

Homework Set J

Due Friday, September 26

1. A particle of mass m lies in a one-dimensional infinite square well in the region $[0, a]$. At $t = 0$, the wave function in the allowed region is given by

$$\langle x | \Psi(t=0) \rangle = \sqrt{30/a^5} (ax - x^2)$$

- (a) Write the wave function in the form

$$|\Psi(t=0)\rangle = \sum_n c_n |\psi_n\rangle$$

where $|\psi_n\rangle$ are the energy eigenstates. Write the wave function $|\Psi(t)\rangle$ at all times.

- (b) Check that the wave function is properly normalized, both in the original coordinate basis, and in the new basis, either analytically or numerically.
 (c) Find the expectation value of the Hamiltonian H for this wave function at $t = 0$, both in the original coordinate basis and the eigenstate basis. Check that they are the same, either analytically or numerically.

2. The Lennard-Jones 6-12 potential is commonly used as a model to describe the potential of an atom in the neighborhood of another atom. Classically, the energy is given by

$$E = \frac{1}{2} m \dot{x}^2 + 4\varepsilon \left[\left(\frac{\sigma}{x} \right)^{12} - \left(\frac{\sigma}{x} \right)^6 \right]$$

- (a) Find the minimum x_{\min} of this potential, and expand the potential to quadratic order in $(x - x_{\min})$
 (b) Determine the classical frequency ω , and calculate the quantum mechanical minimum energy, as a function of the various parameters.