms that receive licenses. In this case, the preferred number of licenses may be greater than one. Increasing the number of licenses reduces rent-seeking expenditures but increases consumer surplus. The preferred number of licenses for the regulator depends on the trade-off between these values. Since rent-seeking expenditures are affected by the number of rent-seeking firms, the preferred number of licenses also depends on N .

To see the effects on k^* , suppose that the regulator can choose a continuous, rather than discrete, number of licenses. Standard comparative statics arguments indicate that if $\partial Z(N, k)/\partial k$ is negative, then k^* is increasing in α . It follows that, in the original discrete problem, if $\partial Z(N, k)/\partial k$ is negative, then k^* is non-decreasing in α . Proposition 1 states conditions under which rent-seeking expenditures are maximized at $k = 1$. These conditions do not imply, however, that the partial derivative of the rent-seeking expenditures is negative. Further restrictions on the rent value function are required. We give a formal statement of such restrictions below in Proposition 3.

Likewise, standard comparative statics arguments indicate that if $\partial^2 Z(N, k)/\partial k \partial N$ is positive, then k^* is non-decreasing in N . Proposition 3 places restrictions on the rent-value function such that the cross-partial derivative of the rent-seeking expenditures is positive.

Proposition 3. *Suppose that the rent-value function satisfies (for $k < N$)*

$$\frac{V(k) - V(k+1)}{V(k+1)} > \frac{1}{k}, \quad (5)$$

then k^* is a non-decreasing function of α . Furthermore, suppose that the rent value function satisfies (for $k < N$)

$$\frac{V(k) - V(k+1)}{V(k+1)} < \gamma, \quad (6)$$

where

$$\gamma = \frac{\frac{1}{N-k} - \frac{1}{N+1}}{\sum_{j=0}^{k-1} \frac{1}{N-j} - \frac{1}{N+1}}.$$

Then k^* is an non-decreasing function of N .

The restrictions on the rent value function in Eqs. (5) and (6) are in terms of a lower and upper bound on the expression:

$$\frac{V(k) - V(k+1)}{V(k+1)}.$$

We can interpret this expression as the relative decrease in the rent value function. It is a measure of the effect of increasing the number of firms that are awarded licenses on the rent earned by any firm that receives a license.

Because Eqs. (5) and (6) utilize bounds in opposite directions, it is useful to verify that they are not mutually exclusive. In Appendix A, we show that $\gamma > 2/k$. Thus, the two equations may be satisfied simultaneously. In the next section, we observe that both requirements are indeed satisfied in a simple linear demand, constant cost industry example with Cournot competitors.

Additional insight into the comparative statics result for α can be obtained by re-writing Eq. (5) as

$$V(k)k > V(k+1)(k+1).$$

This equation indicates that total rent is decreasing in k . If this equation is satisfied, then, as the number of firms increases, competitive interaction increases, and total rent decreases. Proposition 3 shows that total rent decreasing in k is a sufficient condition for k^* to be non-decreasing in α .

Taken together, Propositions 2 and 3 show that the structure of competition in the regulated market influences the outcome of the rent-setting decision through the preferences of the regulator.

2.3. Implications for firm's profits

It is interesting to consider the profits of firms as a function of k^* . We use total expected profit as a measure of firms' collective benefits because the equilibrium

in the rent-seeking game is symmetric. Let $\Pi(N, k)$ be total expected profit. An individual firm's expected profit is equal to expected rent minus the firm's rent-seeking expenditures, and rent-seeking expenditures are at a maximum when $k^* = 1$. This suggests that firms would prefer to have $k^* > 1$, provided that the total rent $V(k)k$ does not decrease "too fast" when k increases. The following proposition makes this observation precise.

Proposition 4. *If the rent value function satisfies (for $k < N$)*

$$\frac{V(k) - V(k+1)}{V(k+1)} < \frac{2}{k}, \quad (7)$$

then $k^ = 1$ is a strict global minimum of total expected profit.*

If Eq. (7) is satisfied, Proposition 4 implies that $k^* = 1$ is the *worst* possible outcome from the point of view of the firms. Once again we express the critical condition in terms of a bound on the relative decrease in the rent value function. Recall that total rent is decreasing in k if and only if the relative decrease in the rent value function is greater than $1/k$. Eq. (7) requires that the relative decrease in the rent value function be *less* than $2/k$. This confirms our intuition that as long as the total rent does not decrease too fast, firms prefer that more than one license be issued by the regulator. If firms that receive licenses are able to collude, Eq. (7) is obviously satisfied because $V(1) = V(k)k$. In the next section, we will see that Eq. (7) is also satisfied in a linear demand, constant cost industry example with Cournot competitors.

We can now compare the firms' profits under different market structures. If firms can collude, then, as derived above, the regulator selects $k^* = 1$ and total industry profit is $\Pi(N, 1)$. Conversely, suppose that firms are Cournot competitors. Since increases in the number of firms in the market increase consumer surplus, the regulator may select $k^* > 1$. Cournot competitors then earn total industry profit $\Pi(N, k^*)$. If Eq. (7) holds, then by Proposition 4, the Cournot firms are at least as well off as their colluding counterparts. They are strictly better off when Eq. (7) holds and the regulator selects $k^* > 1$. This result reverses the normal advantage that collusion on output offers to firms.⁵

⁵ The result that collusion may sometimes make firms worse off has also been observed, in another context, by Dixit and Shapiro (1986). They consider an entry game for an unregulated market, and show that (partial) collusion harms incumbent firms in a contestable market when future entry is probabilistic. In their model, collusion increases the number of firms that the market can profitably sustain, and therefore increases the probability of future entry for each subsequent period, reducing the present value of incumbents' future profit stream.

2.4. The extent of rent dissipation

A primary focus of the rent-seeking literature is the extent of rent dissipation, defined as the ratio of total rent-seeking expenditures to total available rent (Tullock, 1980; Mueller, 1989). In multiple winner rent-seeking contests, total available rent is $V(k)k$. Rent is fully dissipated if $Z(\infty, k) = V(k)k$. The extent of rent dissipation in both single winner and multiple winner contests may depend on a variety of factors, including the risk attitude of firms, the heterogeneity of firms, and the probability function for selecting winning firms as a function of rent-seeking expenditures (Tullock, 1980; Rice and Ulen, 1981; Berry, 1993; Nitzan, 1994; Konrad and Schlesinger, 1997).

The extent of rent dissipation is ambiguous in our model. The number of licenses, and hence their value, is determined endogenously by the regulator. We distinguish between the total potential rent, $V(1)$, and the total available rent $V(k^*)k$. Since it will generally be the case that $V(1) \geq V(k)k$, we interpret potential rent as the absolute upper bound on the expenditures from rent-seeking. Total available rent represents the effective upper bound corresponding to the actual choice of the regulator. Since in our model firms are symmetric, risk-neutral, and the probability function for selecting winners is the same as in Clark and Riis (1996), total available rent is fully dissipated.⁶ The total potential rent is not fully dissipated unless $k^* = 1$. In this case, the total potential rent is made available by the regulator, and then dissipated by the firms through competitive rent-seeking activities.

3. A linear demand, constant cost example

In this section we present a special case to illustrate our general results derived above. Let there be k Cournot competitors in a market with linear demand, constant marginal production costs, and zero fixed costs. Demand is given by $P = a - bQ$ where a and b are positive constants. Let m be marginal production costs. In the Nash equilibrium solution for firms' output choice, profits per firm (before rent-seeking) are

$$V(k) = \frac{(a - m)^2}{b} \frac{1}{(k + 1)^2},$$

⁶ Berry (1993) uses an alternative probability function in which only the first prize is awarded by comparing rent-seeking contributions; subsequent prizes are randomly distributed. With this probability function, total available rent is not fully dissipated.

and consumers' surplus is

$$C(k) = \frac{(a - m)^2}{2b} \frac{k^2}{(k + 1)^2}.$$

We have proposed that $V(1) \geq V(k)k$ is consistent with most market structures. The condition is satisfied in this example. The relative decrease in the rent value function is

$$\frac{V(k) - V(k + 1)}{V(k + 1)} = \frac{2k + 3}{(k + 1)^2}.$$

It is easy to verify that

$$\frac{2k + 3}{(k + 1)^2} < \frac{2}{k}.$$

It follows that Eq. (7) is also satisfied in this example. Hence, we conclude that industry profits (after rent-seeking) are at a minimum if there is only one license.

It is difficult to obtain an analytical solution to the regulator's utility maximization problem because of the summation term in $Z(N, k)$. It is straightforward, however, to calculate the regulator's utility for a given k and N . Since k is bounded by N , a simple computer program that performs an exhaustive search over all k 's can be used to determine the preferred number of licenses for the regulator. This allows us to capture the essential insight of the regulator's problem without specifying the analytical solution. The results of the exhaustive search for a variety of values of the parameters α and k are shown in Table 1.

The rightmost column in Table 1 shows that when the regulator's utility is determined only by rent-seeking expenditures, the preferred choice is to issue only

Table 1
Regulator's preferred number of licenses as a function of α and N

N	α				
	1	3/4	1/2	1/4	0
1	1	1	1	1	1
2	2	2	1	1	1
3	3	3	2	1	1
5	5	5	3	1	1
10	10	10	5	1	1
20	20	20	9	2	1
100	100	98	32	2	1
1000	1000	978	165	2	1
3000	3000	2935	354	2	1
10000	10000	9783	808	2	1

one license. As shown in the $\alpha = 1$ column in Table 1, when the regulator's utility is determined only by consumers' surplus, the preferred choice is to issue a license to each firm. In the comparative statics results, it is easy to verify that

$$\frac{2k + 3}{(k + 1)^2} > \frac{1}{k}.$$

It follows that Eq. (5) is satisfied. Furthermore, since

$$\frac{2k + 3}{(k + 1)^2} < \frac{2}{k},$$

and $\gamma > 2/k$, we know that Eq. (6) is satisfied. Thus, from Proposition 3, k^* is a non-decreasing function of N and α , as can be seen in Table 1.

We can compare the profits of the Cournot firms in this example to a corresponding set of firms that can collude. Recall from Section 2.2 that the regulator selects $k^* = 1$ in the case where firms would subsequently collude in production. Since the conditions of Proposition 4 hold, Cournot firms are better off than the colluding firms when $k^* > 1$. Table 1 illustrates some of the combinations of the parameters that yield this result.

Table 1 also illustrates a somewhat blunt knife-edge result. When $\alpha = 1/4$, the regulator only allows one or two licenses, even when the number of rent-seeking firms is quite large. Conversely, when $\alpha = 3/4$, the regulator allows almost as many licenses as the number of rent-seeking firms. To understand the intuition behind this result, suppose that the first derivative of $C(k)$ is exactly equal to the opposite of the partial derivative of $Z(N, k)$ with respect to k . In this case, we have a true knife-edge result. When $\alpha = 1/2$, any k between 1 and N maximizes utility for the regulator. For smaller α , the preferred number of licenses is one, and for larger α the preferred number of licenses is N . In the linear demand, constant marginal cost example, the derivatives are not exactly equal, but are very "close". To illustrate this, we have plotted the derivatives on the same coordinate system in Fig. 1.⁷ The point at which they cross indicates the preferred number of licenses when $\alpha = 1/2$. Near this point, the curves are virtually identical. Hence, the preferred number of licenses changes very quickly when α changes.

Although we have concentrated on output selection, one can consider other types of market structures in our framework. An interesting case is Bertrand competition, where firms compete on price rather than output.⁸ In the linear demand, constant cost example, the Bertrand equilibrium for $k \geq 2$ firms implies

⁷ To obtain these derivatives, we have used a logarithmic approximation for the summation term in $Z(N, k)$. This approximation does not affect the qualitative features of the derivatives that generate the knife-edge result.

⁸ See Nti (2000) for a model in which Bertrand firms compete in a single winner rent-seeking contest.

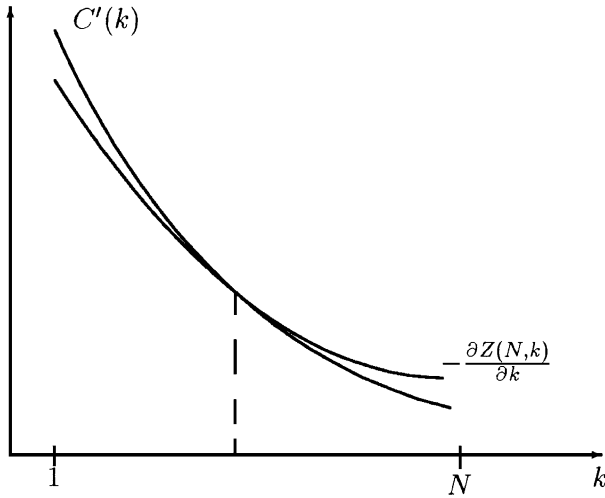


Fig. 1. Derivatives of $C(k)$ and $Z(N,k)$ in the linear demand, constant marginal cost example.

that $V(k) = 0$, because the equilibrium strategy is to set price equal to marginal cost. It follows that Bertrand firms will not devote any resources to rent-seeking when $k^* \geq 2$. This suggests that the regulator has strong benevolent motives if we observe that more than one license is issued in a market with Bertrand competitors.

4. Conclusion

We have presented a model where a regulator chooses the number of licenses and the firms that are granted the licenses. Our results suggest that total rent-seeking expenditures are maximized when the regulator issues only one license. This will occur if the regulator seeks only to maximize personal pecuniary interest, or if all winning firms are expected to collude on their output decisions. If, however, the regulator is also concerned with consumer welfare and firms are not expected to collude on output, more than one license may be offered. While it is obvious that competitive firms are beneficial to consumers, we also find that the firms themselves are likely to have higher profits after rent-seeking when they are not able to collude on output.

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Appendix A

Proof of Proposition 1. By the definition of $Z(N,k)$ and the assumption that $V(1) \geq V(k)k$ we have

$$Z(N,1) = V(1) \left[\frac{N-1}{N} \right] \geq V(k)k \left[\frac{N-1}{N} \right].$$

Since $\sum_{j=1}^{k-1} \frac{k-j}{N-j} > 0$ it follows that, for $k > 1$, we have

$$V(k)k \left[\frac{N-1}{N} \right] > V(k) \left[\frac{k(N-1)}{N} - \sum_{j=1}^{k-1} \frac{k-j}{N-j} \right] = Z(N,k).$$

Combining these two results yields $Z(N,1) > Z(N,k)$ for $k > 1$. □

Proof of Proposition 2. Since $V(1) = V(k)k$, the condition of Proposition 1 is satisfied and hence we have $Z(N,1) > Z(N,k)$. Since $C(k)$ is constant with respect to k , and the regulator's utility function is increasing with respect to both $C(k)$ and $Z(N,k)$, it follows that $k^* = 1$. □

Proof of Proposition 3. To prove the first part of the proposition, we must show that $\partial Z(N,k)/\partial k$ is negative. We can write $Z(N,k)$ more compactly as

$$Z(N,k) = V(k) \left[k - \sum_{j=0}^{k-1} \frac{k-j}{N-j} \right].$$

We use a difference relation to determine the sign of the partial derivative. We have

$$\begin{aligned} & Z(N,k+1) - Z(N,k) \\ &= V(k+1)(k+1) - V(k)k + [V(k) - V(k+1)] \sum_{j=0}^{k-1} \frac{k-j}{N-j} \\ &\quad - V(k+1) \sum_{j=0}^k \frac{1}{N-j} = [V(k) - V(k+1)] \left[\sum_{j=0}^{k-1} \frac{k-j}{N-j} - k \right] \\ &\quad + V(k+1) \left[1 - \sum_{j=0}^k \frac{1}{N-j} \right]. \end{aligned}$$

Since

$$\sum_{j=0}^{k-1} \frac{k-j}{N-j} < k \left(\frac{k}{N} \right) < k,$$

the second term in brackets in the above equation is negative. It follows that

$$Z(N, k + 1) - Z(N, k) < 0,$$

if and only if

$$\frac{V(k) - V(k + 1)}{V(k + 1)} > \frac{\sum_{j=0}^k \frac{1}{N-j} - 1}{\sum_{j=0}^{k-1} \frac{k-j}{N-j} - k}.$$

To prove the first part of the proposition, then, it is sufficient to show that

$$\frac{1}{k} > \frac{\sum_{j=0}^k \frac{1}{N-j} - 1}{\sum_{j=0}^{k-1} \frac{k-j}{N-j} - k}.$$

Re-writing this equation yields

$$\sum_{j=0}^{k-1} \frac{k-j}{N-j} < \sum_{j=0}^{k-1} \frac{k}{N-j} + \frac{k}{N-k},$$

which is clearly satisfied.

To prove the second part of the proposition, we must show that $\partial^2 Z(N, k) / \partial k \partial N$ is positive. It is easier in this case to work with an alternative, but equivalent, expression for $Z(N, k)$ found in Clark and Riis (1998). We have

$$Z(N, k) = \sum_{s=1}^k V(k) \left(1 - \sum_{j=0}^{s-1} \frac{1}{N-j} \right).$$

The appropriate difference relation is

$$\begin{aligned} & Z(N + 1, k + 1) - Z(N + 1, k) - [Z(N, k + 1) - Z(N, k)] \\ &= V(k) \frac{k}{N + 1} - V(k + 1) \frac{k + 1}{N + 1} + V(k + 1) \frac{1}{N - k} \\ &\quad + [V(k + 1) - V(k)] \sum_{j=0}^{k-1} \frac{1}{N - j} \\ &= [V(k) - V(k + 1)] \left[\frac{k}{N + 1} - \sum_{j=0}^{k-1} \frac{1}{N - j} \right] \\ &\quad + V(k + 1) \left[\frac{1}{N - k} - \frac{1}{N + 1} \right]. \end{aligned}$$

Since

$$\frac{k}{N} < \sum_{j=0}^{k-1} \frac{1}{N-j},$$

it follows that the second term in brackets in the above equation is negative. Hence,

$$Z(N + 1, k + 1) - Z(N + 1, k) - [Z(N, k + 1) - Z(N, k)] > 0,$$

if and only if

$$\frac{V(k) - V(k + 1)}{V(k + 1)} < \frac{\frac{1}{N-k} - \frac{1}{N+1}}{\sum_{j=0}^{k-1} \frac{1}{N-j} - \frac{k}{N+1}}.$$

□

Proof of Proposition 4. The firms' total expected profit can be written as

$$\Pi(N, k) = V(k) \left[\sum_{j=0}^{k-1} \frac{k-j}{N-j} \right].$$

The first difference of profit with respect to k is

$$\begin{aligned} \Pi(N, k) - \Pi(N, k + 1) &= V(k) \sum_{j=0}^{k-1} \frac{k-j}{N-j} - V(k + 1) \sum_{j=0}^k \frac{k+1-j}{N-j} \\ &= [V(k) - V(k + 1)] \left[\sum_{j=0}^{k-1} \frac{k-j}{N-j} \right] \\ &\quad - V(k + 1) \left[\sum_{j=0}^k \frac{1}{N-j} \right]. \end{aligned}$$

The firms' profit is strictly increasing in k if and only if

$$\frac{V(k) - V(k + 1)}{V(k + 1)} < \frac{\sum_{j=0}^k \frac{1}{N-j}}{\sum_{j=0}^{k-1} \frac{k-j}{N-j}}.$$

To prove the proposition, then, it is sufficient to show that

$$\frac{2}{k} < \frac{\sum_{j=0}^k \frac{1}{N-j}}{\sum_{j=0}^{k-1} \frac{k-j}{N-j}}.$$

This condition can be re-written as

$$2 \sum_{j=0}^{k-1} \frac{k-j}{N-j} < \sum_{j=0}^{k-1} \frac{k}{N-j} + \frac{k}{N-k},$$

and yet again as

$$\sum_{j=0}^{k-1} \frac{k-2j}{N-j} < \frac{k}{N-k}.$$

Below we use proof by induction to show that

$$\sum_{j=0}^{k-1} \frac{k-2j}{N-j} \leq \frac{k}{N}, \tag{8}$$

and hence the proposition is true.

Now consider the details of the inductive proof. Clearly Eq. (8) is satisfied for $k = 1$. Suppose that it is also satisfied for $k = i$. For $k = i + 1$ we have

$$\sum_{j=0}^i \frac{i+1-2j}{N-j} = \sum_{j=0}^{i-1} \frac{i-2j}{N-j} + \sum_{j=0}^i \frac{1}{N-j} - \frac{i}{N-i}.$$

By assumption, the first term on the right hand side of this equation is less than or equal to i/N . Thus, it is sufficient to show that the last two terms are less than $1/N$. It is easy to verify that

$$\frac{i+1}{N} < \sum_{j=0}^i \frac{1}{N-j} < \frac{i+1}{N-i}.$$

An even lower upper bound can be found by combining the upper and lower bound stated above. We have

$$\sum_{j=0}^i \frac{1}{N-j} < \frac{i+1}{N} + \frac{1}{2} \left(\frac{i+1}{N-i} - \frac{i+1}{N} \right) = \frac{1}{2} \left(\frac{i+1}{N} + \frac{i+1}{N-i} \right). \tag{9}$$

It follows that

$$\sum_{j=0}^i \frac{1}{N-j} - \frac{i}{N-i} < \frac{i+1}{2N} + \frac{1-i}{2(N-i)}.$$

It is straightforward to show that the right hand side of this equation is less than or equal to $1/N$. \square

Finally, we verify that $\gamma > 2/k$. This condition is equivalent to

$$\frac{k}{N-k} - \frac{k}{N+1} > 2 \left[\sum_{j=0}^k \frac{1}{N-j} \right] - \frac{2}{N-k} - \frac{2k}{N+1},$$

which is, in turn, equivalent to

$$\frac{k+2}{N-k} + \frac{k}{N+1} > 2 \left[\sum_{j=0}^k \frac{1}{N-j} \right]. \quad (10)$$

Utilizing the upper bound described by Eq. (9), we have

$$\sum_{j=0}^k \frac{1}{N-j} < \frac{k+1}{N} + \frac{1}{2} \left(\frac{k+1}{N-k} - \frac{k+1}{N} \right) = \frac{1}{2} \left(\frac{k+1}{N} + \frac{k+1}{N-k} \right).$$

Combining this result with Eq. (10) implies $\gamma > 2/k$ if

$$\frac{k+1}{N} + \frac{k+1}{N-k} < \frac{k+2}{N-k} + \frac{k}{N+1}.$$

Placing both sides over a common denominator and simplifying verifies that this equation is indeed satisfied.

References

- Appelbaum, E., Katz, E., 1987. Seeking rents by setting rents: the political economy of rent seeking. *The Economic Journal* 97, 685–699.
- Berry, S.K., 1993. Rent seeking with multiple winners. *Public Choice* 77, 437–443.
- Buchanan, J.M., 1989. *Explorations into Constitutional Economics*. Texas A&M Univ. Press, College Station.
- Clark, D.J., Riis, C., 1996. A multi-winner nested rent-seeking contest. *Public Choice* 87, 177–184.
- Clark, D.J., Riis, C., 1998. Influence and the discretionary allocation of several prizes. *European Journal of Political Economy* 14, 605–625.
- Dixit, A., Shapiro, C., 1986. Entry dynamics with mixed strategies. In: Thomas, L.G. (Ed.), *The Economics of Strategic Planning*. Lexington Books, Lexington, MA.
- Glazer, A., Hassin, R., 1988. Optimal contests. *Economic Inquiry* 26, 133–143.
- Konrad, K., Schlesinger, H., 1997. Risk aversion in rent-seeking and rent-augmenting games. *Economic Journal* 107, 1671–1683.
- Mueller, D.C., 1989. *Public Choice II*. Cambridge Univ. Press, Cambridge.

- Nitzan, S., 1994. Modelling rent-seeking contests. *European Journal of Political Economy* 10, 41–60.
- Nti, K., 1989. More potential entrants may lead to less competition. *Journal of Economics* 49, 47–70.
- Nti, K., 2000. Potential competition and coordination in a market entry game. *Journal of Economics* 71, 149–165.
- Rice, E.M., Ulen, T.S., 1981. Rent seeking and welfare loss. *Research in Law and Economics* 3, 53–65.
- Tollison, R.D., 1997. Rent seeking. In: Mueller, D.C. (Ed.), *Perspectives on Public Choice*. Cambridge Univ. Press, Cambridge, pp. 506–525.
- Tullock, G., 1980. Efficient rent-seeking. In: Buchanan, J.M., Tollison, R.D., Tullock, G. (Eds.), *Towards a Theory of the Rent-seeking Society*. Texas A&M Univ. Press, College Station, pp. 97–112.