Physics 741 – Graduate Quantum Mechanics 1

Homework Set M

Due Wednesday, October 8

1. A particle of mass m and energy E in two dimensions is incident on a plane step function given by

$$V\left(Q_x, Q_y\right) = \begin{cases} 0 & \text{if } Q_x < 0, \\ V_0 & \text{if } Q_x > 0. \end{cases}$$

The incoming wave has wave function $\psi_{\text{in}}\left(x, y\right) = e^{i\left(k_x x + k_y y\right)}$ for $x < 0$.

(a) Write the Hamiltonian. Determine the energy E for the incident wave. Convince yourself that the Hamiltonian has a translation symmetry, and therefore that the transmitted and reflected wave will share something in common with the incident wave (they are all eigenstates of what operator?).

(b) Write the general form of the reflected and transmitted wave. Use Schrödinger's equation to solve for the values of the unknown parts of the momentum for each of these waves (assume $k_x^2 \hbar^2 / 2m > V_0$).

(c) Assume the wave function and its derivative are continuous across the boundary $x = 0$. Find the amplitudes for the transmitted and reflected waves, and find the probability R of the wave being reflected.

2. A particle in three dimensions has Hamiltonian

$$H = \frac{1}{2M}\left(P_x^2 + P_y^2 + P_z^2\right) + \tfrac{1}{4} A\left(Q_x^2 + Q_y^2\right)^2$$

(a) Show that this Hamiltonian has *two* continuous symmetries, and that they commute. Call the corresponding eigenvalues m and k. Are there any restrictions on k and m?

(b) What would be an appropriate set of coordinates for writing the eigenstates of this Hamiltonian? Write the eigenstates as a product of three functions (which I call Z, R, and Φ), and give me the explicit form of two of these functions.