Physics 741 – Graduate Quantum Mechanics 1

Homework Set W

Due Monday, November 10

1. Suppose an electron lies in a region with electric and magnetic fields:

$$\mathbf{B} = B\hat{\mathbf{k}}$$

$$\mathbf{E} = \frac{m\omega_0^2}{e} x\hat{\mathbf{i}}$$

(a) Find the electric potential $U(x)$ such that $\mathbf{E} = -\nabla U(x)$ that could lead to this electric field.

(b) The magnetic field is independent of translations in all three dimensions. However, the electrostatic potential is independent of translations in only two of those dimensions. Find a vector potential **A** with $\mathbf{B} = \nabla \times \mathbf{A}$ which has translation symmetry in the *same* two directions.

(c) Write out the Hamiltonian for this system. Eliminate B in terms of the cyclotron frequency $\omega_B = eB/m$. What two translation operators commute with this Hamiltonian? What spin operator commutes with this Hamiltonian?

(d) Write your wave function in the form

$$\psi(\mathbf{r}) = X(x)Y(y)Z(z)|m_s\rangle$$

Based on some of the operators you worked out in part (c), deduce the form of two of the unknown functions.

(e) Replace the various operators by their eigenvalues in the Hamiltonian. The non-constant terms should be identifiable as a shifted harmonic oscillator.

(f) Make a simple coordinate replacement that shifts it back. If your formulas match mine up to now, they should look like:

$$Q_x = Q_x' - \frac{\hbar k_y \omega_B}{m\left(\omega_B^2 + \omega_0^2\right)}$$

(g) Find the energies of the Hamiltonian

(h) Check that they give sensible answers in the two limits when there is no electric field (pure Landau levels) or no magnetic fields (pure harmonic oscillator plus *y*- and *z*-motion)