Physics 741 – Graduate Quantum Mechanics 1

Homework Set I

Due Wednesday, September 24

1. The commutation relations of the angular momentum operator L_z with the momentum operator $\mathbf{P}$ were worked out explicitly in homework F, problem 2. These solutions are already posted online.
   (a) Using these commutation relations, derive two non-trivial uncertainty relationships.
   (b) Show that if you are in an eigenstate of any observable, the uncertainty in that observable is zero.
   (c) Show that if you are in an eigenstate of L_z, then you must have $\langle P_x \rangle = \langle P_y \rangle = 0$.

2. We will eventually discover that particles have spin, which is described by three operators $\mathbf{S} = (S_x, S_y, S_z)$ with commutation relations

$$[S_x, S_y] = i\hbar S_z, \quad [S_y, S_z] = i\hbar S_x, \quad [S_z, S_x] = i\hbar S_y$$

   A particle in a magnetic field of magnitude B pointing in the z-direction will have Hamiltonian

$$H = -\mu B S_z$$

   where μ and B are constants.
   (a) Derive formulas for the time derivative of all three components of $\langle \mathbf{S} \rangle$
   (b) At time $t = 0$, the expectation values of $\langle \mathbf{S} \rangle$ are given by

$$\langle S_x \rangle_{t=0} = a, \quad \langle S_y \rangle_{t=0} = 0, \quad \langle S_z \rangle_{t=0} = b$$

   Determine the expectation value $\langle \mathbf{S} \rangle_t$ at later times.