

Homework Set F

Due Monday, September 15

1. Prove the following identities about the operators A , B , and C :

(a) Commutators of products:

$$[A, BC] = B[A, C] + [A, B]C \quad \text{and} \quad [AB, C] = A[B, C] + [A, C]B$$

(b) The Jacobi identities:

$$[A, [B, C]] + [B, [C, A]] + [C, [A, B]] = 0 \quad \text{and} \\ [[A, B], C] + [[B, C], A] + [[C, A], B] = 0$$

(c) Show that if A and B are Hermitian, then $i[A, B]$ is also Hermitian

2. Define the angular momentum operators $\mathbf{L} = \mathbf{Q} \times \mathbf{P}$, or in components

$$L_x = Q_y P_z - Q_z P_y, \quad L_y = Q_z P_x - Q_x P_z, \quad L_z = Q_x P_y - Q_y P_x$$

(a) With the help of problem 1(a), work out the six commutators $[L_z, \mathbf{Q}]$ and $[L_z, \mathbf{P}]$.

(b) With the help of problems 1(a) and 2(a), work out the commutators $[L_z, L_x]$ and $[L_z, L_y]$.