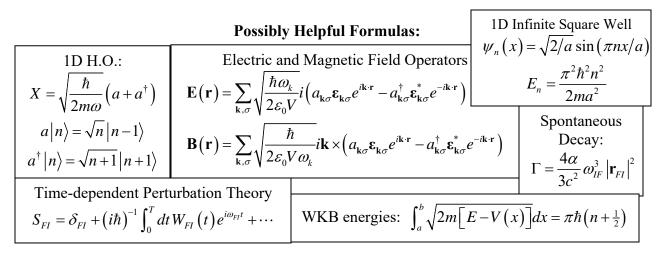
Physics 742 – Graduate Quantum Mechanics 2 Second Exam, Spring 2018

The points for each question are marked. Each question is worth 20 points. Some possibly useful formulas appear at the end of the test.

- 1. A particle of mass *m* is in one dimension in the potential $V(x) = \begin{cases} \frac{1}{2}m\omega_+^2 x^2 & \text{for } x > 0, \\ \frac{1}{2}m\omega_-^2 x^2 & \text{for } x < 0. \end{cases}$ Estimate the energy of the *n*'th eigenstate using the WKB approximation.
- 2. A particle in the ground state of a three-dimensional spherical infinite square well of radius *a* has wave function $\psi(\mathbf{r}) = \frac{1}{r\sqrt{2\pi a}} \sin\left(\frac{\pi r}{a}\right)$ in the allowed region r < a. The radius of this potential well is now increased to 2*a*. What is the probability that the particle remains in the ground state if the radius increases from *a* to 2*a* (a) adiabatically, or (b) suddenly?
- An electron of mass *m* at *t* = 0 is in the ground state |Ψ(*t* = 0)⟩ = |0,0,0⟩ of a three-dimensional harmonic oscillator with frequency ω. In an attempt to excite it to a higher energy state, a small perturbation W = γXYte^{-λt} is turned on starting at *t* = 0 and left on. To leading order, what states |n, p, q⟩ (other than the ground state) can be excited, and what is the probability of it ending in this/these states at *t* = ∞?
- 4. A system consists of a superposition of a zero photon state and a one photon states, so that $|\Psi\rangle = \frac{1}{\sqrt{3}} (|0\rangle i\sqrt{2}|1, \mathbf{q}, \tau\rangle)$ where $\mathbf{q} = q\hat{\mathbf{z}}$, and $\boldsymbol{\varepsilon}_{q\tau} = \hat{\mathbf{x}}$. What are the expectation values of the electric and magnetic field $\langle \mathbf{E}(\mathbf{r}) \rangle$ and $\langle \mathbf{B}(\mathbf{r}) \rangle$?
- 5. An electron is in the |3,1,1⟩ state of the 3D cubical infinite square well with allowed region x, y, z ∈ [0,a]. Show that via the dipole transition, it can only decay to one of the states |1,1,1⟩ or |2,1,1⟩, and calculate the corresponding rate.



Possibly Helpful Integrals: In integrals below, *m* and *n* are non-negative integers

$$\int_{0}^{\infty} x^{n} e^{-\alpha x} dx = n! \alpha^{-(n+1)}, \quad \int_{0}^{y} \sqrt{a - bx^{2}} dx = \frac{a}{2\sqrt{b}} \sin^{-1}\left(y\sqrt{b/a}\right) + \frac{y}{2}\sqrt{a - by^{2}}$$

$$\int_{0}^{a} \sin\left(\pi nx/a\right) \sin\left(\pi mx/a\right) dx = \int_{0}^{a} \sin\left[\pi \left(n + \frac{1}{2}\right)x/a\right] \sin\left[\pi \left(m + \frac{1}{2}\right)x/a\right] dx = \frac{1}{2}a\delta_{nm}$$

$$\int_{0}^{a} \sin\left(\pi nx/a\right) \sin\left[\pi \left(m + \frac{1}{2}\right)x/a\right] dx = 4(-1)^{m+n} an / \left[\pi \left(4m^{2} + 4m + 1 - 4n^{2}\right)\right],$$

$$\int_{0}^{a} x \sin\left(\frac{\pi nx}{a}\right) \sin\left(\frac{\pi mx}{a}\right) dx = \begin{cases} a^{2}/4 & \text{if } n = m, \\ 2a^{2}nm\left[\left(-1\right)^{n+m} - 1\right] / \left[\pi^{2}\left(n^{2} - m^{2}\right)^{2}\right] & \text{if } n \neq m. \end{cases}$$

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