Physics 741 – Graduate Quantum Mechanics 1

Homework Set X

Due Wednesday, November 12

1. Two non-identical spinless particles are in the asymmetric infinite square well, defined by

$$H = \frac{P_1^2 + P_2^2}{2m} + V(Q_1, Q_2) \quad \text{where} \quad V(Q_1, Q_2) = \begin{cases} 0 & \text{if } 0 < Q_1 < Q_2 < a \\ \infty & \text{otherwise} \end{cases}$$

   The ground state wave function is given by

$$\psi(x_1, x_2) = \begin{cases} N \sin(\pi x_1/a) \sin(\pi x_2/a)\left[\cos(\pi x_1/a) - \cos(\pi x_2/a)\right] & \text{for } 0 < x_1 < x_2 < a \\ 0 & \text{otherwise} \end{cases}$$

   (a) Check that this wave function is a solution of Schrödinger's equation, and determine the ground state energy. Also check that it is continuous.
   (b) Find the normalization constant *N*.
   (c) The position of the two particles is measured in this state. What is the probability that $x_1 < \frac{1}{2}a$, that $\frac{1}{2}a < x_2$, and that both are true?

2. A system of more than two particles is an eigenstate of every pair switching operator, that is,

$$P(i \leftrightarrow j)|\psi\rangle = \lambda_{ij}|\psi\rangle \quad \text{for every } i \neq j$$

   (a) For *i*, *j*, and *k* all different, simplify the product

$$P(i \leftrightarrow j) P(i \leftrightarrow k) P(i \leftrightarrow j)$$

   (b) Demonstrate that $\lambda_{ik} = \lambda_{jk}$ for any *i*, *j*, and *k* all different
   (c) Argue that for any pair λ_{ij} and λ_{kl}, $\lambda_{ij} = \lambda_{kl}$, whether there are any matches or not. Hence there is only one common eigenvalue for all pair switchings.