

## Homework Set T

Due Wednesday, October 29

1. Suppose that  $\mathbf{U}$  and  $\mathbf{V}$  are vector operators with respect to some angular momentum operator  $\mathbf{J}$ ; that is,

$$[J_i, U_j] = \sum_k i\hbar \varepsilon_{ijk} U_k, \quad \text{and} \quad [J_i, V_j] = \sum_k i\hbar \varepsilon_{ijk} V_k$$

Show that

(a)  $S = \mathbf{U} \cdot \mathbf{V}$  is a scalar operator.

(b)  $\mathbf{W} = \mathbf{U} \times \mathbf{V}$  is a vector operator

2. Suppose that  $\mathbf{U}$  is a vector operator with respect to some angular momentum operator  $\mathbf{J}$ ; that is,

$$[J_i, U_j] = \sum_k i\hbar \varepsilon_{ijk} U_k.$$

Define the operators  $U_q$  as

$$U_0 = U_z, \quad U_{\pm 1} = \mp \sqrt{\frac{1}{2}} U_x - i \sqrt{\frac{1}{2}} U_y$$

Show explicitly that this is a spherical tensor with  $k = 1$ , so that

$$[J_z, U_q] = \hbar q U_q \quad \text{and} \quad [J_{\pm}, U_q] = \hbar \sqrt{2 - q^2} \mp q U_{q \pm 1}$$