

Homework Set S

Due Monday, October 27

1. Suppose we have two particles with spin $\frac{1}{2}$, and they are in the total spin-0 state.

$$|10\rangle = \frac{1}{\sqrt{2}}(|+-\rangle - |-+\rangle) = \frac{1}{\sqrt{2}}(|+_{-z}\rangle - |-_{+z}\rangle)$$

where we have introduced the subscripts z to make clear that the states are eigenstates of the two operators S_{1z} and S_{2z} . Let's define the spin operators $S_{i\theta}$ as

$$S_{i\theta} = \frac{1}{2}\hbar(\cos\theta\sigma_{iz} + \sin\theta\sigma_{ix})$$

where $i = 1, 2$ simply denotes which of the two spins we are measuring.

- (a) Write out explicitly and demonstrate that the state

$$\frac{1}{\sqrt{2}}(|+_{\theta}\rangle - |-_{\theta}\rangle) = |10\rangle$$

where these states are eigenstates of the two operators $S_{i\theta}$. You may find tensor notation helpful. You start by writing out

$$\begin{aligned} |+_{\theta}\rangle &= |+_{\theta}\rangle \otimes |-_{\theta}\rangle = \left[\cos\left(\frac{1}{2}\theta\right)|+_{z}\rangle + \sin\left(\frac{1}{2}\theta\right)|-_{z}\rangle \right] \otimes \left[-\sin\left(\frac{1}{2}\theta\right)|+_{z}\rangle + \cos\left(\frac{1}{2}\theta\right)|-_{z}\rangle \right] \\ &= -\cos\left(\frac{1}{2}\theta\right)\sin\left(\frac{1}{2}\theta\right)|+_{z}+_{z}\rangle + \dots \end{aligned}$$

- (b) Suppose you measure the spin of particle 1 at an angle θ , so you measure $S_{1\theta}$. What is the probability that you get each of the possible results? In each case, what is the wave function afterwards?
- (c) After completing the measurement in part (b), you perform a *second* measurement, at a different angle, on the other spin, $S_{2\theta'}$. Show in each case that the probability of the second measurement yielding the same result as the first is

$$P(m_1 = m_2) = \sin^2\left[\frac{1}{2}(\theta' - \theta)\right]$$

- (d) Spin one is going to be measured on one of the two axes **a** and **c** sketched at right. Spin two is going to be measured on one of the two axes **b** and **d** sketched at right. What is the probability, in all four combinations, that the spins match? I want the answer as a percent. This computation will play an important role in our discussion of hidden variables and Bell's inequality later.

